8|10|202

P. INDUZIONE

① $P(n=1)$ vera

② $P(n) \Rightarrow P(n+1)$ vera

- $$\sum_{k=1}^{n+1} f(k) = f(k=1)+f(k=2)+\ldots+f(k=n)+f(k=n+1) = \sum_{k=1}^{n} f(k) + f(k=n+1)$$

Dimostrare mediante il principio di induzione:

$\boxed{1.77}$ $\sum_{k=1}^{n} 2k = n(n+1)$ $P(n)$

① $P(n=1)$: $\sum_{k=1}^{1} 2k = 1(1+1)$

$$2\cdot(1) = 1\cdot(2)$$
$$2 = 2 \quad \text{vera} \quad \checkmark$$

② $P(n) \Rightarrow P(n+1)$ vera

$P(n+1)$: $\sum_{k=1}^{n+1} 2k = (n+1)((n+1)+1)$

$$\sum_{k=1}^{n} 2k + 2(n+1) = (n+1)(n+2)$$
$$n(n+1) + 2(n+1) = (n+1)(n+2)$$
* $$(n+1)(n+2) = (n+1)(n+2) \quad \text{vera} \quad \checkmark$$

* uso $P(n)$ vera

$\boxed{1.74}$ $\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$: $P(n)$

① $P(n=1)$: $\sum_{k=0}^{1} 2^k = 2^{1+1} - 1$

$$2^0 + 2^1 = 2^2 - 1$$
$$1 + 2 = 4 - 1$$
$$3 = 3 \quad \text{vera} \quad \checkmark$$

② $P(n+1)$: $\sum_{k=0}^{n+1} 2^k = 2^{(n+1)+1} - 1$

$$\sum_{k=0}^{n} 2^k + 2^{(n+1)} = 2^{n+2} - 1$$
$$2^{n+1} - 1 + 2^{n+1} = 2^{n+2} - 1$$
* $$2 \cdot 2^{n+1} - 1 = 2^{n+2} - 1$$
$$2^1\, 2^{n+1} - 1 = 2^{n+2} - 1 \qquad a^x a^y = a^{x+y}$$
$$2^{1+(n+1)} - 1 = 2^{n+2} - 1$$
$$2^{n+2} - 1 = 2^{n+2} - 1 \quad \text{vera}$$

* uso $P(n)$ vera

SUP/INF + MIN|MAX

$$M_1 = \sup X \longleftrightarrow \begin{cases} M_1 \geq x & \forall x \in X \\ \forall \varepsilon > 0 \ \exists x \in X : M_1 - \varepsilon < x \end{cases}$$

$$m_1 = \inf X \longleftrightarrow \begin{cases} m_1 \leq x & \forall x \in X \\ \forall \varepsilon > 0 \ \exists x \in X : m + \varepsilon > x \end{cases}$$

$$M = \max X \longleftrightarrow \begin{cases} M \geq x & \forall x \in X \\ M \in X \end{cases} \quad !!$$

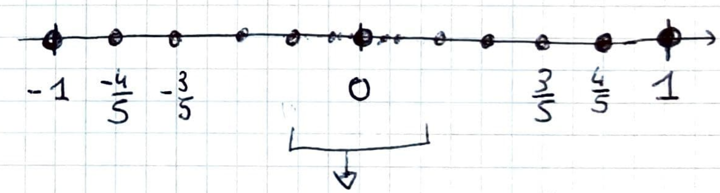
$$m = \min X \longleftrightarrow \begin{cases} m \leq x & \forall x \in X \\ m \in X \end{cases} \quad !!$$

$\boxed{1.40}$ Determinare estremo superiore ed inferiore ed eventualmente il massimo e minimo dell'insieme:

$$A = \left\{ \frac{2n}{n^2+1} : n \in \mathbb{Z} \right\}.$$

$$\mathbb{Z} = \{0, \pm 1, \pm 2, \dots\}$$

$$A = \left\{ \underset{n=0}{0};\ \underset{n=\pm 1}{\pm 1};\ \underset{n=\pm 2}{\pm \tfrac{4}{5}};\ \underset{n=\pm 3}{\pm \tfrac{3}{5}};\ \dots \right\}$$



n grande positivo $\dfrac{2/n}{1+1/n^2} \sim \varepsilon = 0 + \varepsilon > 0$ piccolo e positivo

n grande negativo $\dfrac{2/n}{1+1/n^2} \sim -\varepsilon = 0 - \varepsilon < 0$ piccolo negativo

l'insieme è limitato

$\inf A = \min A = -1$

$\sup A = \max A = 1$

Verifichiamolo:

Se $M = \max A \rightarrow \forall x \in A \quad x \leq M$

$m = \min A \rightarrow \forall x \in A \quad x \geq m$

x è generico elemento di A : $x = \frac{2n}{n^2+1}$

$x \leq M$ diventa $\frac{2n}{n^2+1} \leq 1$ $\quad (n^2+1 > 0)$

$2n \leq n^2+1$

$n^2 - 2n + 1 \geq 0$

$\underbrace{(n-1)^2}_{\text{quadrato}} \geq 0$ VERA

$x \geq m$ diventa $\frac{2n}{n^2+1} \geq -1$ $\quad (n^2+1 > 0)$

$2n \geq -n^2 - 1$

$n^2 + 2n + 1 \geq 0$

$\underbrace{(n+1)^2}_{\text{quadrato}} \geq 0$ VERA

NUMERI COMPLESSI

$i = \sqrt{-1} \qquad i^2 = -1$

$z = a + ib \qquad a, b \in \mathbb{R} \qquad z \in \mathbb{C}$

$\begin{cases} a = Re(z) \\ b = Im(z) \end{cases}$

complesso coniugato di z: $\bar{z} = a - ib$

$Re(\bar{z}) = Re(z)$

$Im(\bar{z}) = -Im(\bar{z})$

$z \cdot \bar{z} = (a+ib)(a-ib) =$

$= a^2 - \cancel{iab} + \cancel{iab} - i^2b^2 =$

$= a^2 - i^2b^2 = a^2 - (-1)b^2 = a^2 + b^2$

$\boxed{4.2}$ $\frac{i}{1-i}$ Scriverlo nella forma $a+ib$

$$\frac{i}{\underbrace{1-i}_{z}} \cdot \frac{1+i}{\underbrace{1+i}_{\bar{z}}} = \frac{i+i^2}{\underbrace{(1)^2+(1)^2}_{a^2+b^2}} = \frac{1}{2}\left(i + (-1)\right) =$$

$$= -\frac{1}{2} + \frac{1}{2} i$$

PER LA PROSSIMA VOLTA (LUNEDÌ 12/10)

- Principio di induzione. Dimostrare

$$\sum_{k=1}^{n} (2k-1) = n^2$$

$$\sum_{k=1}^{n} (2k-1)^2 = \frac{n(4n^2-1)}{3}$$

- Sup/Inf/Max/min

$$B = \left\{ \frac{3n+2}{n} : n \in \mathbb{N} \right\}$$

$$C = \left\{ \frac{(-1)^n}{n} : n \in \mathbb{N} \right\}$$ Studia per n pari e n dispari

- scrivere nella forma $z = a + ib$:

$$\frac{1}{i} \quad , \quad \frac{1-i}{1+i} \quad , \quad 13 \cdot \frac{1+i}{2-3i}$$