PER LUNEDÌ 14 DICEMBRE : calcolare:

① $\int \cos(3x)\cos(2x)\,dx$

② $\int \lg(1+x^3)^{x^2}\,dx$

③ $\int x^3 \,\mathrm{sen}(x^2)$

④ $\int \frac{x^2-7x+12}{(x-2)^3}\,dx$

⑤ $\int \frac{\sqrt{x}}{2+\sqrt{x}}\,dx$

⑥ $\int \frac{e^x}{3e^{2x}+e^x+2}\,dx$

Soluzioni ES. 14 DIC. 2020

① $\int \cos(3x)\cos(2x)\,dx =$

$2\cos(ax)\cos(bx) = \cos[(a+b)x] + \cos[(a-b)x]$

$= \frac{1}{2}\int \cos(5x)\,dx + \frac{1}{2}\int \cos x\,dx =$

$= \frac{1}{10}\,\mathrm{sen}(5x) + \frac{1}{2}\,\mathrm{sen}x + c$

② $\int \lg(1+x^3)^{x^2}\,dx =$

$= \int \underset{g'}{x^2}\,\underbrace{\lg(1+x^3)}_{f}\,dx$ $\quad g = x^3/3$ $\quad f' = \frac{3x^2}{1+x^3}$

$= \lg(1+x^3)\frac{x^3}{3} - \int \frac{3x^2}{1+x^3}\frac{x^3}{3}\,dx =$

$= \lg(1+x^3)\frac{x^3}{3} - \int \frac{x^5}{1+x^3}\,dx$

$x^5 = x^2(x^3+1) - x^2$

$\int \frac{x^5}{1+x^3}\,dx = \int x^2\,dx - \int \frac{x^2}{1+x^3}\,dx = \frac{x^3}{3} - \frac{1}{3}\lg(1+x^3)$

$= \lg(1+x^3)\frac{x^3}{3} - \frac{x^3}{3} + \frac{1}{3}\lg(1+x^3) + C =$

$= \frac{1}{3}\left[(x^3+1)\lg(1+x^3) - x^3\right] + C$

③ $\int x^3 \operatorname{sen}(x^2)\, dx =$

$= \frac{1}{2}\int \underset{f}{x^2}\ \underbrace{2x\operatorname{sen}(x^2)}_{g'}\, dx =$

$f' = 2x$

$g = -\cos(x^2)$

$= \frac{1}{2}\left[-x^2\cos(x^2)\right] - \frac{1}{2}\int 2x(-\cos(x^2))\, dx =$

$= -\frac{1}{2}x^2\cos(x^2) + \frac{1}{2}\int D[\operatorname{sen}(x^2)]\, dx =$

$= -\frac{1}{2}x^2\cos(x^2) + \frac{1}{2}\operatorname{sen}(x^2) + c$

④ $\int \frac{x^2-7x+12}{(x-2)^3}\, dx$

$\frac{x^2-7x+12}{(x-2)^3} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3}$

$= \frac{A(x-2)^2 + B(x-2) + C}{(x-2)^3}$

$= \frac{Ax^2 + (-4A+B)x + (4A-2B+C)}{(x-2)^3}$

$\begin{cases} A = 1 \\ -4A+B = -7 \\ 4A-2B+C = 12 \end{cases}$ $\begin{cases} A = 1 \\ B = -7+4A = -7+4 = -3 \\ C = 12-4A+2B = 12-4-6 = 2 \end{cases}$

i) $\int \frac{A}{x-2}\, dx = \int \frac{1}{x-2}\, dx = \lg|x-2| + c$

ii) $\int \frac{B}{(x-2)^2}\, dx = -3\int (x-2)^{-2}\, dx = \frac{3}{x-2} + c$

iii) $\int \frac{C}{(x-2)^3}\, dx = \int 2(x-2)^{-3}\, dx = -\frac{1}{(x-2)^2} + c$

