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More numbers

When 1, 2, 3... is not enough

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Arguments over what counts as a number

Illustration by Jon Berkley

EVERY now and then mathematics has been convulsed by a row, not over where numbers come from—but over what should be allowed to count as one. Two millennia ago, inspired by such discoveries as the relationship between musical pitch and the lengths of vibrating strings (double the length of the string and the note falls by an octave), the followers of Pythagoras decided that all of Nature must be expressible as ratios of whole numbers. Their discovery that one very simple geometric ratio—that of the length of a square's diagonal to the length of its side—could not was, according to legend, so shocking that it was kept a secret on pain of death.

Such "irrational" numbers were bad enough, but what to make of negative ones? Although they had been widely employed since at least the mid-1500s, in particular to represent debts, many mathematicians refused to use them, claiming that quantities less than zero were an absurdity. A number "submits to be taken away from another number greater than itself but to attempt to take it away from a number less than itself is ridiculous," wrote William Frend, a Cambridge mathematician, in 1796.

And what, too, to make of the square roots of negative numbers? Since both negative and positive numbers, when multiplied by themselves, give positive answers, such numbers were labelled "imaginary", and regarded by many as meaningless. "The symbol -1 is beyond the power of arithmetical computation," wrote Robert Woodhouse, another Cambridge mathematician, in 1801. It took the brilliant idea, of Carl Friedrich Gauss and others, of regarding imaginary numbers as perpendicular to "real" ones before the latest variety of number could be accepted into the swelling menagerie.

By the end of the 19th century irrational, negative and imaginary numbers were widely accepted—not least because they were so very useful; turning one's back on such delights meant voluntarily abstaining from doing some very interesting mathematics. But still the numerical controversies raged. Georg Cantor, a German mathematician, had developed a "transfinite arithmetic" to calculate with the infinitely many infinities he had discovered, each infinitely larger than the previous one. Leopold Kronecker, a prominent German mathematician (and one of Cantor's teachers) described his student as a "scientific charlatan", a "renegade" and a "corrupter of youth"; his work was a "disease" from which mathematics would surely be cured some day, thought French mathematician Henri Poincaré.

Between 1910 and 1913 Bertrand Russell and Alfred North Whitehead published their three-volume "Principia Mathematica", in which, among other things, they sought to solve certain paradoxes that arose from Cantor's work. Their main aim, though, was to provide a firm foundation for all of mathematics—a hopeless quest, it turned out, when Kurt Gödel published his "incompleteness theorem" in 1931.

Russell and Whitehead suggested no new numbers or arithmetical rules, but they did try to show how the simplest numbers—integers—could be built using the principles of logic. But the methods they proposed for even the simplest sums were desperately cumbersome. And for the proof that 1+1=2, readers had to wait until volume II, page 83.