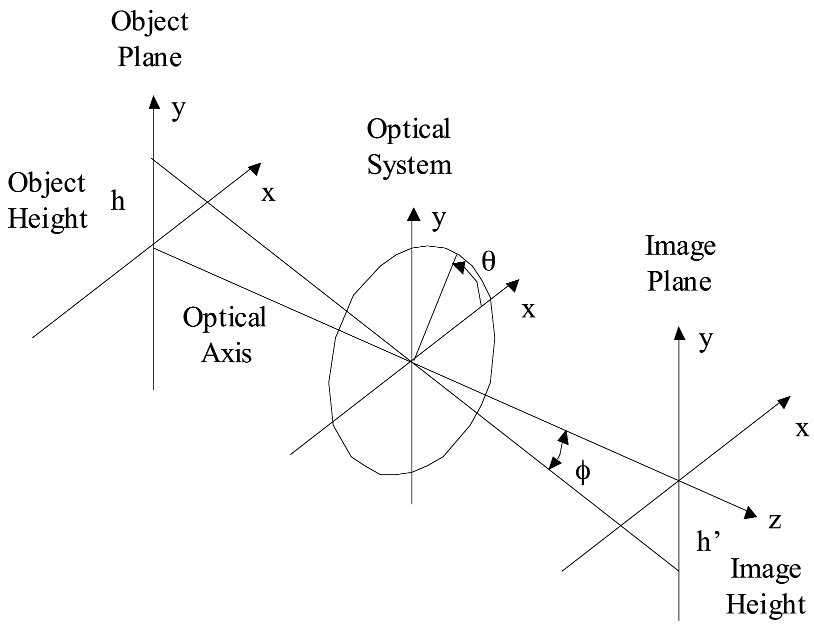
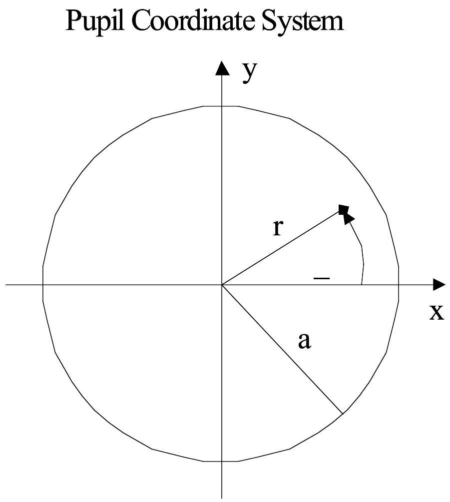
Polinomi di Zernike

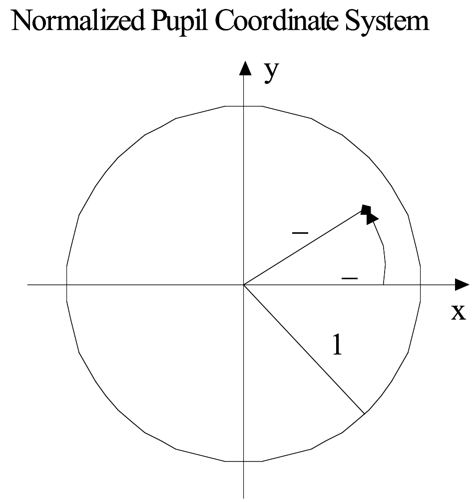
Bibliografia: roorda e Maeda

Sistema di coordinate



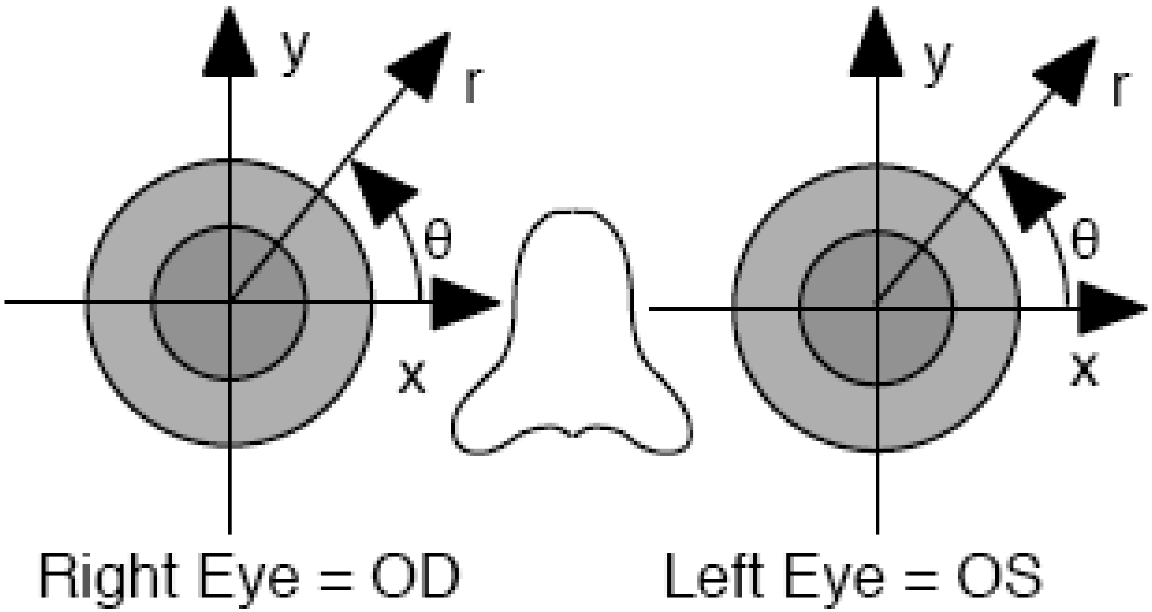


x = r cos(_)
y = r sin(_)
_ = tan $^{-1}$(x/y)
r = (x ^{2}+y^{2})$^{1/2}$



x = _ cos(_)
y = _ sin(_)
_ = tan $^{-1}$(x/y)
_ = r/a = (x ^{2}+y^{2})$^{1/2}$

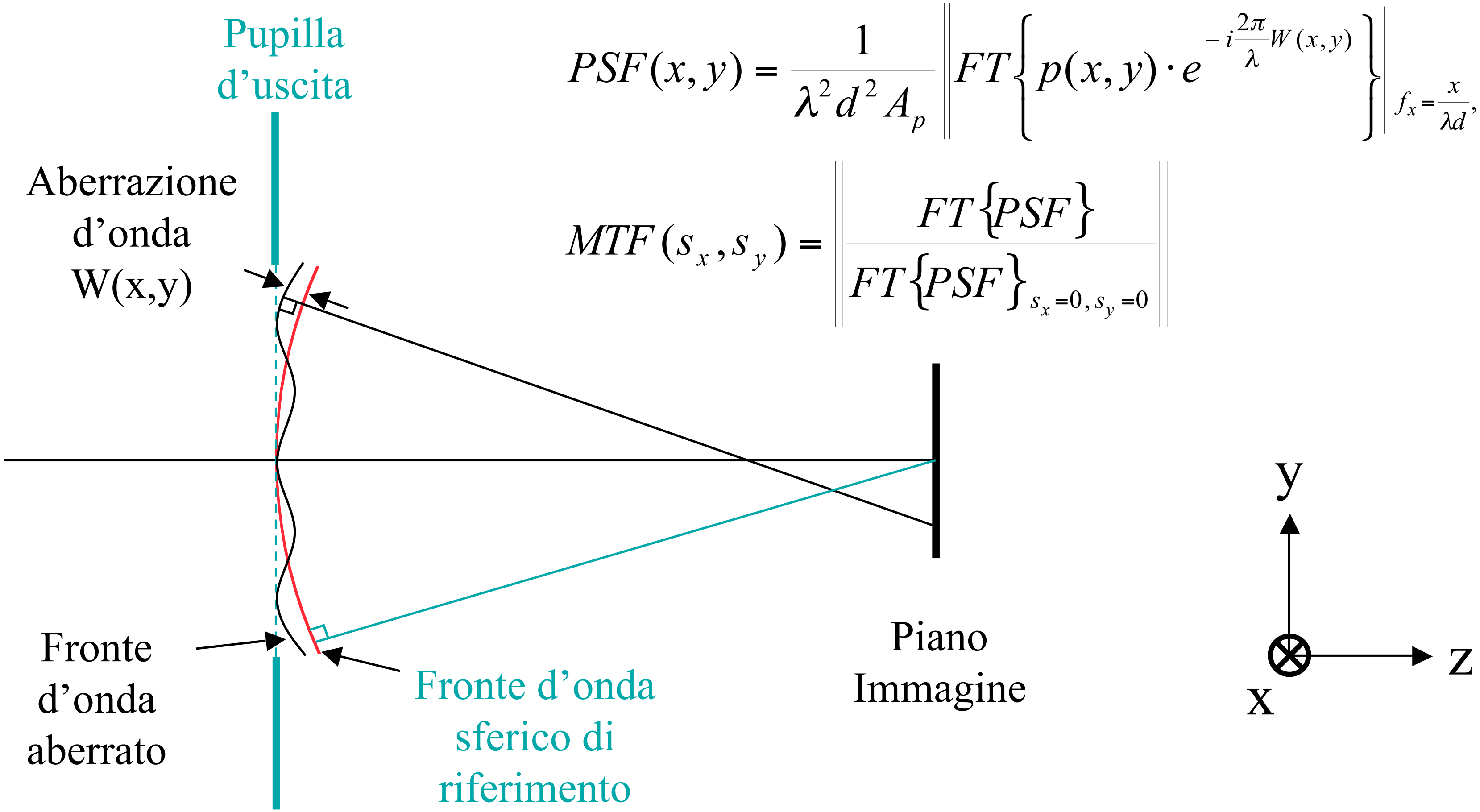
Sistema di coordinate per l’occhio



Aberrazione d'onda

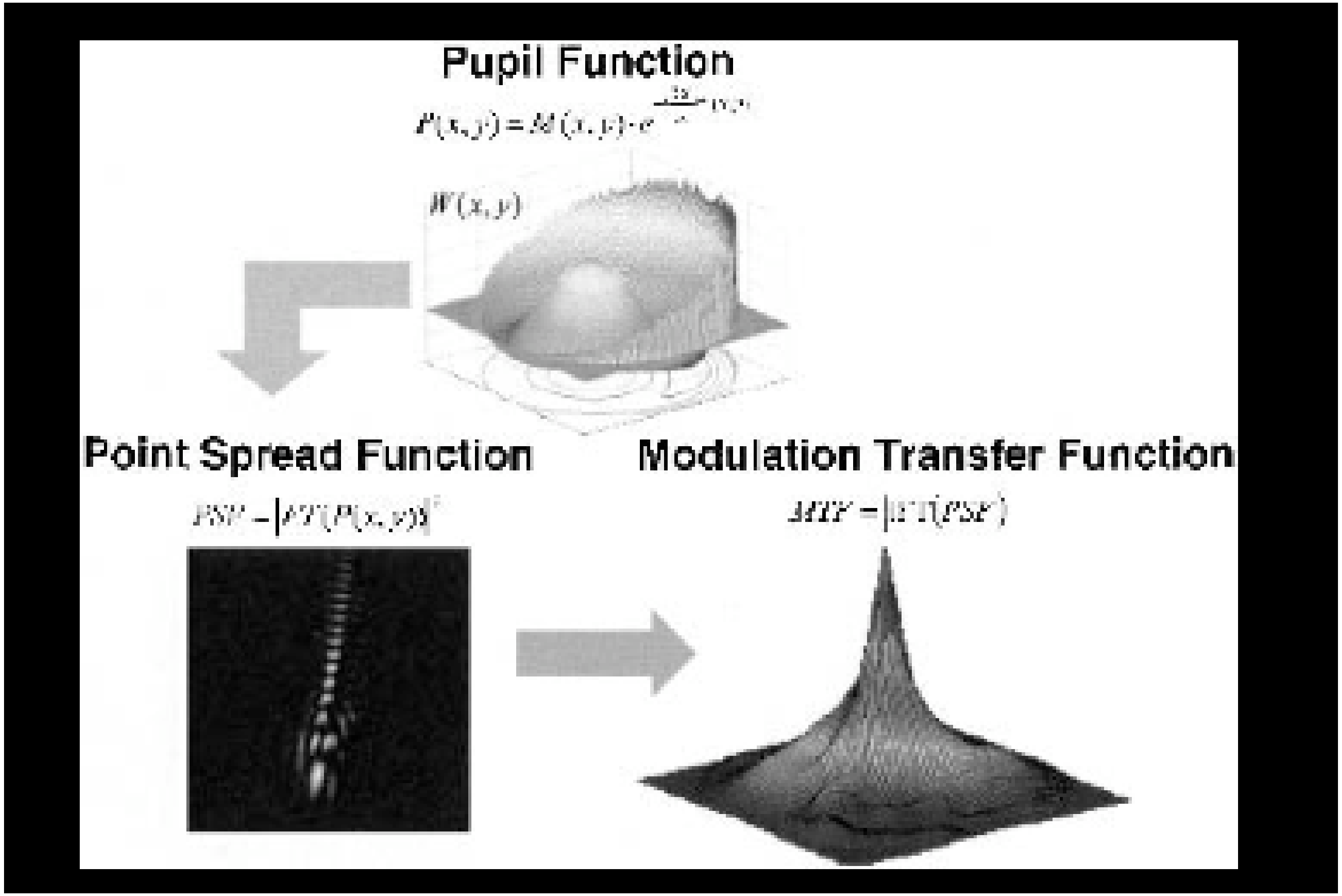
$$PSF(x,y) = \frac{1}{\lambda^2 d^2 A_p} \left\| FT\left\{ p(x,y) \cdot e^{-i\frac{2\pi}{\lambda}W(x,y)} \right\}\Big|_{f_x=\frac{x}{\lambda d},\, f_y=\frac{y}{\lambda d}} \right\|^2$$

$$MTF(s_x, s_y) = \left\| \frac{FT\{PSF\}}{FT\{PSF\}\big|_{s_x=0,\, s_y=0}} \right\|$$



L'aberrazione del fronte d'onda, W(x,y), è la distanza, in termini di cammino ottico OPD (prodotto tra indice di rifrazione e cammino fisico), tra la sfera di riferimento e il fronte d'onda "reale".