$\int \frac{x^2-7x+12}{(x-2)^3}\, dx = \lg|x-2| + \frac{3}{x-2} - \frac{1}{(x-2)^2} + C$

⑤ $\int \frac{\sqrt{x}}{2+\sqrt{x}}\, dx$

$t = \sqrt{x}$; $t^2 = x$

$dt = \frac{1}{2}\frac{1}{\sqrt{x}}\, dx$; $2t\, dt = dx$

$dt = \frac{1}{2t}\, dx$

$= \int \frac{t}{2+t}\, 2t\, dt = \int \frac{2t^2}{2+t}\, dt =$

$2t^2 = (t+2)(2t-4) + 8$

$= \int (2t-4)\, dt + \int \frac{8}{2+t}\, dt =$

$= t^2 - 4t + 8\lg|2+t| + c =$

$= x - 4\sqrt{x} + 8\lg(2+\sqrt{x}) + C$

⑥ $$\int \frac{e^x}{3e^{2x}-e^x+2}\,dx \qquad t=e^x \quad dt=e^x\,dx=t\,dx \quad \to dx=\frac{dt}{t}$$

$$=\int \frac{t}{3t^2-t+2}\cdot\frac{dt}{t}=$$

$$=\frac{1}{3}\int \frac{1}{t^2-\frac{1}{3}t+\frac{2}{3}}\,dt \qquad t^2-\frac{1}{3}t+\frac{2}{3}=p(t) \quad \Delta=\frac{1}{9}-\frac{8}{3}=-\frac{23}{9}<0$$

$$t^2-\frac{1}{3}t+\frac{2}{3}=\left(t-\frac{1}{6}\right)^2-\frac{1}{36}+\frac{2}{3}=\left(t-\frac{1}{6}\right)^2+\frac{23}{36}=$$

$$=\frac{23}{36}\left[\frac{36}{23}\left(t-\frac{1}{6}\right)^2+1\right]=\frac{23}{36}\left[\left(\frac{6}{\sqrt{23}}\left(t-\frac{1}{6}\right)\right)^2+1\right]$$

$$=\frac{1}{3}\,\frac{36}{23}\int\frac{dt}{\left[\frac{6}{\sqrt{23}}\left(t-\frac{1}{6}\right)\right]^2+1}= \qquad y=\frac{6}{\sqrt{23}}\left(t-\frac{1}{6}\right) \quad dy=\frac{6}{\sqrt{23}}\,dt$$

$$=\frac{12}{23}\int\frac{\frac{\sqrt{23}}{6}\,dy}{y^2+1}=\frac{2}{\sqrt{23}}\,\mathrm{arctg}(y)+c=$$

$$=\frac{2}{\sqrt{23}}\,\mathrm{arctg}\left(\frac{6}{\sqrt{23}}\left(t-\frac{1}{6}\right)\right)+c=\frac{2}{\sqrt{3}}\,\mathrm{arctg}\left(\frac{6t-1}{\sqrt{3}}\right)+c=$$

$$=\frac{2}{\sqrt{23}}\,\mathrm{arctg}\left(\frac{6e^x-1}{\sqrt{23}}\right)+c$$

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① $$\int\frac{x^6+2x^2-1}{x^4-1}\,2x\,dx= \qquad t=x^2 \quad dt=2x\,dx$$

$$=\int\frac{t^3+2t-1}{t^2-1}\,dt=$$

$$\begin{array}{cccc|l} t^3 & 0 & 2t & -1 & t^2-1 \\ -t^3 & & t & & t \\ \hline / & & 3t & -1 & \end{array}$$

$$\to t^3+2t-1=t(t^2-1)+3t-1$$

$$=\int t\,dt+\int\frac{3t-1}{t^2-1}\,dt=\frac{1}{2}t^2+\int\frac{3t-1}{t^2-1}\,dt$$

$$\frac{3t-1}{(t-1)(t+1)}=\frac{A}{t-1}+\frac{B}{t+1}=\frac{(A+B)t+(A-B)}{(t-1)(t+1)}\to\begin{cases}A+B=3\\A-B=-1\end{cases}\to\begin{cases}A=1\\B=2\end{cases}$$

