

Am1c – Soluzioni Tutorato I

Ripasso di Am1

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Esercizio 1

$$(1) \lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = \lim_{y \rightarrow \infty} y \ln\left(1 + \frac{1}{y}\right) = \lim_{y \rightarrow \infty} \ln\left(1 + \frac{1}{y}\right)^y = 1$$

$$(2) \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \lim_{y \rightarrow 1} \frac{y-1}{\ln y} = \lim_{z \rightarrow 0} \frac{z}{\ln(z+1)} = 1$$

$$(3) \lim_{x \rightarrow \infty} \frac{x^6 - 6x^5 + 4x + 3}{x^3 - 4} = \lim_{x \rightarrow \infty} \frac{x^6 \left(1 - \frac{6}{x} + \frac{4}{x^5} + \frac{3}{x^6}\right)}{x^3 \left(1 - \frac{4}{x^3}\right)} = \lim_{x \rightarrow \infty} \frac{x^3 \left(1 - \frac{6}{x} + \frac{4}{x^5} + \frac{3}{x^6}\right)}{\left(1 - \frac{4}{x^3}\right)} = \infty$$

$$(4) \lim_{x \rightarrow \infty} \frac{x^5 - 7x^4 + 12x^3 + 4}{x^8 + 14x^5 - 80} = \lim_{x \rightarrow \infty} \frac{x^5 \left(1 - \frac{7}{x} + \frac{12}{x^2} + \frac{4}{x^5}\right)}{x^8 \left(1 + \frac{14}{x^3} - \frac{80}{x^8}\right)} = \lim_{x \rightarrow \infty} \frac{\left(1 - \frac{7}{x} + \frac{12}{x^2} + \frac{4}{x^5}\right)}{x^3 \left(1 + \frac{14}{x^3} - \frac{80}{x^8}\right)} = 0$$

$$(5) \lim_{x \rightarrow \infty} \frac{12x^6 - 14x^3 + 24}{(6x^3 + 2)^2} = \lim_{x \rightarrow \infty} \frac{12x^6 - 14x^3 + 24}{36x^6 + 24x^3 + 4} = \lim_{x \rightarrow \infty} \frac{x^6 \left(12 - \frac{14}{x^3} + \frac{24}{x^6}\right)}{x^6 \left(36 + \frac{24}{x^3} + \frac{4}{x^6}\right)} = \lim_{x \rightarrow \infty} \frac{\left(12 - \frac{14}{x^3} + \frac{24}{x^6}\right)}{\left(36 + \frac{24}{x^3} + \frac{4}{x^6}\right)} = \frac{1}{3}$$

$$(6) \lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{x^2} = \lim_{x \rightarrow 0} \frac{e^{-x} (e^{2x} - 2e^x + 1)}{x^2} = \lim_{x \rightarrow 0} \frac{e^{-x} (e^x - 1)^2}{x^2} = 1$$

$$(7) \lim_{x \rightarrow 0} \frac{\arcsin x}{x} = \lim_{y \rightarrow 0} \frac{y}{\sin y} = 1$$

$$(8) \lim_{x \rightarrow \pi} \frac{\sin x}{x - \pi} = \lim_{x \rightarrow \pi} \frac{\sin(x - \pi)}{x - \pi} = \lim_{y \rightarrow 0} \frac{\sin y}{y} = 1$$

$$(9) \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \frac{1 + \cos x}{1 + \cos x} \right) = \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x^2 (1 + \cos x)} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} \frac{1}{1 + \cos x} = \frac{1}{2}$$

$$(10) \lim_{x \rightarrow 0} \frac{\arctan x}{x} = \lim_{y \rightarrow 0} \frac{y}{\tan y} = \lim_{y \rightarrow 0} \left(\frac{y}{\sin y} \cos y \right) = 1$$

$$(11) \lim_{x \rightarrow 4} \frac{x^4 - 4x^3 - 3x^2 + 14x - 8}{x^6 - 4x^5 + x^4 - 4x^3} = \lim_{x \rightarrow 4} \frac{(x^3 - 3x + 2)(x - 4)}{(x^5 + x^3)(x - 4)} = \frac{27}{544}$$