I passaggi matematici



Sviluppo polinomiale

- L'aberrazione del fronte d'onda W(x,y) può essere sviluppata in termini di polinomi di Zernike
- Il contributo di ogni termine è indipendente dall'altro

Formule matematiche

The Zernike polynomials are defined as[3] :

$$Z_n^m(\rho,\theta) = N_n^m R_n^{|m|}(\rho)\cos(m\theta) \qquad \text{for } m \ge 0,\ 0 \le \rho \le 1,\ 0 \le \theta \le 2\pi$$

$$= -N_n^m R_n^{|m|}(\rho)\sin(m\theta) \qquad \text{for } m < 0,\ 0 \le \rho \le 1,\ 0 \le \theta \le 2\pi$$

for a given n : m can only take on values of $-n, -n+2, -n+4, \ldots, n$

N_n^m is the normalization factor

$$N_n^m = \sqrt{\frac{2(n+1)}{1+\delta_{m0}}} \qquad \delta_{m0} = 1 \text{ for } m = 0, \quad \delta_{m0} = 0 \text{ for } m \ne 0$$

factorial.m

zernike.m

$R_n^{|m|}(\rho)$ is the radial polynomial

$$R_n^{|m|}(\rho) = \sum_{s=0}^{(n-|m|)/2} \frac{(-1)^s (n-s)!}{s!\,[0.5(n+|m|)-s]\,[0.5(n-|m|)-s]}\,\rho^{n-2s}$$

Lista dei polinomi di Zernike

mode j	order n	frequency m	$Z_n^m(\rho,\theta)$	Meaning
0	0	0	1	Constant term, or Piston
1	1	-1	$2\rho\sin(\theta)$	Tilt in y-direction, Distortion
2	1	1	$2\rho\cos(\theta)$	Tilt in x-direction, Distortion
3	2	-2	$\sqrt{6}\rho^2\sin(2\theta)$	Astigmatism with axis at $\pm 45^\circ$
4	2	0	$\sqrt{3}(2\rho^2-1)$	Field curvature, Defocus
5	2	2	$\sqrt{6}\rho^2\cos(2\theta)$	Astigmatism with axis at 0° or 90°
6	3	-3	$\sqrt{8}\rho^3\sin(3\theta)$	
7	3	-1	$\sqrt{8}(3\rho^3-2\rho)\sin(\theta)$	Coma along y-axis
8	3	1	$\sqrt{8}(3\rho^3-2\rho)\cos(\theta)$	Coma along x-axis
9	3	3	$\sqrt{8}\rho^3\cos(3\theta)$	
10	4	-4	$\sqrt{10}\rho^4\sin(4\theta)$	
11	4	-2	$\sqrt{10}(4\rho^4-3\rho^2)\sin(2\theta)$	Secondary Astigmatism
12	4	0	$\sqrt{5}(6\rho^4-6\rho^2+1)$	Spherical Aberration, Defocus
13	4	2	$\sqrt{10}(4\rho^4-3\rho^2)\cos(2\theta)$	Secondary Astigmatism
14	4	4	$\sqrt{10}\rho^4\cos(4\theta)$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	

Aberrazione del fronte d’onda

The wave aberration is expressed as a weighted sum of Zernike polynomials[7] :

$$W(\rho,\theta) = \sum_{n}^{k} \sum_{m=-n}^{n} W_n^m Z_n^m(\rho,\theta)$$

$$= \sum_{n}^{k} \left\{ \sum_{m=-n}^{-1} W_n^m (-N_n^m R_n^{|m|}(\rho)\sin(m\theta)) + \sum_{m=0}^{n} W_n^m (N_n^m R_n^{|m|}(\rho)\cos(m\theta)) \right\}$$

$$W(x,y) = \sum_{j=0}^{j\max} W_j Z_j(x,y)$$

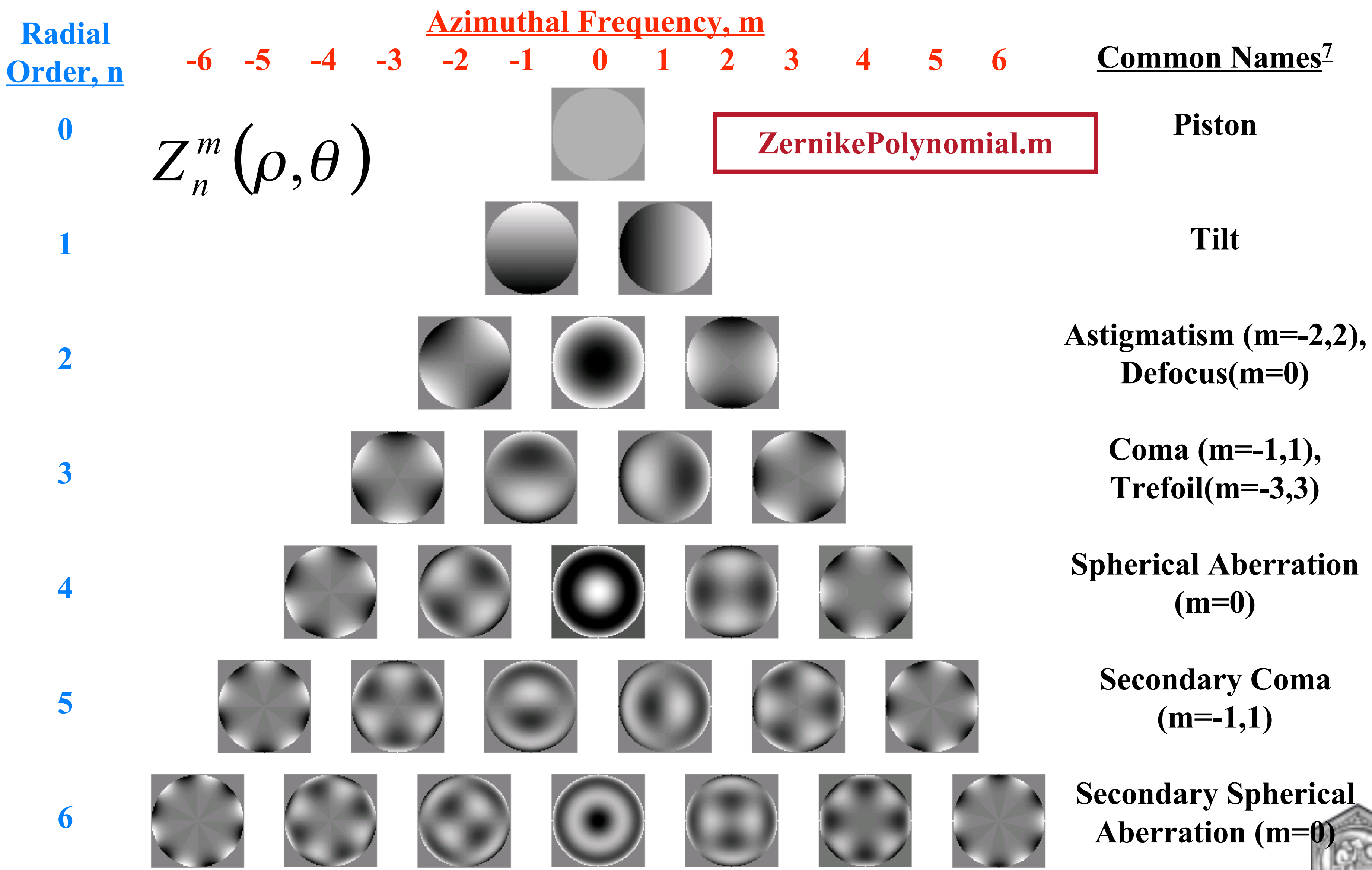
Calcola_aberrazione_onda.m

Utilizzo di un solo indice

n\m	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5
0						j=0					
1					1		2				
2				3		4		5			
3			6		7		8		9		
4		10		11		12		13		14	
5	15		16		17		18		19		20

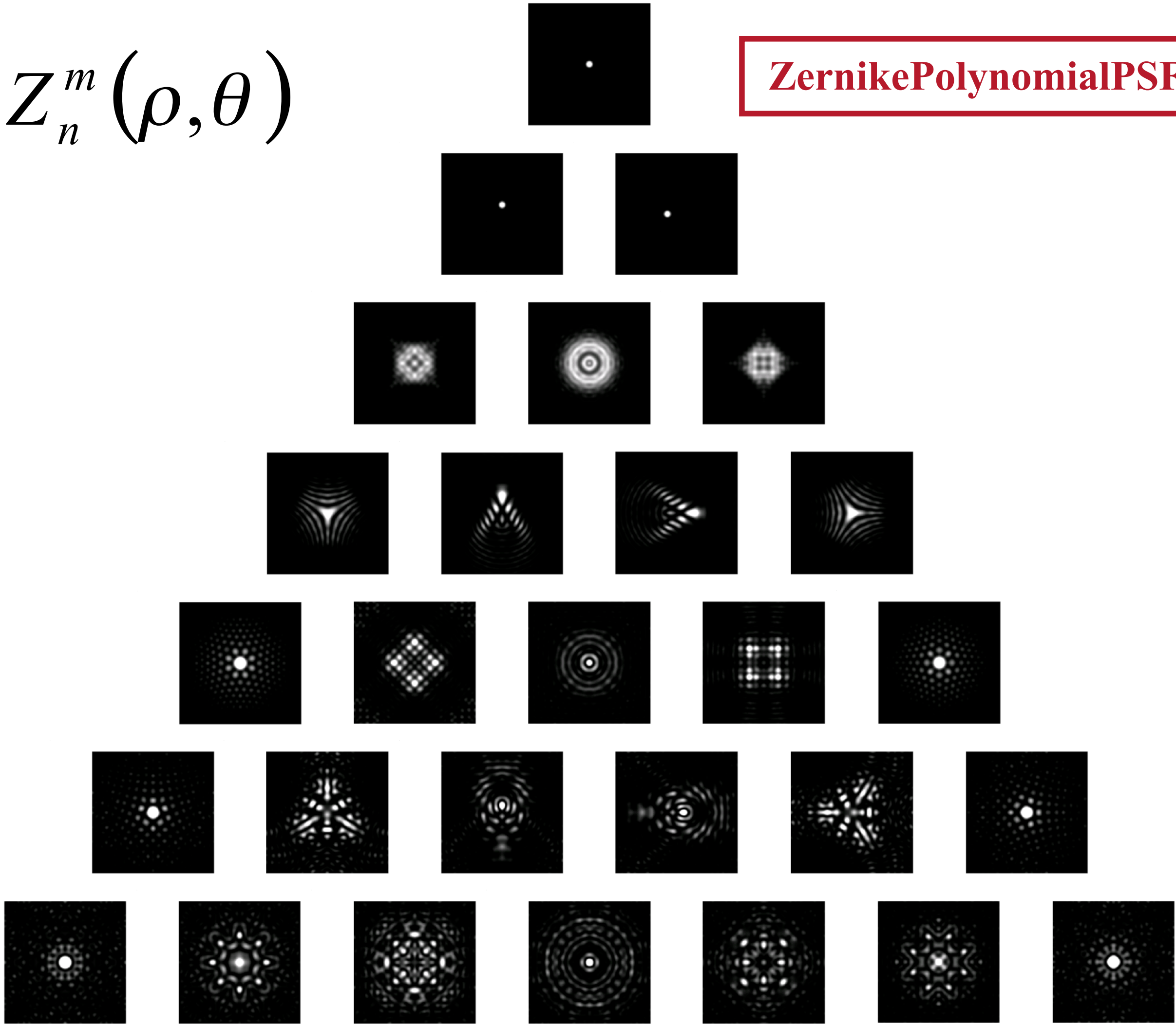
Talvolta si ricorre all’utilizzo di un solo indice *j*. In questo caso questa tabella indica l’equivalenza tra le due notazioni

