ESERCITAZIONE 17 DIC 2020

① $$\int \frac{1}{\sqrt{-x^2-2x+3}}\,dx \qquad -x^2-2x+3=0 \qquad x=-3,1$$

$$= \int \frac{1}{\sqrt{-(x+3)(x-1)}}\,dx = \int \frac{1}{\sqrt{(x+3)(1-x)}}\,dx =$$

$$= \int \frac{1}{(x+3)\sqrt{\frac{1-x}{x+3}}}\,dx =$$

$$t=\sqrt{\frac{1-x}{x+3}} \qquad t^2=\frac{1-x}{x+3} \qquad t^2x+3t^2=1-x$$

$$\to x = \frac{1-3t^2}{t^2+1}$$

$$dx = \frac{-6t(t^2+1)-2t(1-3t^2)}{(t^2+1)^2}dt = \frac{-8t}{(t^2+1)^2}\,dt$$

$$x+3 = \frac{1-3t^2}{t^2+1}+3 = \frac{4}{t^2+1}$$

$$= \int \frac{1}{\frac{4}{t^2+1}\cdot \cancel{t}} \cdot \frac{-8\cancel{t}}{(t^2+1)^{\cancel{2}}}\,dt = -2\int \frac{1}{t^2+1}\,dt =$$

$$= -2\,\mathrm{arctg}(t)+c = -2\,\mathrm{arctg}\sqrt{\frac{1-x}{x+3}}+c$$

② $$\int \frac{1}{\sqrt{x^2+px+q}}\,dx$$

$$\sqrt{x^2+px+q}=t-x \qquad \cancel{x^2}+px+q=t^2-2tx+\cancel{x^2}$$

$$\to x=\frac{t^2-q}{p+2t} \qquad dx=\frac{2t(p+2t)-2(t^2-q)}{(p+2t)^2}\,dt$$

$$dx = \frac{2(t^2+pt+q)}{(p+2t)^2}\,dt$$

$$\sqrt{x^2+px+q} = t-x = t-\frac{t^2-q}{p+2t} = \frac{t^2+pt+q}{p+2t}$$

$$= \int \frac{p+2t}{t^2+pt+q}\;\frac{2(t^2+pt+q)}{(p+2t)^2}\,dt = \int \frac{2}{p+2t}\,dt =$$

$$= \lg|p+2t|+c = \lg\left|p+\sqrt{x^2+px+q}+x\right|+c$$

③ $$\int \frac{dx}{x+\sqrt{1+x^2}} \qquad \sqrt{x^2+1}=t-x \quad \to t = x+\sqrt{x^2+1}$$

$$\cancel{x^2}+1 = t^2-2tx+\cancel{x^2} \quad \to x=\frac{t^2-1}{2t}$$

$$dx = \frac{2t(2t)-2(t^2-1)}{4t^2}\,dt = \frac{t^2+1}{2t^2}\,dt \quad \to$$

$$= \int \frac{1}{t} \frac{t^2+1}{2t^2}\, dt = \frac{1}{2}\int \frac{t^2+1}{t^3}\, dt =$$

$$= \frac{1}{2}\left[\int \frac{t^2}{t^3}\, dt + \int \frac{1}{t^3}\, dt\right] = \frac{1}{2}\left[\int \frac{1}{t}\, dt + \int \frac{1}{t^3}\, dt\right] =$$

$$= \frac{1}{2}\left[\lg|t| - \frac{1}{2} t^{-2}\right] + c = \frac{1}{2} \lg(\sqrt{x^2+1} + x| - \frac{1}{4}\frac{1}{(x+\sqrt{x^2+1})^2} + c$$

(4) $\displaystyle\int \frac{1}{\sqrt{x}\,(\sqrt[4]{x}-1)}\, dx \qquad t = \sqrt[4]{x} \qquad t^2 = \sqrt{x}$

$$t^4 = x \rightarrow 4t^3\, dt = dx$$

$$= \int \frac{1}{t^2(t-1)} 4t^3\, dt = 4\int \frac{t}{t-1}\, dt = \qquad \frac{t}{t-1} = \frac{t+1-1}{t-1} = \frac{t-1}{t-1} + \frac{1}{t-1} = 1 + \frac{1}{t-1}$$

o usuale divisione

$$= 4\left[\int dt + \int \frac{1}{t-1}\, dt\right] =$$

$$= 4\left[t + \lg|t-1|\right] + c =$$

$$= 4\sqrt[4]{x} + 4\lg|\sqrt[4]{x} - 1| + c$$

(5) $\displaystyle\int \sqrt{x^2-1}\, dx \qquad t - x = \sqrt{x^2-1}$

$$t^2 - 2tx + \cancel{x^2} = \cancel{x^2} - 1 \rightarrow x = \frac{t^2+1}{2t} \quad ■$$

$$dx = \frac{4t^2 - 2t^2 - 2}{4t^2}\, dt = \frac{t^2-1}{2t^2}\, dt$$

$$\sqrt{x^2-1} = t - x = t - \frac{t^2+1}{2t} = \frac{t^2-1}{2t} \quad ⊛$$

$$= \int \frac{t^2-1}{2t} \frac{t^2-1}{2t^2}\, dt = \int \frac{t^4 - 2t^2 + 1}{4t^3}\, dt =$$

$$= \int \frac{t}{4}\, dt - \int \frac{1}{2t}\, dt + \int \frac{1}{4t^3}\, dt =$$

$$= \frac{1}{8} t^2 - \frac{1}{2}\lg|t| - \frac{1}{8t^2} + C = \frac{1}{8}\frac{t^4-1}{t^2} - \frac{1}{2}\lg|t| + c$$

$$\frac{t^4-1}{t^2} = \frac{(t^2-1)}{t} \frac{(t^2+1)}{t} = 2\sqrt{x^2-1} \cdot 2x = 4x\sqrt{x^2-1}$$