$$(12) \lim_{x \rightarrow e} \frac{x - e}{\sqrt{x} - \sqrt{e}} = \lim_{x \rightarrow e} \left(\frac{x - e}{\sqrt{x} - \sqrt{e}} \frac{\sqrt{x} + \sqrt{e}}{\sqrt{x} + \sqrt{e}} \right) = \lim_{x \rightarrow e} \sqrt{x} + \sqrt{e} = 2\sqrt{e}$$

$$(13) \lim_{x \rightarrow 0^+} x^x = \lim_{x \rightarrow 0^+} e^{x \ln x} = e^0 = 1$$

$$(14) \lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos x} = \lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x} \frac{\sin x}{x} = 2$$

$$(15) \lim_{x \rightarrow 0} \frac{7^{11x} - 1}{x} = \lim_{x \rightarrow 0} \frac{e^{x \ln 7^{11}} - 1}{x} = \lim_{x \rightarrow 0} \frac{e^{x \ln 7^{11}} - 1}{x \ln 7^{11}} \ln 7^{11} = \ln 7^{11}$$

$$(16) \lim_{x \rightarrow 0} \frac{A^x - B^x}{x} = \lim_{x \rightarrow 0} \frac{B^x \left[\left(\frac{A}{B} \right)^x - 1 \right]}{x} = \lim_{x \rightarrow 0} \frac{B^x \left[e^{x \ln \frac{A}{B}} - 1 \right]}{x} = \lim_{x \rightarrow 0} \frac{B^x \left[e^{x \ln \frac{A}{B}} - 1 \right]}{x \ln \frac{A}{B}} \ln \frac{A}{B} = \ln \frac{A}{B}$$

$$(17) \lim_{x \rightarrow 0} \frac{\sqrt{3+x} - \sqrt{3-x}}{\sqrt{5+x} - \sqrt{5-x}} = \lim_{x \rightarrow 0} \frac{\sqrt{3+x} - \sqrt{3-x}}{\sqrt{5+x} - \sqrt{5-x}} \frac{\sqrt{3+x} + \sqrt{3-x}}{\sqrt{3+x} + \sqrt{3-x}} \frac{\sqrt{5+x} + \sqrt{5-x}}{\sqrt{5+x} + \sqrt{5-x}} =$$

$$= \lim_{x \rightarrow 0} \frac{\sqrt{5+x} + \sqrt{5-x}}{\sqrt{3+x} + \sqrt{3-x}} = \sqrt{\frac{5}{3}}$$

$$(18) \lim_{x \rightarrow 0} \frac{\ln(\tan^4 x + 1)}{e^{2 \sin^4 x} - 1} = \lim_{x \rightarrow 0} \frac{\ln(\tan^4 x + 1)}{\tan^4 x} \frac{2 \sin^4 x}{e^{2 \sin^4 x} - 1} \frac{1}{2 \cos^4 x} = \frac{1}{2}$$

$$(19) \lim_{x \rightarrow 0} \frac{\ln \sin^2 x^2}{\ln x^4} = \lim_{x \rightarrow 0} \frac{\ln \sin^2 x^2 - \ln x^4 + \ln x^4}{\ln x^4} = \lim_{x \rightarrow 0} \frac{\ln \frac{\sin^2 x^2}{x^4} + \ln x^4}{\ln x^4} = 1$$

$$(20) \lim_{x \rightarrow 0} (\sin x^2)^{\frac{1}{\log_{17} x^2}} = \lim_{x \rightarrow 0} 17^{\frac{\log_{17} \sin x^2}{\log_{17} x^2}} = 17$$