Polinomi di Zernike a doppio indice

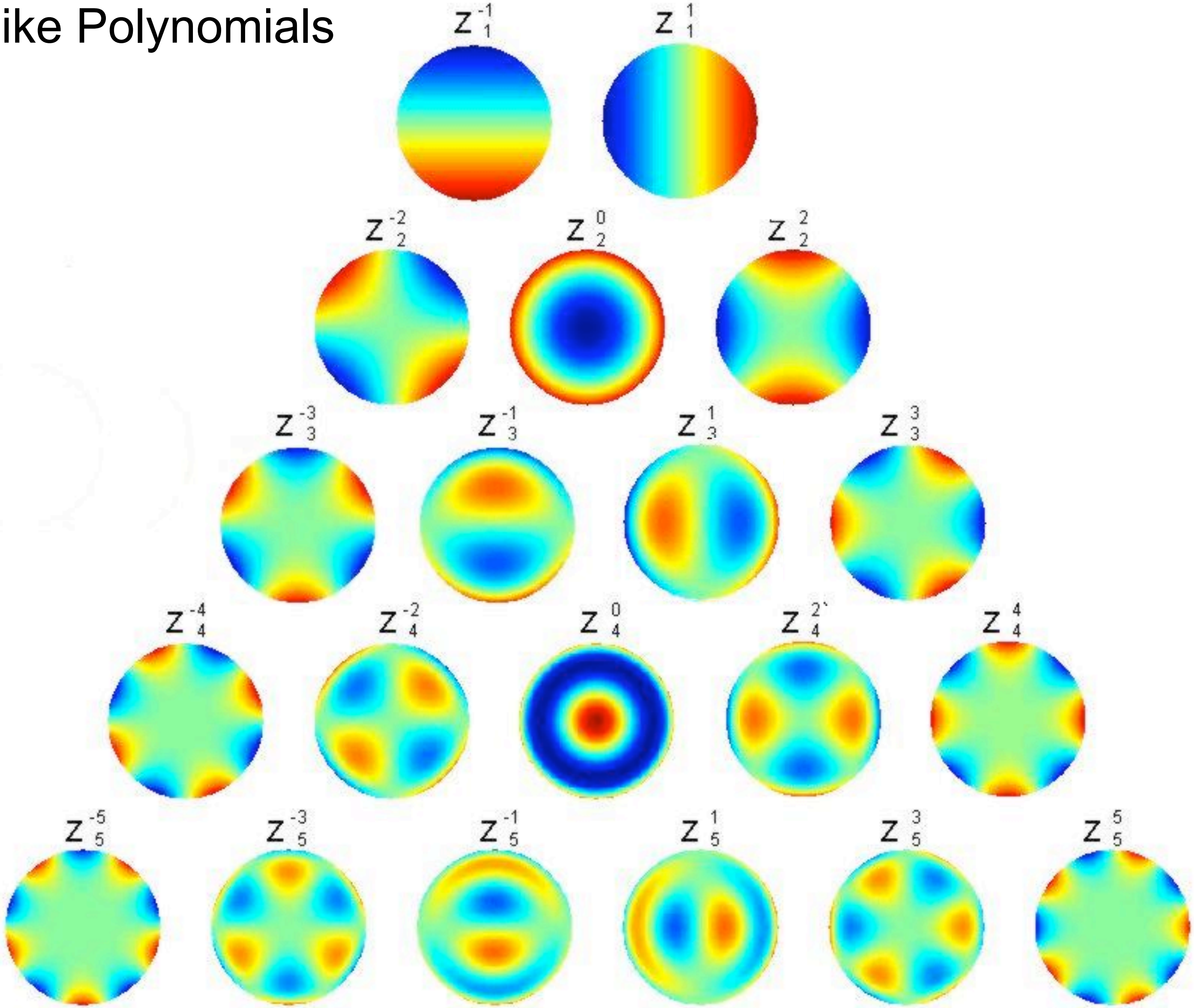


PSF per i vari termini di Zernike

$$Z_n^m(\rho,\theta)$$



Zernike Polynomials



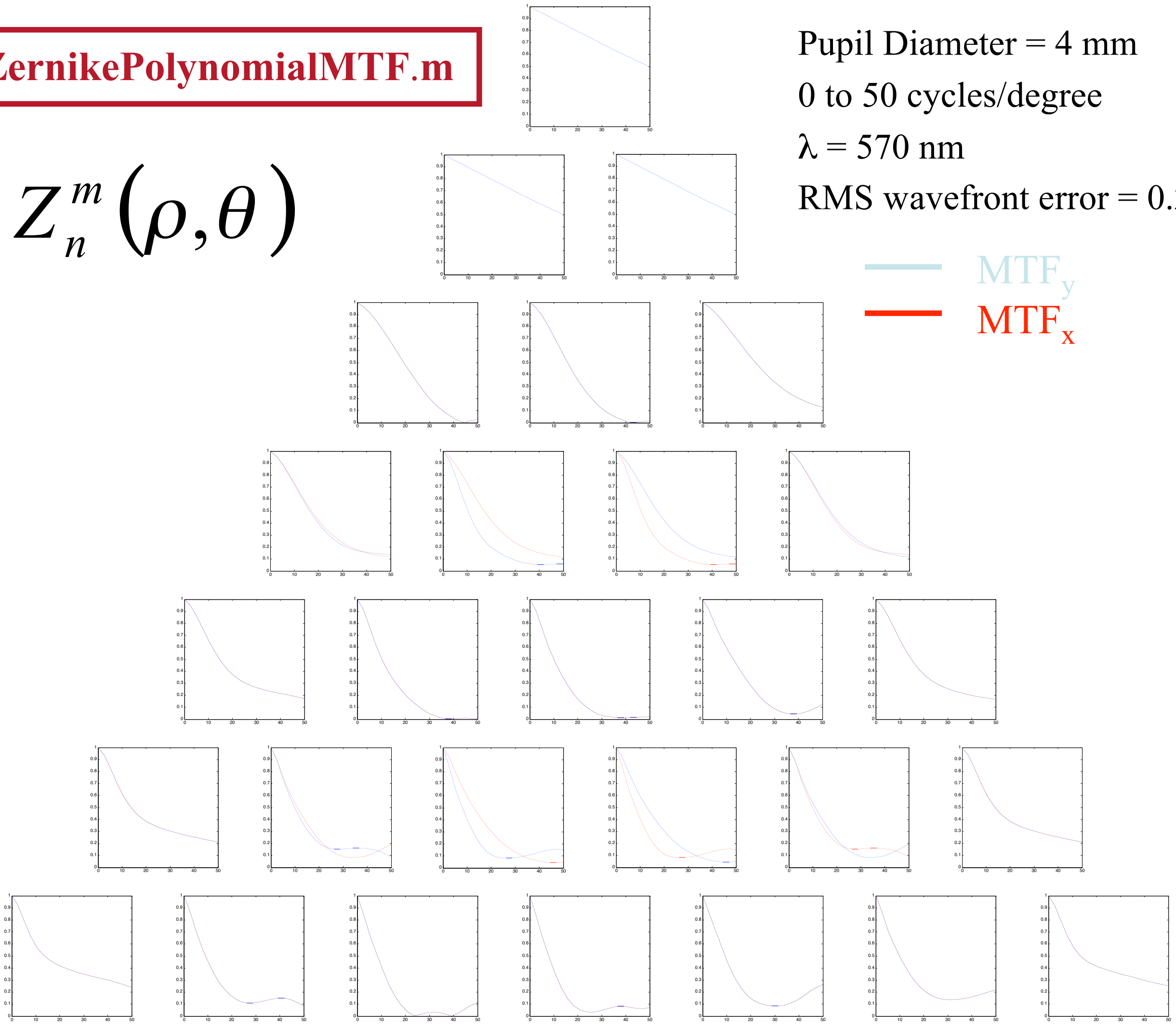
Double-Index Zernike Polynomial MTFs

Azimuthal Frequency, m

Radial Order, n	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	Common Names[7]
0														Piston
1														Tilt
2														Astigmatism (m=-2,2), Defocus(m=0)
3														Coma (m=-1,1), Trefoil(m=-3,3)
4														Spherical Aberration (m=0)
5														Secondary Coma (m=-1,1)
6														Secondary Spherical Aberration (m=0)

ZernikePolynomialMTF.m

$$Z_n^m(\rho,\theta)$$



Pupil Diameter = 4 mm

0 to 50 cycles/degree

λ = 570 nm

RMS wavefront error = 0.2λ

MTF_y

MTF_x

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Double-Index Zernike Polynomial MTFs

Radial Order, n

Azimuthal Frequency, m

Common Names[7]

