ESERCITAZIONE 7 GEN 21

$\boxed{6.10}$ Calcolare la somma delle serie geometriche

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad \text{con } |x|<1$$

a) $\sum_{0}^{\infty} \frac{1}{2^n} = \frac{1}{1-1/2} = \frac{2}{1} = 2$

b) $\sum_{1}^{\infty} \frac{1}{2^n} = \sum_{0}^{\infty} \frac{1}{2^n} - \frac{1}{2^0} = 2-1=1$

c) $\sum_{n=0}^{\infty} 3^n \to \infty$

d) $\sum_{0}^{\infty} \left(\frac{4}{5}\right)^n = \frac{1}{1-4/5} = \frac{5}{1} = 5$

$\boxed{6.16}$ Dopo aver dimostrato per induzione la formula

$$\sum_{n=1}^{k} \lg \frac{(n+1)^2}{n(n+2)} = \lg \frac{2(k+1)}{k+2}$$

calcolare la somma infinita

INDUZ: ① $P(1)$ vera

② $P(n+1)$ vera date $P(n)$ vera

① $\sum_{n=1}^{1} \lg \frac{(n+1)^2}{n(n+2)} = \lg \frac{4}{3}$

$\lg \frac{2(k+1)}{k+2}\Big|_{k=1} = \lg \frac{4}{3}$

② $S_{k+1} = \lg \frac{2(k+2)}{k+3}$

$$S_{k+1} = S_k + \lg \frac{(k+2)^2}{(k+1)(k+3)} = \lg \frac{2(k+1)}{k+2} + \lg \frac{(k+2)^2}{(k+1)(k+3)} =$$

$$= \lg \frac{2\cancel{(k+1)}}{\cancel{k+2}} \frac{(k+2)^{\cancel{2}}}{\cancel{(k+1)}(k+3)} = \lg \frac{2(k+2)}{k+3} \quad \blacksquare$$

$\boxed{6.18}$ Verificare che $\sum_{1}^{\infty} 2^{1/n}$ diverge

CN $\lim_{n\to\infty} a_n = 0$: $\lim_{n\to\infty} 2^{1/n} = 1$

TERMINI NON NEG

$\boxed{6.20}$

$\sum_{1}^{\infty} \frac{1}{n^2}$ è convergente

$n^2 < n(n+1)$

$2n^2 > n(n+1)$ $\to$ $\frac{1}{2n^2} < \frac{1}{n(n+1)}$ $\qquad \frac{1}{n^2} < \frac{2}{n(n+1)}$

$\sum_{1}^{\infty} \frac{1}{n(n+1)} = 1$ VISTO IN CLASSE $\to$ CONVERGE (limitata sup.)

$\boxed{6.21}$ stabilire il carattere

b) $\sum_{n=1}^{\infty} \sqrt[n]{n}$ $\qquad \lim_{n\to\infty} \sqrt[n]{n} = \lim_{n\to\infty} e^{\frac{1}{n}\ln n} = e^0 = 1$ DIVERGE

c) $\sum_{1}^{\infty} \frac{n}{n^3+1}$ $\qquad \lim_{n\to\infty} a_n = 0$ $\qquad \frac{n}{n^3+1} < \frac{n}{n^3} = \frac{1}{n^2} \to$ conv

$\to$ converge

d) $\sum_{1}^{\infty} \frac{n}{n^2+1}$ $\quad \frac{n}{n^2+1} = \frac{1}{n+1/n} \geq \frac{1}{2n}$ ⇝ DIVERGE

$(n^2+1 > n^2+n^2 = 2n^2)$

$\boxed{6,23}$ $\sum_{1}^{\infty} \frac{\ln n}{n^2}$ $\quad \lim_{n\to\infty} a_n = 0$

m1) CONFRONTO LIMITI

$\lim_{n\to\infty} \frac{a_n}{b_n} = \ell$ $\quad a_n \geq 0$, $b_n > 0$

$b_n = \frac{1}{n^{3/2}}$ $\quad \lim \frac{a_n}{b_n} = \lim \frac{\ln n}{\sqrt{n}} = 0$

se $\ell = 0$ e $\sum b_n < \infty \Rightarrow \sum a_n$ converge

m2) CONFRONTO INFINITESIMI

$a_n \geq 0$ $\quad \lim_{n\to\infty} n^p a_n = \ell$

$\lim_{n\to\infty} n^{3/2} \frac{\ln n}{n^2} = \lim \frac{\ln n}{\sqrt{n}} = 0$ $\quad$ se $\ell = 0$ e $p > 1$

$\sum a_n$ conv.

$\boxed{6,27}$ $\sum_{1}^{\infty} \left(\frac{2}{n} - \operatorname{sen}\frac{1}{n}\right)$

$n^p\left(\frac{2}{n} - \operatorname{sen}\frac{1}{n}\right) \overset{p=1}{=} (2-1) = 1$ $\Rightarrow$ DIV.

$\boxed{6,29}$ $\sum_{1}^{\infty} \left(\frac{1}{n} - \operatorname{sen}\frac{1}{n}\right)$

con $p=1$ $\ell = 0$ MA NON CONCLUDIAMO NULLA

$\operatorname{sen} t = t - \frac{t^3}{6}$ $\quad \operatorname{sen}\frac{1}{n} = \frac{1}{n} - \frac{1}{6n^3}$

$\lim n^p\left(\frac{1}{6n^3}\right) = \frac{1}{6}$ $\quad p=3$

CONV.

$\boxed{6,28}$ A) $\sum_{0}^{\infty} (\pi - 2\operatorname{arctg} n)$

$\lim_{n\to\infty} \frac{\pi - 2\operatorname{arctg} n}{n^{-p}} \overset{\text{De l'Hopital}}{=} \lim_{n\to\infty} \frac{-\frac{2}{1+n^2}}{-p\, n^{-p-1}} =$

$= \lim_{n\to\infty} \frac{2}{p} \frac{n^{p+1}}{1+n^2} = 2$ $\quad$ con $p=1$ $\Rightarrow$ DIV.

b) $\sum_{1}^{\infty} \frac{(n-1)^n}{n!}$ $\quad \frac{n^{n+1}}{(n+1)!} \cdot \frac{n!}{(n-1)^n} = \frac{n}{n+1}\left(\frac{n}{n-1}\right)^n$

$\frac{n}{n-1} = 1 + \frac{1}{y} \qquad \frac{n \pm 1}{n-1} = 1 + \frac{1}{n-1} \qquad y = n-1$

$$\lim \underbrace{\frac{n}{n+1}}_{1} \Big[\underbrace{\left(1 + \frac{1}{n-1}\right)^{n-1}}_{e} \Big]^{\underbrace{\frac{n}{n-1}}_{1}} = e > 1 \text{ diverge}$$

Per lunedì 11 gennaio 2021

6.24 $\sum_{1}^{\infty} \frac{1}{3^n - n}$

6.25

a) $\sum_{n=1}^{\infty} \frac{3n^2+1}{n^4+n+1}$

b) $\sum_{1}^{\infty} \frac{5n-1}{3n^2+2}$

6.26 $\sum_{1}^{\infty} \frac{\lg n}{n}$

32) $\sum_{1}^{\infty} \left(1 - n^2 \operatorname{sen}^2 \frac{1}{n}\right)$

37) a) $\sum_{1}^{\infty} \frac{x^{n-1}}{(n+1)!}$ $\qquad$ b) $\sum_{1}^{\infty} \frac{x^n}{n}$

38) $\sum_{1}^{\infty} \frac{n^2}{n!}$

39) a) $\sum_{1}^{\infty} \frac{n^n}{(n+1)!}$