$$=\frac{1}{2}t^2+\int\frac{dt}{t-1}+2\int\frac{dt}{t+1}=\frac{1}{2}t^2+\lg|t-1|+2\lg|t+1|+c=$$

$$=\frac{1}{2}x^4+\lg|x^2-1|+2\lg(x^2+1)+c$$

② $$\int tg^3x\,dx \qquad t=tg\,x \qquad dt=\frac{1}{\cos^2x}\,dx$$

$$\frac{1}{\cos^2x}=\frac{\cos^2x+sen^2x}{\cos^2x}=1+\left(\frac{sen\,x}{\cos x}\right)^2=1+tg^2x=1+t^2$$

$$\to dt=(1+t^2)\,dx \qquad \to dx=\frac{dt}{1+t^2}$$

$$=\int t^3\,\frac{dt}{1+t^2}=\int t\,dt-\int\frac{t}{t^2+1}\,dt=$$

$$=\frac{1}{2}t^2-\frac{1}{2}\lg(t^2+1)+c=\frac{1}{2}tg^2x-\frac{1}{2}\lg(tg^2x+1)+c$$

$$t^3=t(t^2+1)-t$$

③ $\int \frac{dx}{\cos^6 x}$ $tg x = t$ $dx = \frac{dt}{1+t^2}$ $\frac{1}{\cos^2 x} = 1+t^2$

$= \int (1+t^2)^3 \frac{dt}{1+t^2} = \int (1+t^2)^2 dt = \int (1+2t^2+t^4)\, dt =$

$= t + \frac{2}{3}t^3 + \frac{1}{5}t^5 = tg x + \frac{2}{3} tg^3 x + \frac{1}{5} tg^5 x + c$

④ $\int_1^2 (e^x - 1)^{-1/2} dx$

$t = \sqrt{e^x - 1}$ $t^2 = e^x - 1 \leftarrow$ o derivate questa

$dt = \frac{1}{2} \frac{1}{\sqrt{e^x-1}} e^x dx = \frac{1}{2} \frac{1}{t} (t^2+1)\, dx \to dx = \frac{2t\,dt}{t^2+1}$ *

estremi: $x = 1$ $t = \sqrt{e-1}$

$x = 2$ $t = \sqrt{e^2-1}$

$= \int_{\sqrt{e-1}}^{\sqrt{e^2-1}} t^{-1} \frac{2t\,dt}{t^2+1} = \int_{\sqrt{e-1}}^{\sqrt{e^2-1}} \frac{2}{t^2+1}\, dt =$

$= \left[2 \, arctg\, t\right]_{\sqrt{e-1}}^{\sqrt{e^2-1}} = 2\left[arctg\sqrt{e^2-1} - arctg\sqrt{e-1}\right]$

⑤ $\int_1^4 \frac{lg x}{\sqrt{x}} dx$ $t = \sqrt{x}$ $t^2 = x$ $2t\,dt = dx$

estremi $x = 1$ $t = 1$

$x = 4$ $t = 2$

$= \int_1^2 \frac{lg t^2}{t} 2t\,dt = \int_1^2 2 \, lg t^2 \, dt = 4 \int_1^2 \underbrace{lg t}_{f} \, \underbrace{dt}_{g'=1} =$

$\int lg t\, dt = t\, lg t - t + c$

$= 4\left[t\, lg t - t\right]_1^2 = 4\left[2 lg 2 - 2 - \cancel{lg 1} + 1\right] =$

$= 4\left[2 lg 2 - 1\right] = 4 lg 4 - 4$

⑥ $\int_0^{\pi/2} \frac{sen x}{\cos^2 x - 6\cos x + 9} dx =$

$\cos x = t$

$-sen x\, dx = dt$

$x = 0$ $t = 1$

$x = \pi/2$ $t = 0$

$= \int_1^0 \frac{-dt}{t^2 - 6t + 9} = \int_0^1 \frac{dt}{t^2 - 6t + 9} =$ $\left[\int_a^b = -\int_b^a\right]$