ZernikePolynomialMTF.m

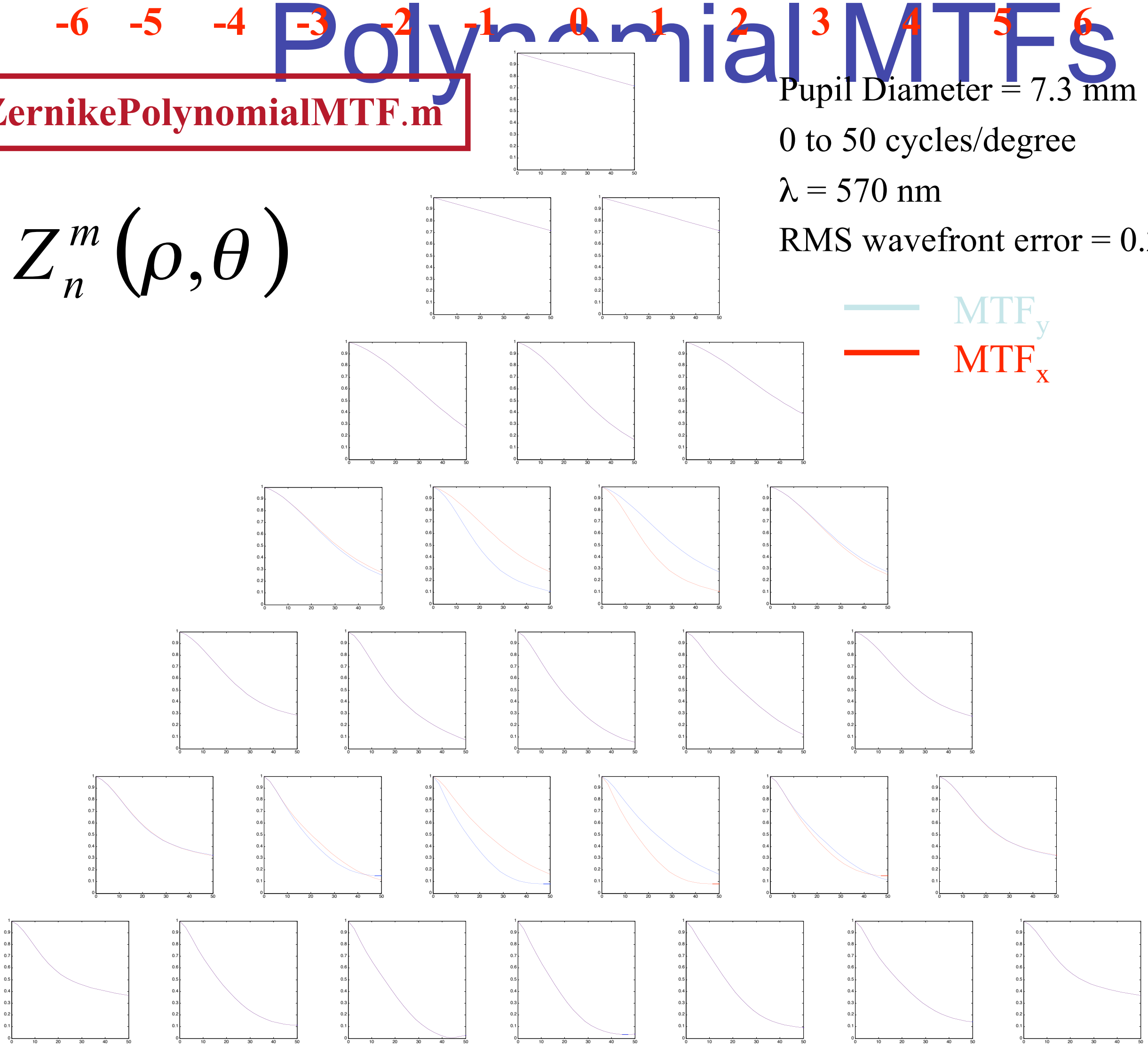
$$Z_n^m(\rho,\theta)$$

Pupil Diameter = 7.3 mm

0 to 50 cycles/degree

λ = 570 nm

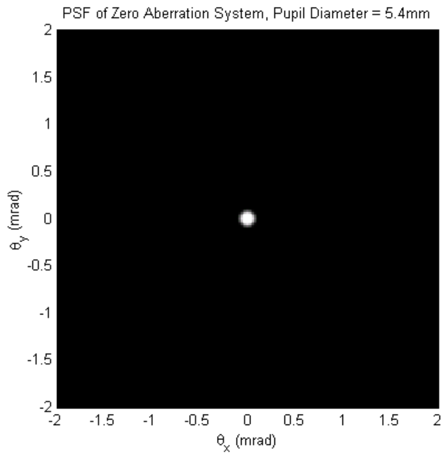
RMS wavefront error = 0.2λ

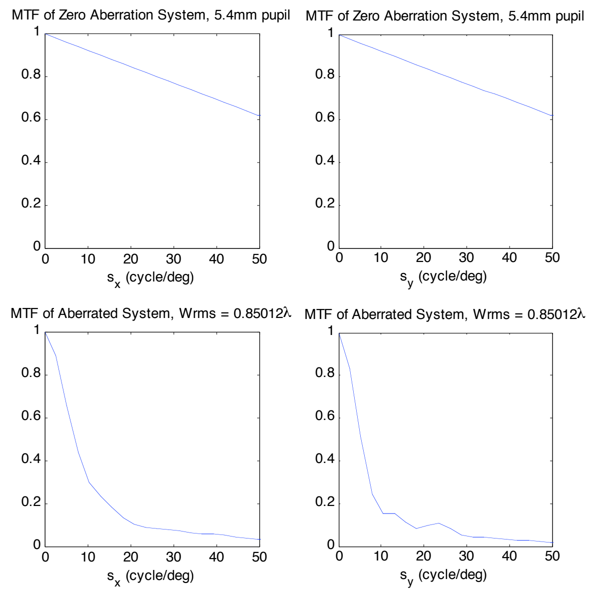


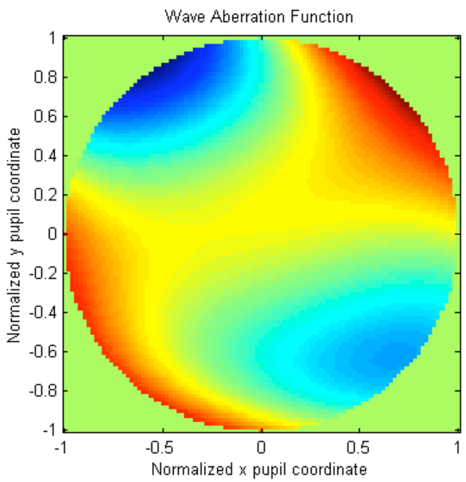
n	Common Names
0	Piston
1	Tilt
2	Astigmatism (m=-2,2), Defocus(m=0)
3	Coma (m=-1,1), Trefoil(m=-3,3)
4	Spherical Aberration (m=0)
5	Secondary Coma (m=-1,1)
6	Secondary Spherical Aberration (m=0)

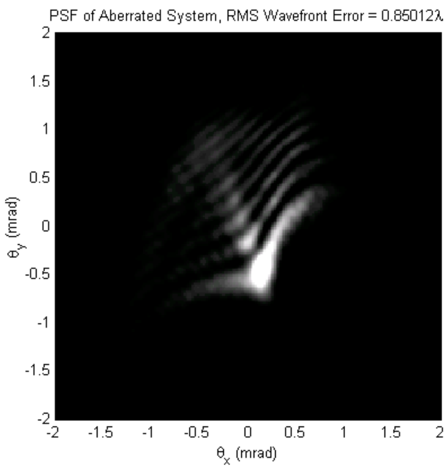
Simulation based on Human

Mode j	Coefficient (µm)	RMS Coefficient (µm)
0	0	0
1	0	0
2	0	0
3	1.02	0.416413256
4	0	0
5	0.33	0.134721936
6	0.21	0.074246212
7	-0.26	-0.091923882
8	0.03	0.010606602
9	-0.34	-0.120208153
10	-0.12	-0.037947332
11	0.05	0.015811388
12	0.19	0.084970583
13	-0.19	-0.060083276
14	0.15	0.047434165
Total RMS Wavefront Error (µm)		0.484608089









WaveAberrationMTF.m

WaveAberration.m

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WaveAberrationPSF.m

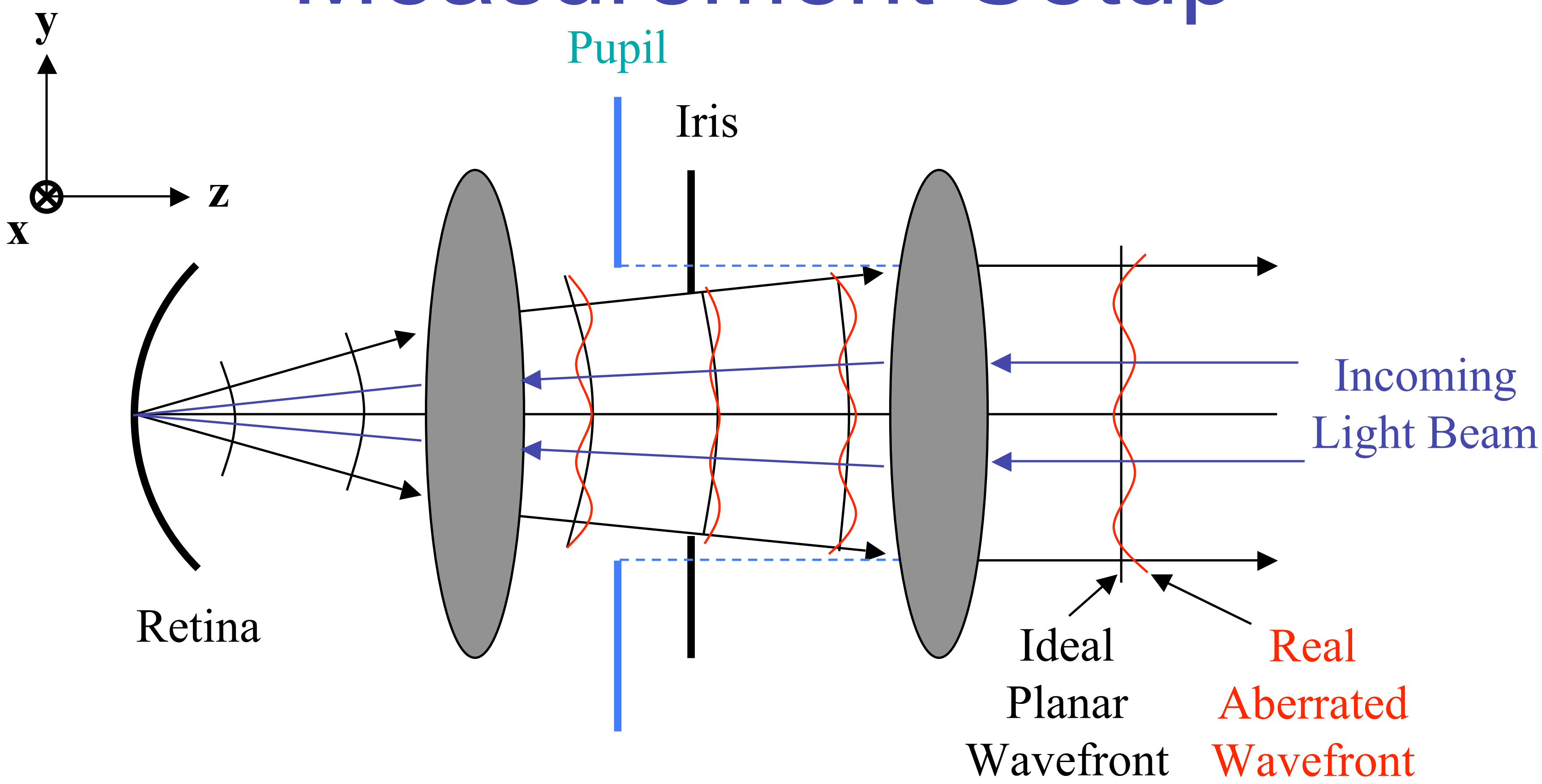
What are Zernike Polynomials?

- set of basic shapes that are used to fit the wavefront
- analogous to the parabolic x^2 shape that can be used to fit 2D data

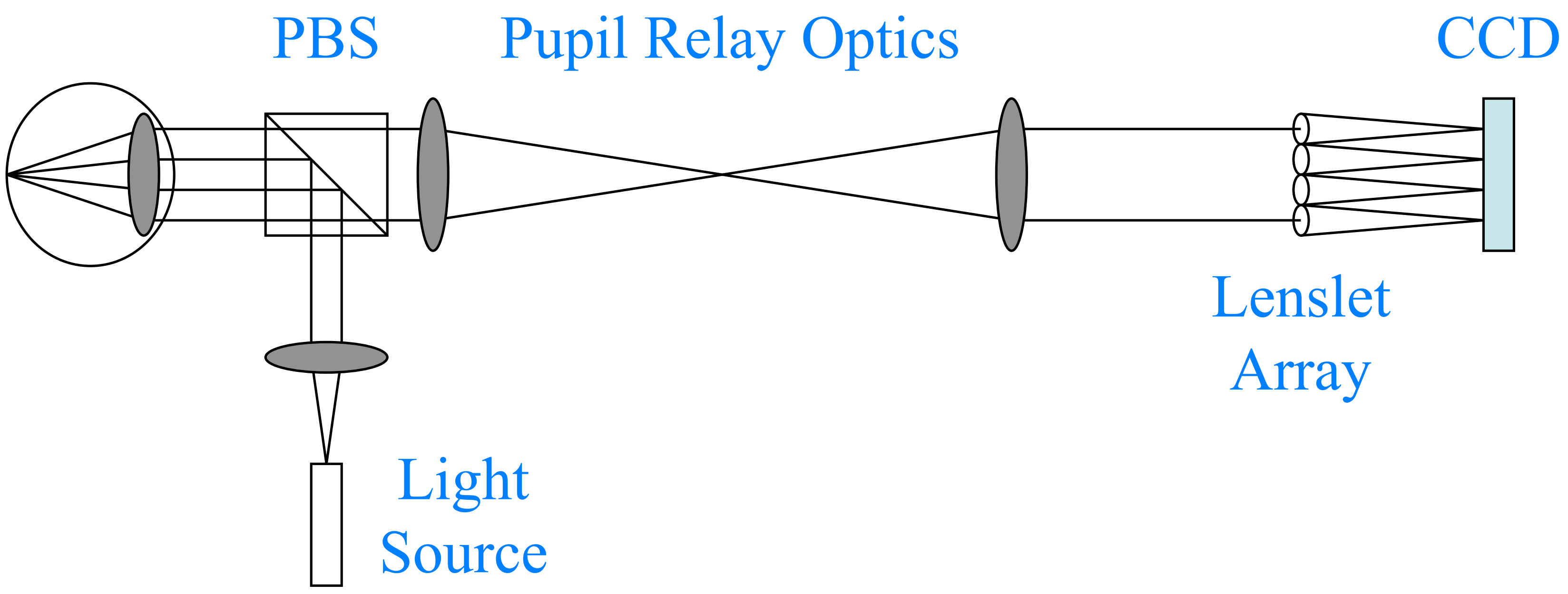
Properties of Zernike Polynomials

- orthogonal
  - terms are not similar in any way, so the weighting of one terms does not depend on whether or not other terms are being fit also
- normalized
  - the RMS wave aberration can be simply calculated as the vector of all or a subset of coefficients
- efficient
  - Zernike shapes are very similar to typical aberrations found in the eye