⊛ ■

$$= \frac{1}{2} x\sqrt{x^2-1} - \frac{1}{2}\lg|x + \sqrt{x^2-1}| + c$$

(6) $\displaystyle\int \frac{x^2}{\sqrt{x^2-1}}\, dx \qquad t - x = \sqrt{x^2-1}$ (come es. 5)

$$x = \frac{t^2+1}{2t} \qquad \sqrt{x^2-1} = \frac{t^2-1}{2t} \qquad dx = \frac{t^2-1}{2t^2}\, dt$$

$$= \int \underbrace{\frac{(t^2+1)^2}{4t^2}}_{x^2} \underbrace{\frac{2t}{t^2-1}}_{1/\sqrt{x^2-1}} \underbrace{\frac{t^2-1}{2t^2}\, dt}_{dx} = \int \frac{(t^2+1)^2}{4t^3}\, dt = \rightarrow$$

$$= \int \frac{t^4+2t^2+1}{4t^3}\,dt = \int \left(\frac{t}{4}+\frac{1}{2t}+\frac{1}{4t^3}\right)dt =$$

$$= \frac{1}{8}t^2 + \frac{1}{2}\lg|t| - \frac{1}{8t^2} + c = \frac{1}{2}\lg|t| + \frac{1}{8}\frac{t^4-1}{t^2}$$

$\hookrightarrow$ es 5

$$= \frac{1}{2}\lg\left|x+\sqrt{x^2-1}\right| + \frac{1}{2}x\sqrt{x^2+1} + c$$

⑦ $$\int \frac{1}{\sqrt{1-4x^2}}\,dx = \int \frac{1}{\sqrt{1-(2x)^2}}\,dx = \frac{1}{2}\operatorname{arcsen} 2x + c$$

oppure: $2x = t \quad 2\,dx = dt \quad dx = \frac{dt}{2}$

$$= \int \frac{1}{\sqrt{1-t^2}}\frac{dt}{2} = \frac{1}{2}\int \frac{1}{\sqrt{1-t^2}}\,dt = \frac{1}{2}\operatorname{arcsen} t + c$$

$$= \frac{1}{2}\operatorname{arcsen} 2x + c$$

⑧ $$\int \frac{dx}{x\sqrt{4x^2+2x+1}}$$

$$\sqrt{4x^2+2x+1} = 2x+t$$

$$\cancel{4x^2}+2x+1 = \cancel{4x^2}+4tx+t^2$$

$$(4t-2)x = 1-t^2 \quad \rightarrow \quad x = \frac{1-t^2}{4t-2} = \frac{1-t^2}{2(2t-1)}$$

$$dx = \frac{-2t(4t-2)-4(1-t^2)}{4(2t-1)^2}dt = \frac{-8t^2+4t-4+4t^2}{4(2t-1)^2}\,dt =$$

$$= \frac{-t^2+t-1}{(2t-1)^2}\,dt$$

$$\sqrt{4x^2+2x+1} = 2x+t = 2\,\frac{1-t^2}{2(2t-1)} + t = \frac{t^2-t+1}{2t-1}$$

$$= \int \frac{2(\cancel{2t-1})}{1-t^2}\,\frac{\cancel{2t-1}}{\cancel{t^2-t+1}}\,\frac{\overset{(-1)}{-\cancel{t^2+t-1}}}{(\cancel{2t-1})^2}\,dt =$$

$$= -\int \frac{2}{1-t^2}\,dt = \int \frac{2}{t^2-1}\,dt = \qquad \frac{1}{t^2-1} = \frac{A}{t-1} + \frac{B}{t+1}$$

$A = 1/2 \quad B = -1/2$

$$= \int \frac{1}{t-1}\,dt - \int \frac{1}{t+1}\,dt =$$

$$= \lg|t-1| - \lg|t+1| + c =$$

$$= \lg\left|\sqrt{4x^2+2x+1} - 2x - 1\right| - \lg\left|\sqrt{4x^2+2x+1} - 2x + 1\right| + c$$

PER LUNEDI 21 DIC

1) $\displaystyle\int \frac{dx}{\sqrt{x^2-1}-x}$

2) $\displaystyle\int \frac{dx}{x\sqrt{x^2+2x}}$

3) $\displaystyle\int \frac{dx}{(x-1)\sqrt{x^2-2}}$

4) $\displaystyle\int \frac{dx}{\sqrt{2x^2+3x-2}}$

5) $\displaystyle\int \frac{x}{\sqrt{x^2-3x+2}}\,dx$

6) $\displaystyle\int \frac{dx}{x\sqrt{x^2+x+1}}$

hints: $\sqrt{\ } = t \pm x$

$= \alpha t \pm \beta x$