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From Fish to Infinity

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Steven Strogatz on math, from basic to baffling.

Tags:

[mathematics](#), [numbers](#)

I have a friend who gets a tremendous kick out of science, even though he's an artist. Whenever we get together all he wants to do is chat about the latest thing in evolution or quantum mechanics. But when it comes to math, he feels at sea, and it saddens him. The strange symbols keep him out. He says he doesn't even know how to pronounce them.

In fact, his alienation runs a lot deeper. He's not sure what mathematicians do all day, or what they mean when they say a proof is elegant. Sometimes we joke that I just should sit him down and teach him everything, starting with $1 + 1 = 2$ and going as far as we can.

Crazy as it sounds, over the next several weeks I'm going to try to do something close to that. I'll be writing about the elements of mathematics, from pre-school to grad school, for anyone out there who'd like to have a second chance at the subject — but this time from an adult perspective. It's not intended to be remedial. The goal is to give you a better feeling for what math is all about and why it's so enthralling to those who get it.

So, let's begin with pre-school.

The best introduction to numbers I've ever seen — the clearest and funniest explanation of what they are and why we need them — appears in a "Sesame Street" video called "123 Count With Me." Humphrey, an amiable but dim-witted fellow with pink fur and a green nose, is working the lunch shift at The Furry Arms hotel, when he takes a call from a room full of penguins. Humphrey listens carefully and then calls out their order to the kitchen: "Fish, fish, fish, fish, fish, fish." This prompts Ernie to enlighten him about the virtues of the number six.

Children learn from this that numbers are wonderful shortcuts. Instead of saying the word "fish" exactly as many times as there are penguins, Humphrey could use the more powerful concept of "six."

As adults, however, we might notice a potential downside to numbers. Sure, they are great time savers, but at a serious cost in abstraction. Six is more ethereal than six fish, precisely because it's more general. It applies to six of anything: six plates, six penguins, six utterances of the word "fish." It's the ineffable thing they all have in common.

Viewed in this light, numbers start to seem a bit mysterious. They apparently exist in some sort of Platonic realm, a level above reality. In that respect they are more like other lofty concepts (e.g., truth and justice), and less like the ordinary objects of daily life. Upon further reflection, their philosophical status becomes even murkier. Where exactly do numbers come from? Did humanity invent them? Or discover them?

A further subtlety is that numbers (and all mathematical ideas, for that matter) have lives of their own. We can't control them. Even though they exist in our minds, once we decide what we mean by them we have no say in how they behave. They obey certain laws and have certain properties, personalities, and ways of combining with one another, and there's nothing we can do about it except watch and try to understand. In that sense they are eerily reminiscent of atoms and stars, the things of this world, which are likewise subject to laws beyond our control ... except that those things exist outside our heads.

This dual aspect of numbers — as part- heaven, and part- earth — is perhaps the most paradoxical thing about them, and the feature that makes them so useful. It is what the physicist Eugene Wigner had in mind when he wrote of "the unreasonable effectiveness of mathematics in the natural sciences."

In case it's not clear what I mean about the lives of numbers and their uncontrollable behavior, let's go back to the The Furry Arms. Suppose that Humphrey suddenly gets a call on another line, from a room occupied by as many penguins as before, also clamoring for fish. After taking both calls, what should Humphrey yell out to the kitchen? If he hasn't learned anything, he could shout "fish" once for each penguin. Or, using his numbers, he could tell the cook he needs six orders of fish for the first room and six more for the second room. But what he really needs is a new concept: addition. Once he's mastered it, he'll proudly say he needs six plus six (or, if he's a show-off, 12) fish.

The creative process here is the same as the one that gave us numbers in the first place. Just as numbers are a shortcut for counting by ones, addition is a shortcut for counting by any amount. This is how mathematics grows. The right abstraction leads to new insight, and new power.

Before long, even Humphrey might realize he can keep counting forever.

Yet despite this infinite vista, there are always constraints on our creativity. We can decide what we mean by things like 6 and +, but once we do, the results of equations like $6 + 6$ are beyond our control. In mathematics, we'll see in the coming weeks, our freedom lies in the questions we ask — and in how we pursue them — but not in the answers awaiting us.

Notes:

For the Sesame Street video, see “Sesame Street – 123 Count With Me (1997).” It is available for purchase online in either VHS or DVD format.

For the famous essay on the unreasonable effectiveness of mathematics, see:

E. Wigner, “The unreasonable effectiveness of mathematics in the natural sciences,” Communications in Pure and Applied Mathematics, vol. 13, No. 1 (February 1960), pp. 1-14. A pdf version is [here](#).

For a passionate presentation of the ideas that numbers have lives of their own and that mathematics can be viewed as a form of art, see: P. Lockhart, “A Mathematician’s Lament: How School Cheats Us Out of Our Most Fascinating and Imaginative Art Form” (Bellevue Literary Press, 2009).

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