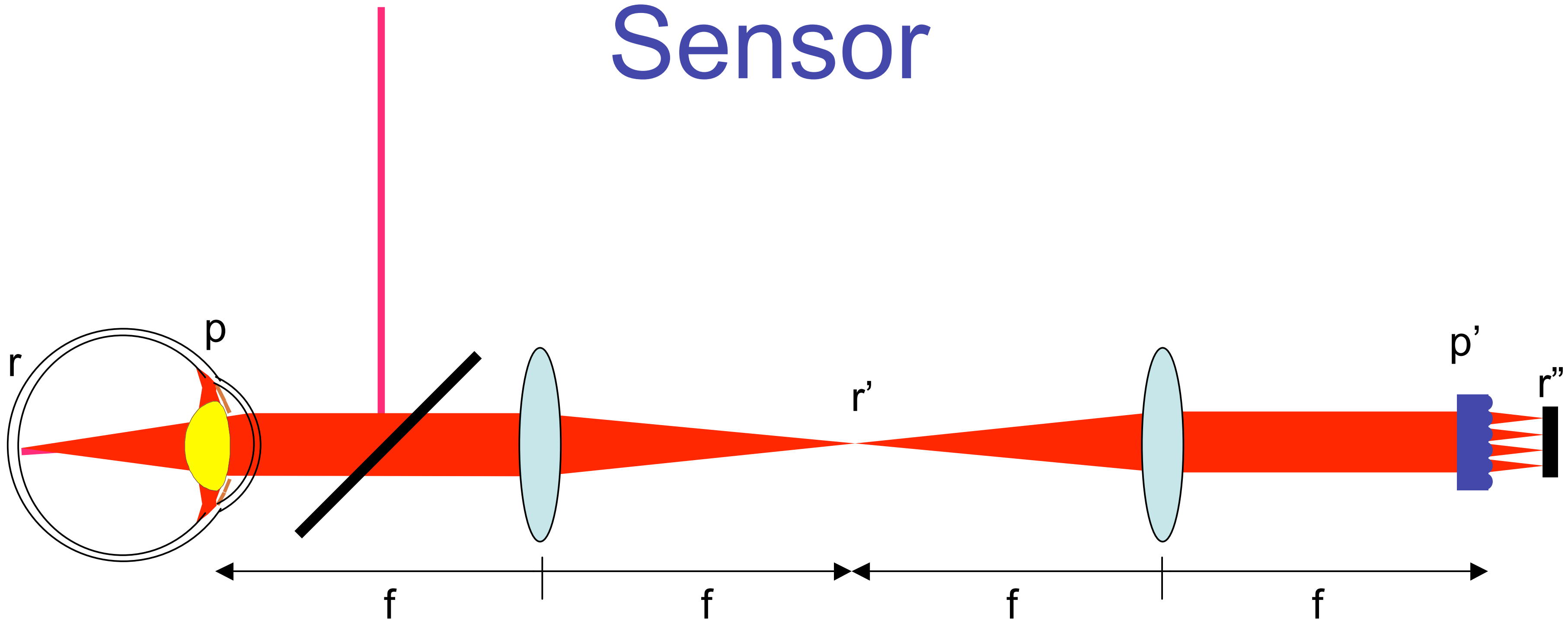
Measurement Setup



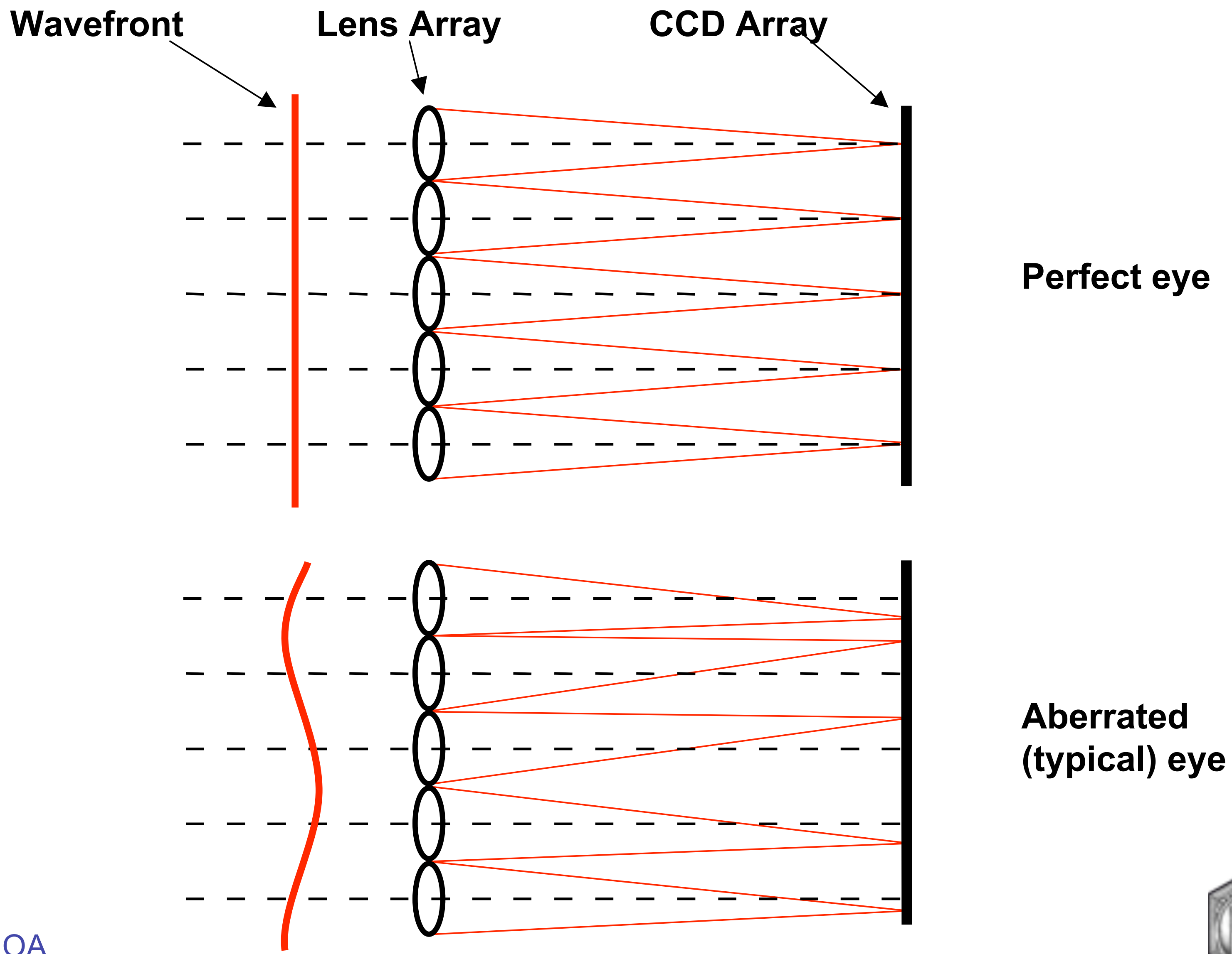
Shack-Hartmann Sensor Layout



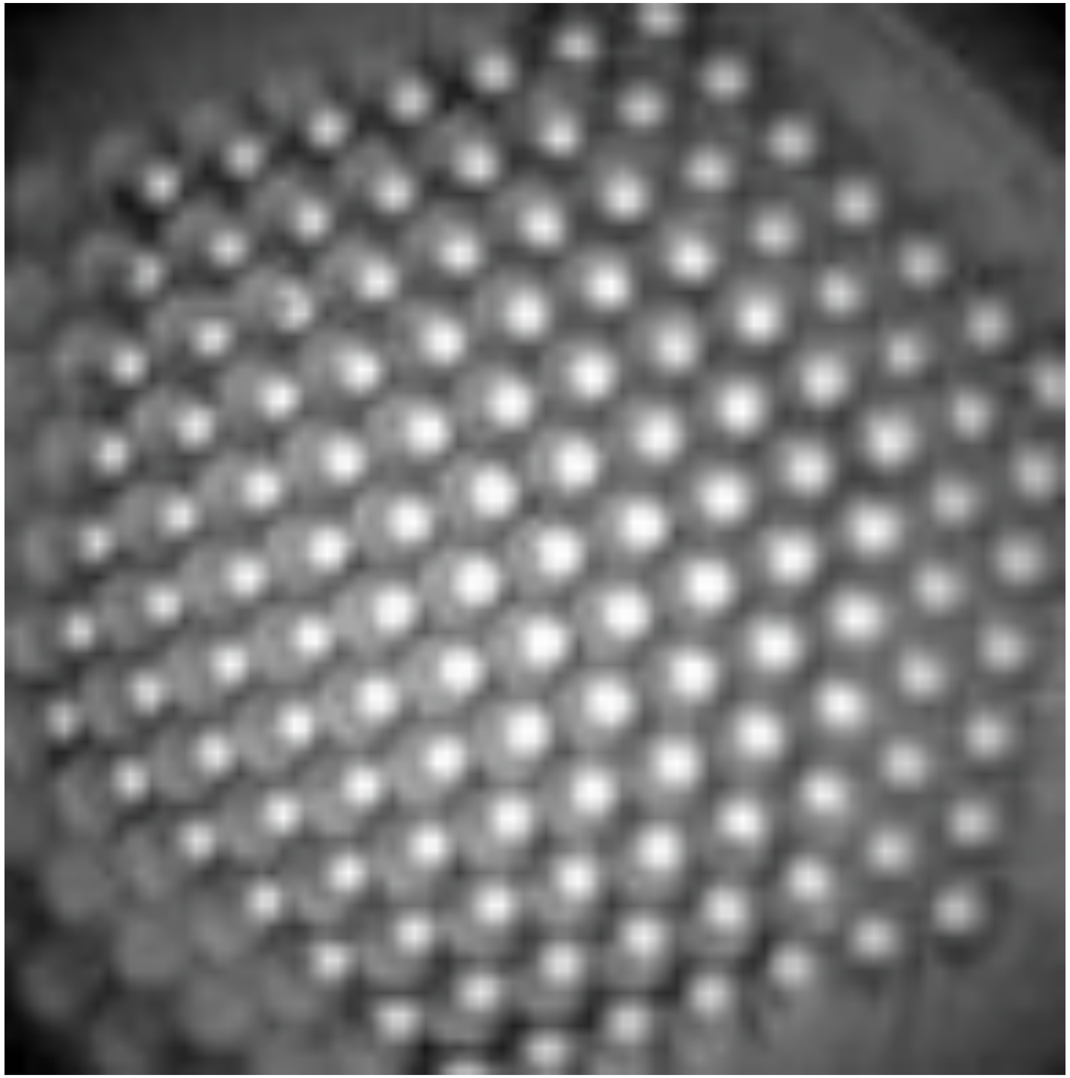
Shack-Hartmann Wavefront Sensor



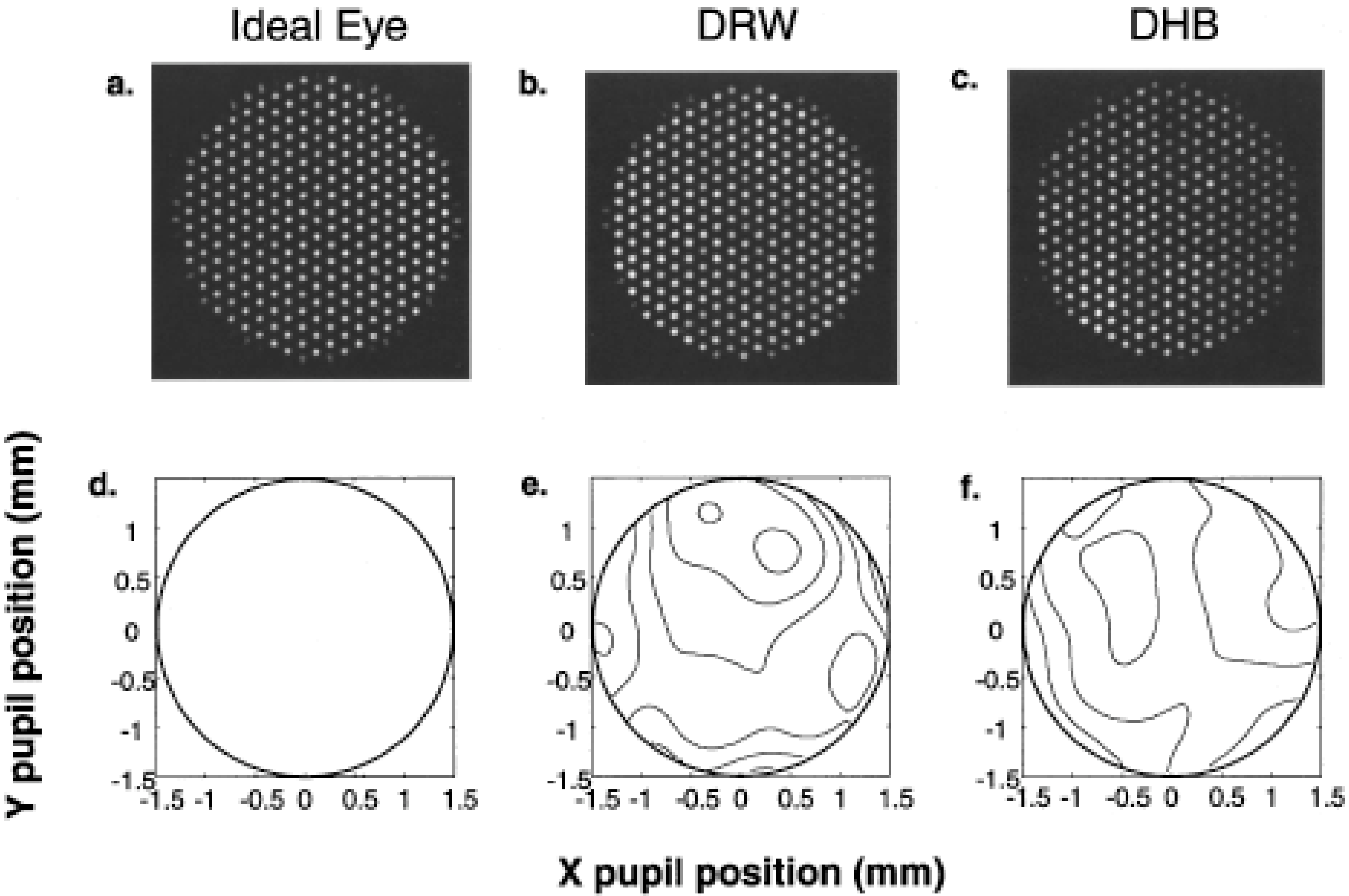
Shack-Hartmann Wavefront Sensor



Lenslet Array



Alcune immagini



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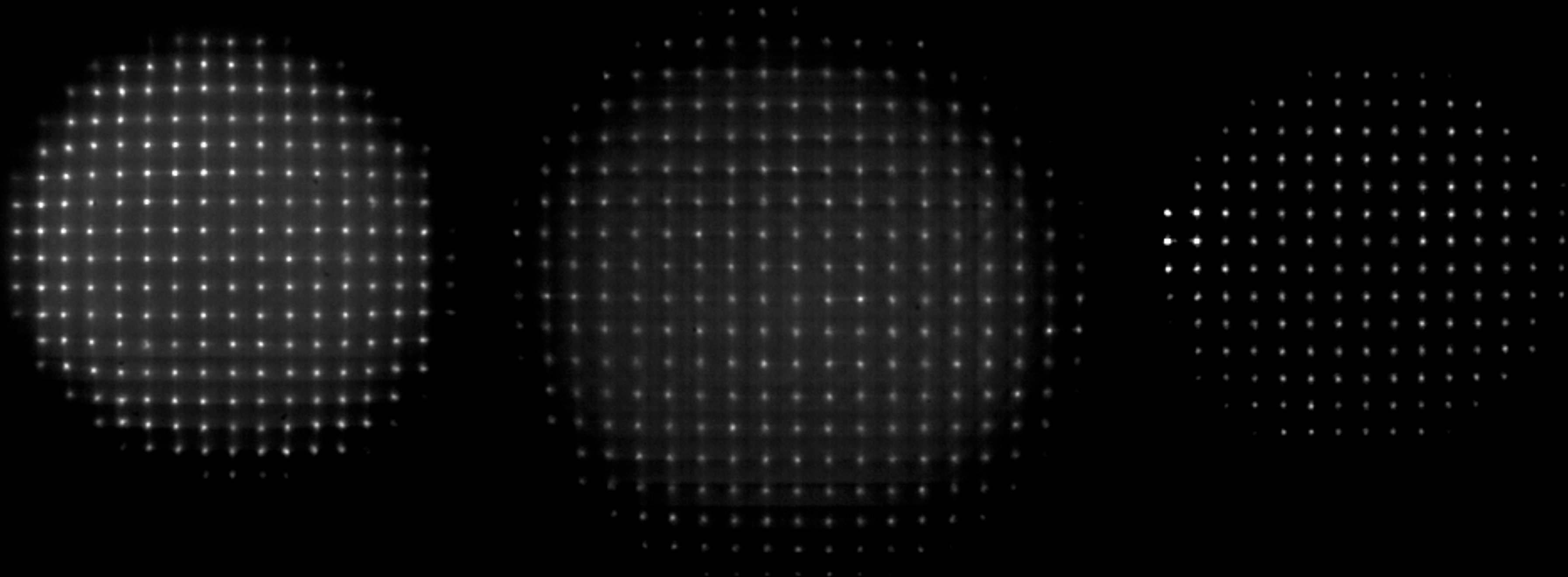
Shack-Hartmann Images