$= \int_0^1 \frac{dt}{(t-3)^2} = \left[-(t-3)^{-1}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

⑦ $\int_2^3 lg(x^2 - x)\, dx =$

$\int \underbrace{lg(x^2-x)}_{f} \, \underbrace{dx}_{g'=1} =$ $f' = \frac{2x-1}{x^2-x}$ $g = x$

$= x\, lg(x^2-x) - \int \frac{2x-1}{x(x-1)} x\, dx =$ $\frac{2x-1}{x-1} = 2 + \frac{1}{x-1}$

$= x\, lg(x^2-x) - 2x - lg|x-1| + c \rightarrow$

$$\int_2^3 \lg(x^2-x)\,dx =$$

$$= \left[x\lg(x^2-x) - 2x - \lg|x-1| \right]_2^3 =$$

$$= 3\lg 6 - 6 - \lg 2 - 2\lg 2 + 4 + \cancel{\lg 1} =$$

$$= 3\lg 6 - 3\lg 2 - 2 = 3\lg 6/2 - 2 = 3\lg 3 - 2 = \lg 27 - 2$$

(8) $$\int \frac{1}{x^2(x^2+1)^2}\,dx$$

$\longrightarrow x^2+1=0 \; : \; x = \pm i$

$$\frac{1}{x^2(x^2+1)^2} = \frac{A_1}{x} + \frac{A_2}{x^2} + \frac{h_1x+k_1}{x^2+1} + \frac{h_2x+k_2}{(x^2+1)^2}$$

minimo comune multiplo ↑ è $x^2(x^2+1)^2$, a numeratore:

$$A_1 x(x^2+1)^2 + A_2(x^2+1)^2 + (h_1x+k_1)(x^2+1)x^2 + (h_2x+k_2)x^2 =$$

$$= A_1x^5 + 2A_1x^3 + A_1x + A_2x^4 + 2A_2x^2 + A_2 + h_1x^5 + h_1x^3 + k_1x^4 + k_1x^2 + h_2x^3 + k_2x^2 =$$

$$= (A_1+h_1)x^5 + (A_2+k_1)x^4 + (2A_1+h_1+h_2)x^3 + (2A_2+k_1+k_2)x^2 + (A_1)x + (A_2) = N(x)$$

$$\frac{1}{x^2(x^2+1)^2} = \frac{N(x)}{x^2(x^2+1)^2}$$

scriviamo il sistema:

$$\begin{cases} A_1+h_1=0 \\ A_2+k_1=0 \\ 2A_1+h_1+h_2=0 \\ 2A_2+k_1+k_2=0 \\ A_1=0 \\ A_2=1 \end{cases} \rightarrow \begin{cases} A_1=0 \\ A_2=1 \\ h_1=0 \\ k_1=-1 \\ h_2=0 \\ k_2=-1 \end{cases}$$

$$\frac{1}{x^2(x^2+1)^2} = \frac{1}{x^2} - \frac{1}{x^2+1} - \frac{1}{(x^2+1)^2}$$

- $\displaystyle\int \frac{dx}{x^2} = -\frac{1}{x} + c$
- $\displaystyle\int \frac{dx}{x^2+1} = \operatorname{arctg} x + 1$
- $\displaystyle\int \frac{dx}{(x^2+1)^2} = \frac{x}{2(x^2+1)} + \frac{1}{2}\operatorname{arctg} x + c$ fatto a lezione + pag 153 es. 4.21/22

$$\int \frac{1}{x^2(x^2+1)^2}\,dx = -\frac{1}{x} - \operatorname{arctg} x - \frac{x}{2(x^2+1)} - \frac{1}{2}\operatorname{arctg} x + c =$$

$$= -\frac{1}{x} - \frac{3}{2}\operatorname{arctg} x - \frac{x}{2(x^2+1)} + c$$