(21) Dalle formule di addizione della tangente $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$ si ottiene

$$\lim_{x \rightarrow 0} x \tan \left(xa + \arctan \frac{b}{x} \right) = \lim_{x \rightarrow 0} x \frac{\tan ax + \frac{b}{x}}{1 - \frac{b}{x} \tan ax} = \lim_{x \rightarrow 0} \frac{x \tan ax + b}{1 - ab \frac{\tan ax}{ax}} = \begin{cases} \frac{b}{1-ab} & ab \neq 1 \\ \infty & ab = 1 \end{cases}$$

$$(22) \lim_{x \rightarrow 4} \frac{\sin(x^3 - 4x^2 + x - 4)}{\sin(x^2 - 6x + 8)} = \lim_{x \rightarrow 4} \frac{\sin((x^2 + 1)(x - 4))}{(x^2 + 1)(x - 4)} \frac{(x - 2)(x - 4)}{\sin((x - 2)(x - 4))} \frac{(x^2 + 1)}{(x - 2)} = \frac{17}{2}$$

$$(23) \lim_{x \rightarrow 0} \frac{\ln(1 - x + x^2)}{x} = \lim_{x \rightarrow 0} \frac{\ln(1 + x(x - 1))}{x(x - 1)} (x - 1) = -1$$

$$(24) \lim_{x \rightarrow -\infty} (1 + e^x)^x = \lim_{x \rightarrow -\infty} \left[\left(1 + \frac{1}{e^{-x}} \right)^{e^{-x}} \right]^{xe^x} = 1$$

$$(25) \lim_{x \rightarrow \infty} \left(\frac{x+2}{x+1} \right)^x = \lim_{x \rightarrow \infty} \left[\left(1 + \frac{1}{x+1} \right)^{x+1} \right]^{\frac{x}{x+1}} = e$$

$$(26) \lim_{x \rightarrow \infty} \sin x \left[\ln(\sqrt{x+1}) - \ln(\sqrt{x+1}) \right] = \lim_{x \rightarrow \infty} \sin x \cdot \ln \left(\frac{\sqrt{x+1}}{\sqrt{x+1}} \right) = \lim_{x \rightarrow \infty} \sin x \cdot \ln \left(\frac{1 + \frac{1}{\sqrt{x}}}{\sqrt{1 + \frac{1}{x}}} \right) = 0$$

$$(27) \lim_{x \rightarrow 0^+} x^{\ln x} = \lim_{x \rightarrow 0^+} e^{\ln^2 x} = +\infty$$

$$(28) \lim_{x \rightarrow \infty} 3^x - x^4 = \lim_{x \rightarrow \infty} 3^x \left(1 - \frac{x^4}{3^x} \right) = \infty$$

$$(29) \lim_{x \rightarrow \infty} \sqrt{x} \sin \frac{1}{x} = \lim_{y \rightarrow 0} \frac{\sin y}{\sqrt{y}} = \lim_{y \rightarrow 0} \frac{\sin y}{y} \sqrt{y} = 0$$

$$(30) \lim_{x \rightarrow 0} \frac{x \sin(e^x - 1)}{\ln(2 - \cos x)} = \lim_{x \rightarrow 0} \frac{\sin(e^x - 1)}{e^x - 1} \frac{e^x - 1}{x} \frac{1 - \cos x}{\ln(2 - \cos x)} \frac{x^2}{1 - \cos x} \frac{x \cdot x}{x^2} = 2$$

Esercizio 2 Usiamo in entrambi i casi il teorema ponte:

(1) Basta notare che, posto $f(x) = \frac{1}{\sin \frac{1}{x}}$, $f(x_n) \xrightarrow{n \rightarrow +\infty} \infty$ se $x_n = \frac{1}{n\pi}$ mentre $f(x_n) \xrightarrow{n \rightarrow +\infty} 1$ se

$$x_n = \frac{1}{\frac{\pi}{2} + 2n\pi}.$$

(2) Posto $f(x) = \frac{|5-x|}{5-x}$ si ha $f(x_n) \xrightarrow{n \rightarrow +\infty} -1$ se $x_n = \frac{1}{n} + 5$ mentre $f(x_n) \xrightarrow{n \rightarrow +\infty} 1$ se

$x_n = -\frac{1}{n} + 5$. Infine si ha che $\lim_{x \rightarrow 5^+} \frac{|5-x|}{5-x} = -1$ e $\lim_{x \rightarrow 5^-} \frac{|5-x|}{5-x} = 1$.