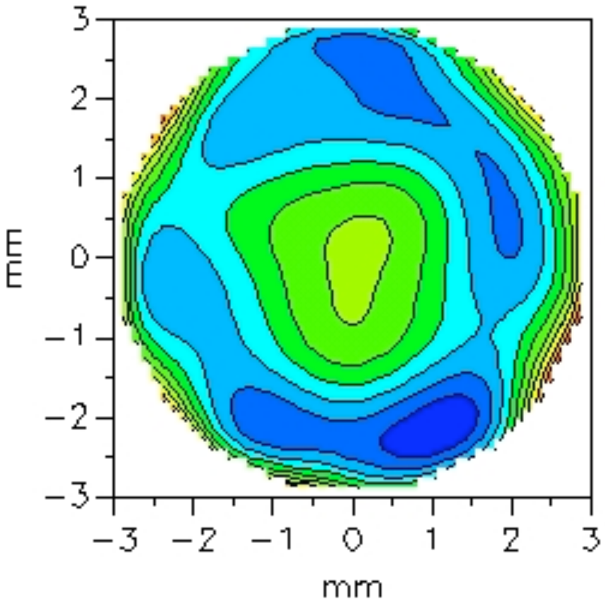
BD **KW** **SM**



Wavefront Maps

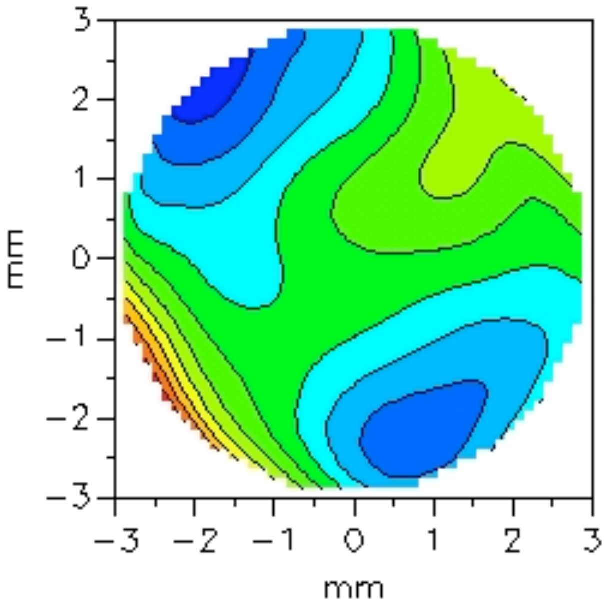
(at best focal plane)

BD



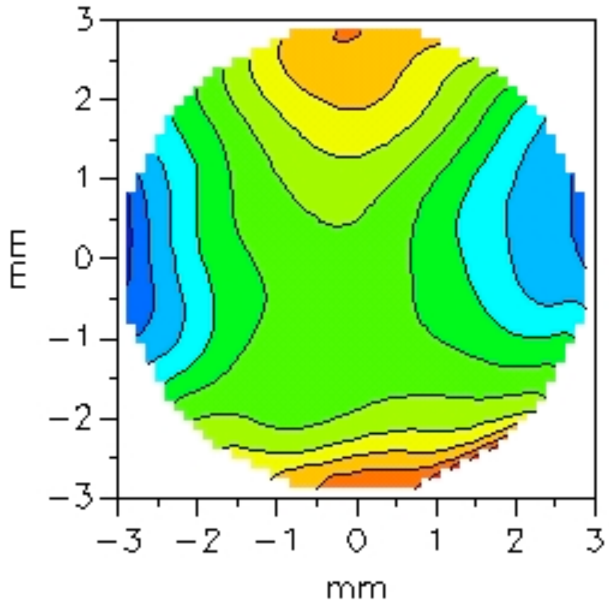
0.33 DS
–0.17 DC X 87

KW



6.42 DS
–0.6 DC X 126

SM



0 DS
–1 DC X 3

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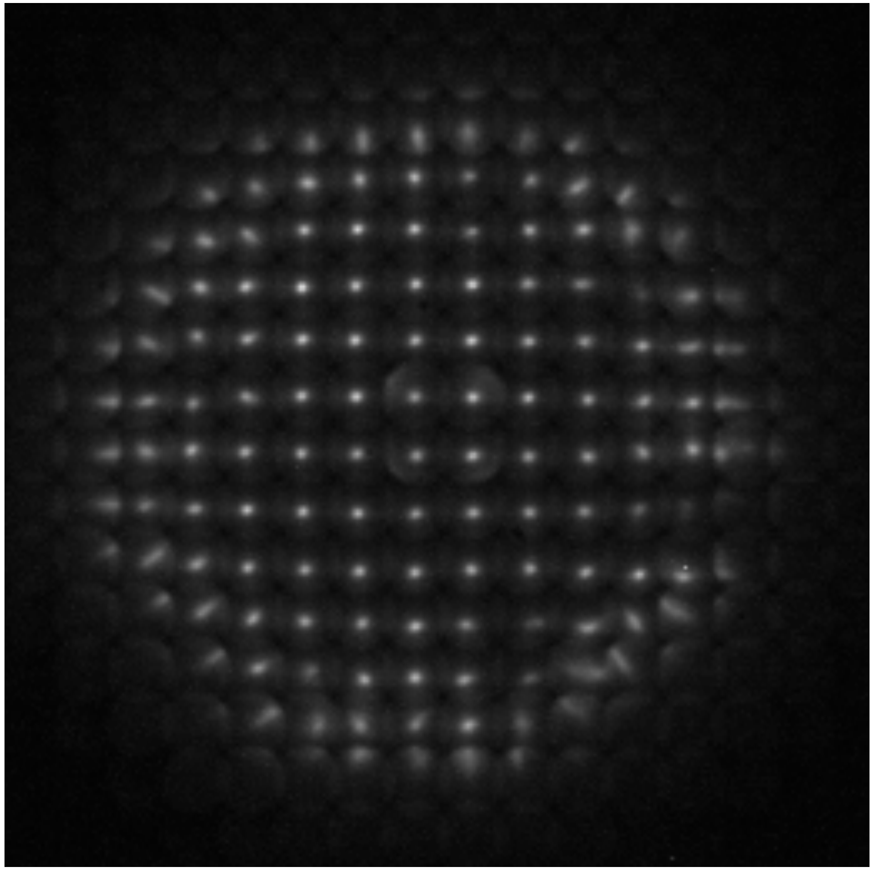
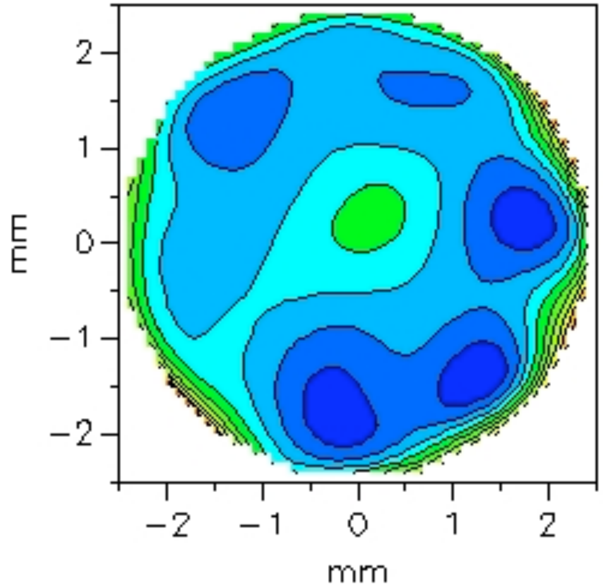
Aberrations of an RK patient

