Università degli Studi Roma Tre
Dipartimento di Matematica

I Prova di Esonero
FS2, Elettromagnetismo
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SOLUZIONI

1-a) $$V_B = \frac{1}{4\pi\varepsilon_0 l}\left(2Q_1 - \frac{Q_2}{\sqrt{2}}\right) = \frac{1}{4\cdot 3.142\cdot 8.854\cdot 10^{-12}\cdot 0.5}\left(2\cdot 45\cdot 10^{-10} - \frac{97\cdot 10^{-10}}{1.414}\right) = 38.5\ \text{V}$$

1-b) $$F_{BD} = \underset{Q'Q_2}{F_{BD}} - \sqrt{\underset{Q'Q_1}{F_{BC}^2} + \underset{Q'Q_1}{F_{BA}^2}} = \frac{Q'}{4\pi\varepsilon_0}\left[\frac{Q_2}{\left(\sqrt{2}\, l\right)^2} - \frac{\sqrt{2}\, Q_1}{l^2}\right] = \frac{Q'}{4\pi\varepsilon_0 l^2}\left(\frac{Q_2}{2} - \sqrt{2}\, Q_1\right) =$$

$$= \frac{15\cdot 10^{-10}}{4\cdot 3.142\cdot 8.854\cdot 10^{-12}\cdot 0.5^2}\left(-\frac{97\cdot 10^{-10}}{2} - 1.414\cdot 45\cdot 10^{-10}\right) = -6.04\cdot 10^{-7}\ \text{N}$$

$$V_B = \frac{2}{4\pi\varepsilon_0}\cdot\frac{Q_1}{l} - \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_2}{\sqrt{2}\, l} = 0$$

1-c) $$2Q_1 = \frac{Q_2}{\sqrt{2}}$$

$$\frac{Q_2}{Q_1} = 2\sqrt{2} = 2.83$$

$$U = \frac{1}{2} C\,\Delta V^2 = \frac{1}{2}\cdot\frac{\varepsilon_0 S}{d}\cdot\Delta V^2$$

2-a) $$U' = \frac{1}{2} C'\,\Delta V^2 = \frac{1}{2}\cdot\frac{\varepsilon_0\varepsilon_r S}{d}\cdot\Delta V^2$$

$$\varepsilon_r = \frac{U'}{U} = \frac{14.4\cdot 10^{-4}}{4.5\cdot 10^{-4}} = 3.2$$

$$C = \frac{\varepsilon_0 S}{d} \qquad C'' = \frac{\varepsilon_0\varepsilon_r S}{d+x}$$ e poichè $C = C''$ ciò implica

2-b) $$\frac{\varepsilon_0 S}{d} = \frac{\varepsilon_0\varepsilon_r S}{d+x}$$

$$d = \frac{d+x}{\varepsilon_r}$$

$$x = d\cdot(\varepsilon_r - 1) = 0.04\cdot 2.2 = 0.088\ \text{m}$$

$$\tau = S\cdot(d+x) = 2.5\cdot 10^{-4}\cdot 1.28\cdot 10^{-1} = 3.2\cdot 10^{-5}\ \text{m}^3$$

2-c) $$U = \int_\tau u\, d\tau = u\cdot\tau$$

$$u = \frac{U}{\tau} = \frac{4.5\cdot 10^{-4}}{3.2\cdot 10^{-5}} = 14.06\ \text{J}/\text{m}^3$$

3-a)

$$\mathrm{tg}\,\theta_2 = \frac{\varepsilon_2}{\varepsilon_1}\,\mathrm{tg}\,\theta_1 = \frac{3.5}{2.4}\cdot 0.424 = 0.618$$

$$\theta_2 = 31°\,43'$$

3-b)

$$|\mathbf{E_1}|\sin\theta_1 = |\mathbf{E_2}|\sin\theta_2$$

$$|\mathbf{E_2}| = |\mathbf{E_1}|\,\frac{\sin\theta_1}{\sin\theta_2} = 100\cdot\frac{0.391}{0.526} = 74.3\ \mathrm{V/m}$$

3-c)

Dalle relazioni

$$|\mathbf{D_1}| = \varepsilon_0\ \varepsilon_1\ |\mathbf{E_1}| \qquad\qquad |\mathbf{D_2}| = \varepsilon_0\ \varepsilon_2\ |\mathbf{E_2}| = \varepsilon_0\ \varepsilon_2\ |\mathbf{E_1}|\frac{\sin\theta_1}{\sin\theta_2}$$

si ha :

$$\frac{\mathrm{D}_2 - \mathrm{D}_1}{\mathrm{D}_1} = \frac{\varepsilon_0\ \mathrm{E}_1\left(\varepsilon_2\ \dfrac{\sin\theta_1}{\sin\theta_2} - \varepsilon_1\right)}{\varepsilon_0\ \varepsilon_1\ \mathrm{E}_1} = \frac{\varepsilon_2\ \dfrac{\sin\theta_1}{\sin\theta_2} - \varepsilon_1}{\varepsilon_1} = \frac{\varepsilon_2}{\varepsilon_1}\cdot\frac{\sin\theta_1}{\sin\theta_2} - 1 = \frac{3.5\cdot 0.391}{2.4\cdot 0.526} - 1 = 0.084$$

La variazione percentuale sarà 8.4%