Wavefront sensor image

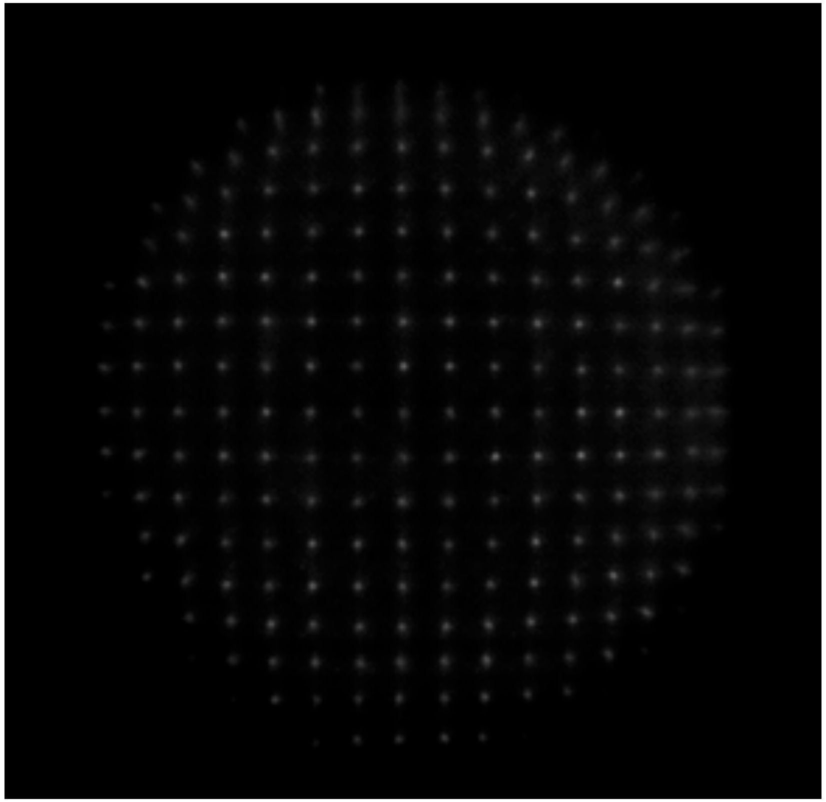


Wavefront aberration

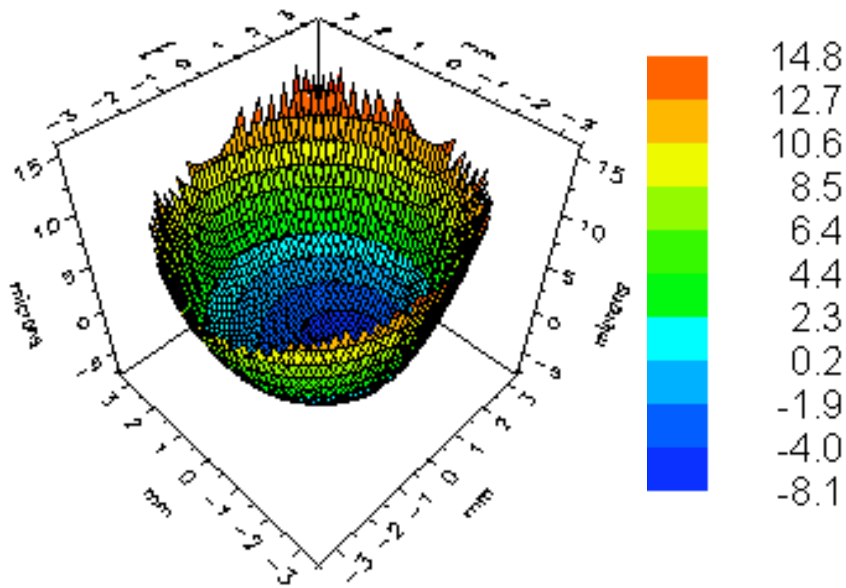


Aberrations of a LASIK patient

Wavefront sensor image



Wavefront aberration



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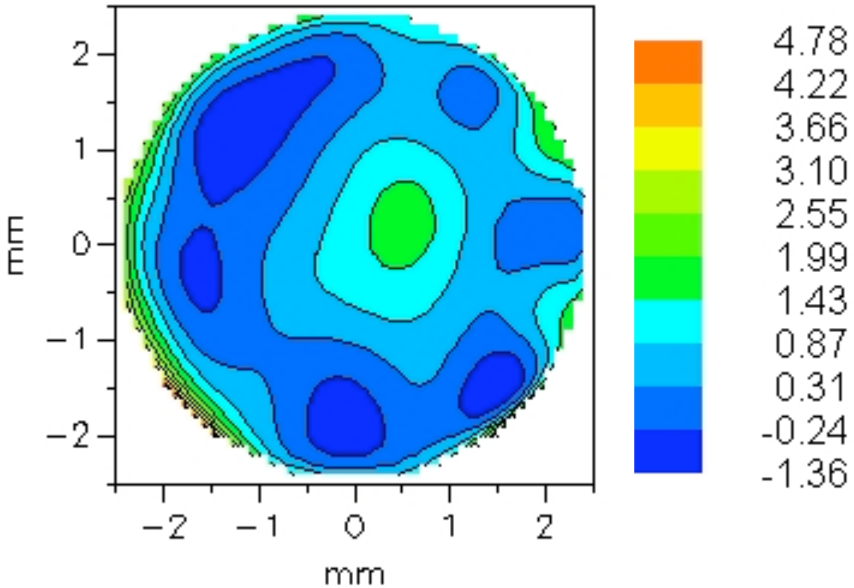
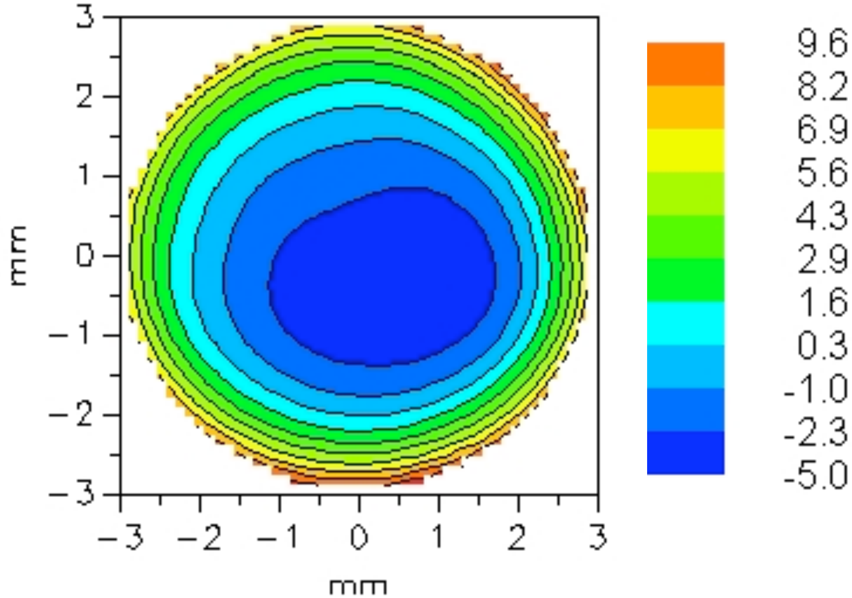
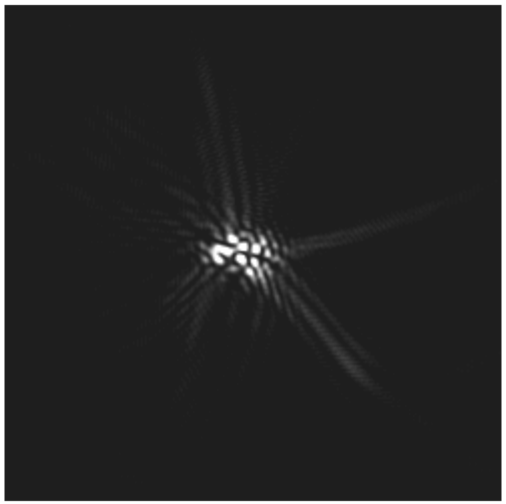
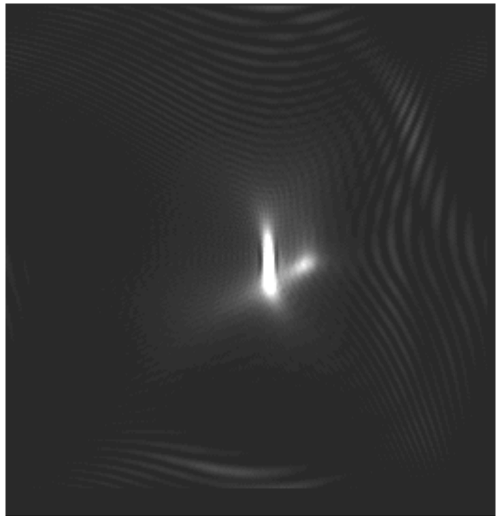
Post - RK

Post - LASIK



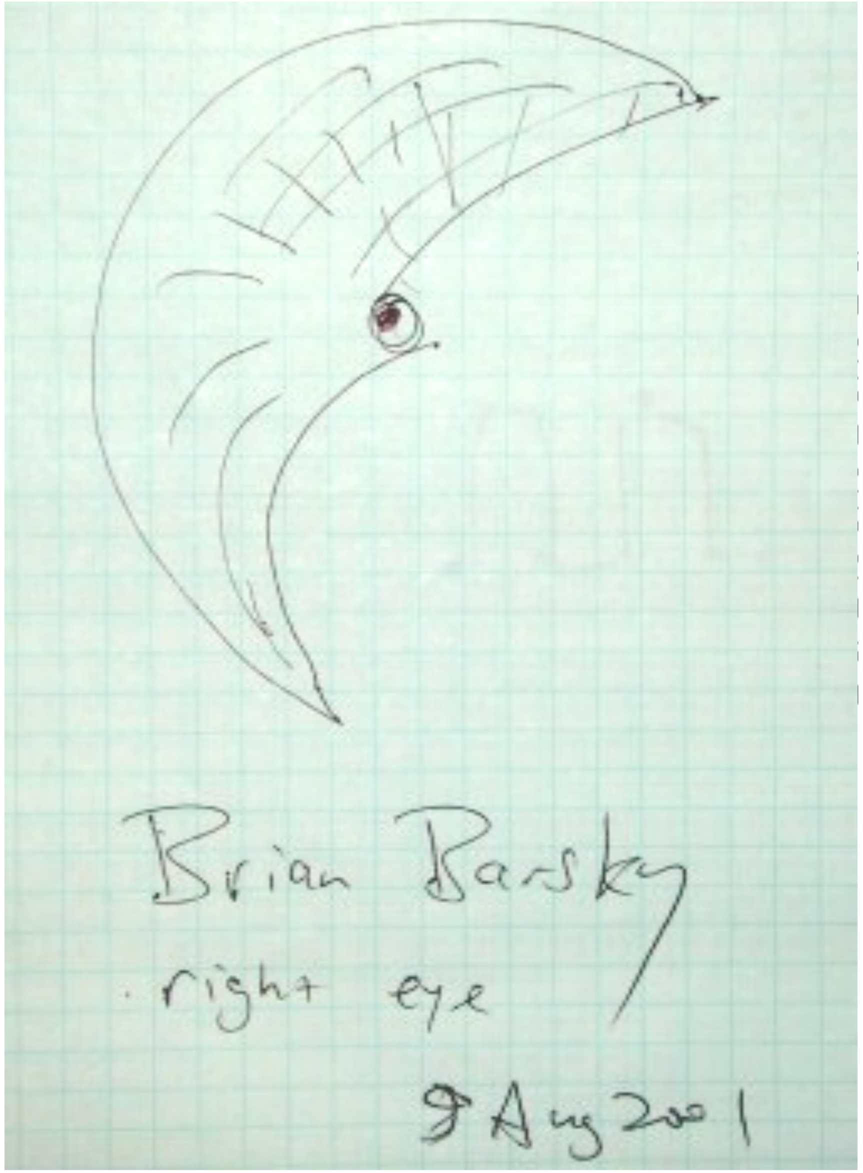


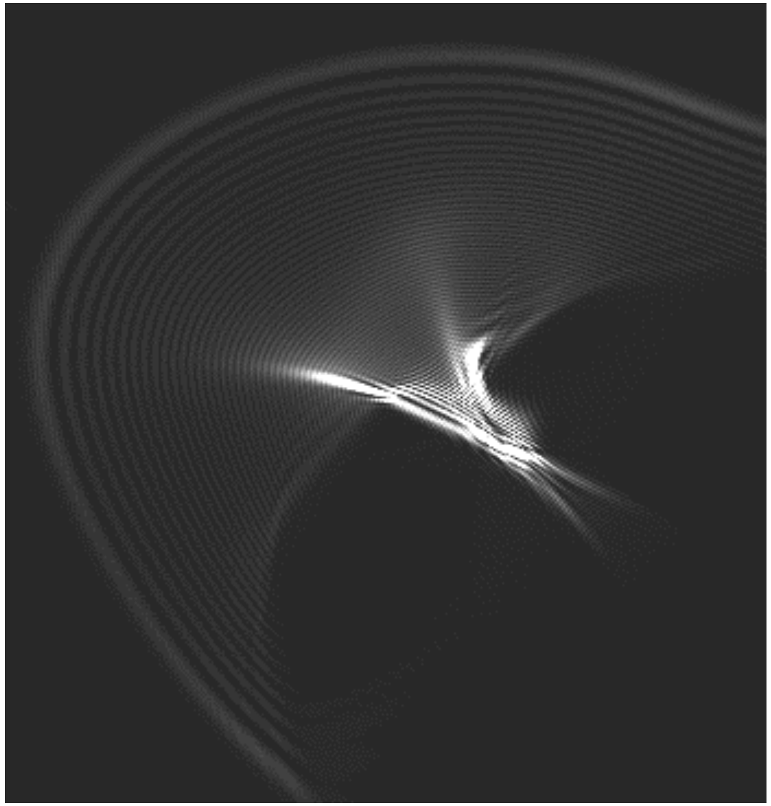




CNR-INOA

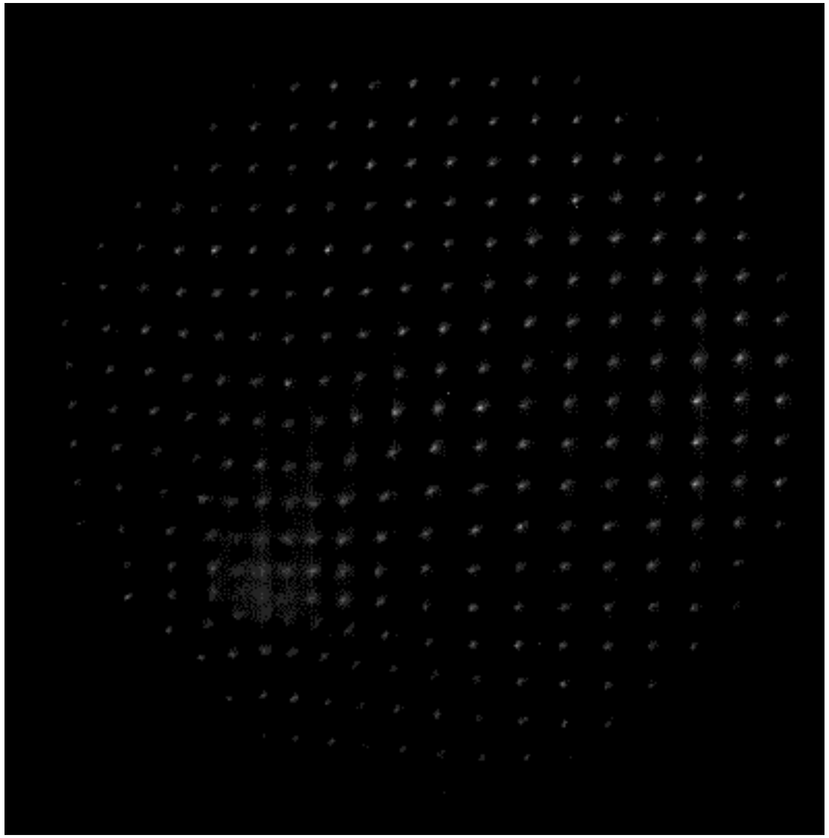
Cheratocono



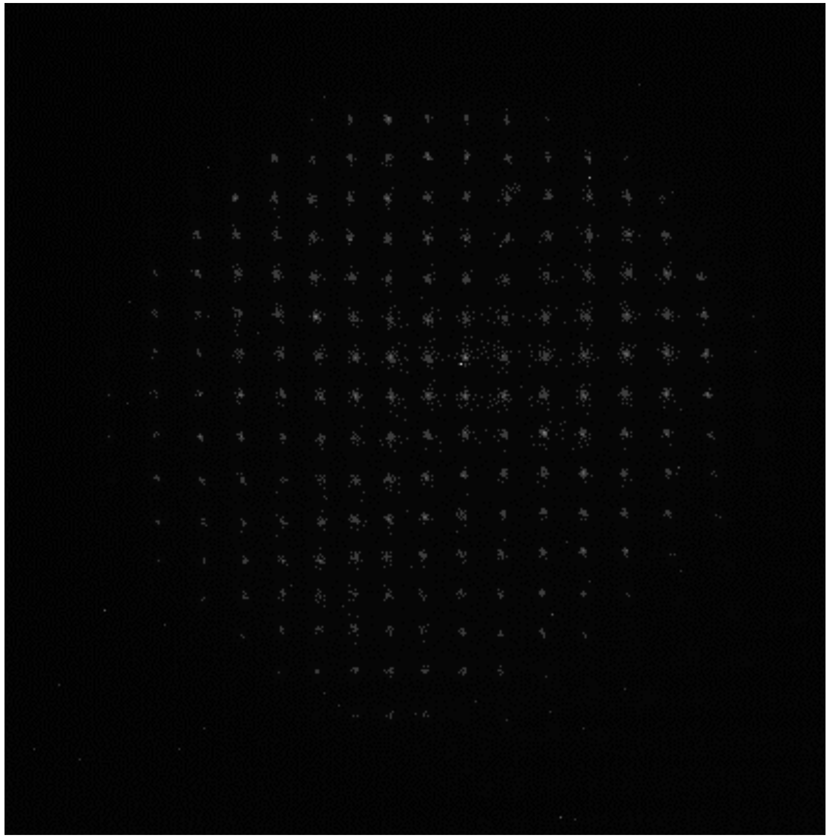


LAC per cheratocono

unaided eye

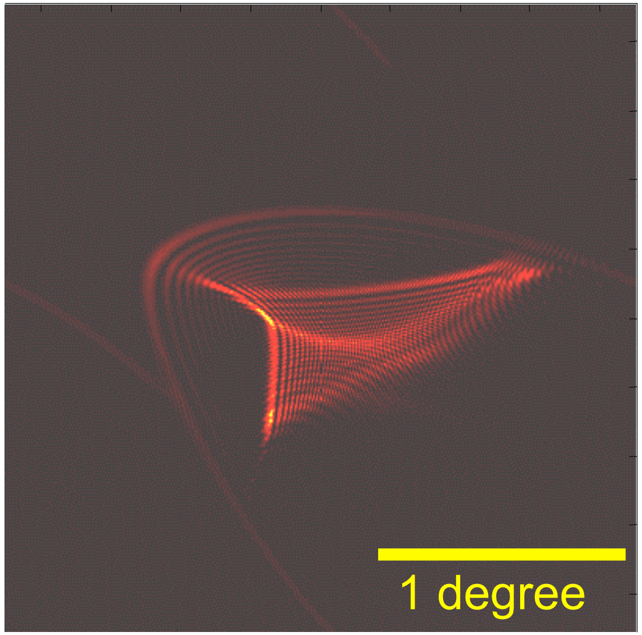


custom contact lens



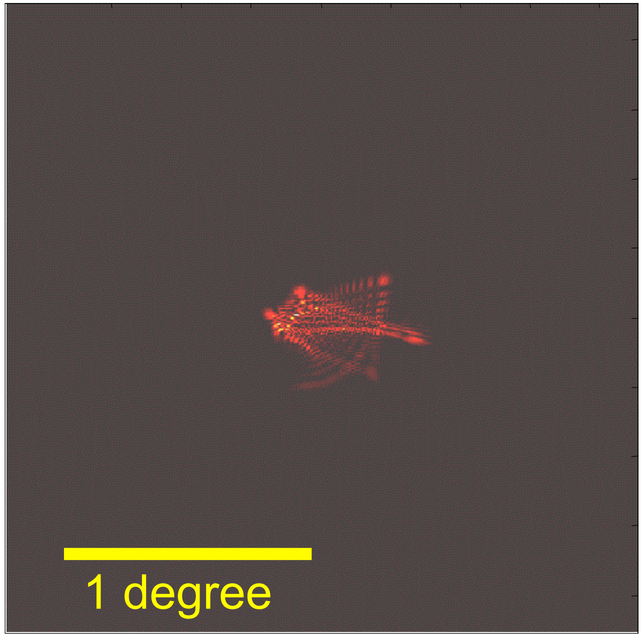
PSF (pupilla 5 mm)

unaided eye



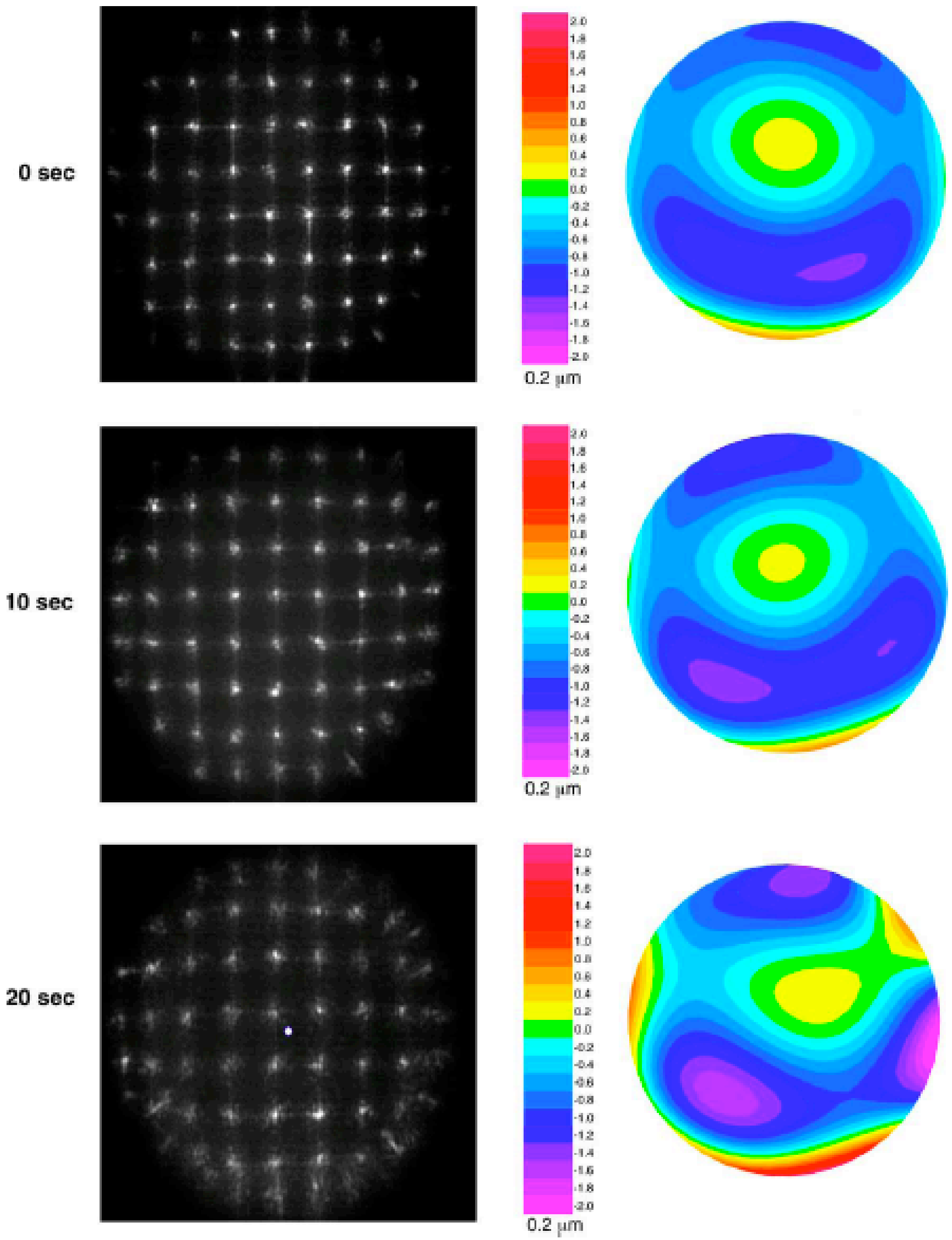
rms = 4.16
strehl ratio = 0.0008

custom contact lens



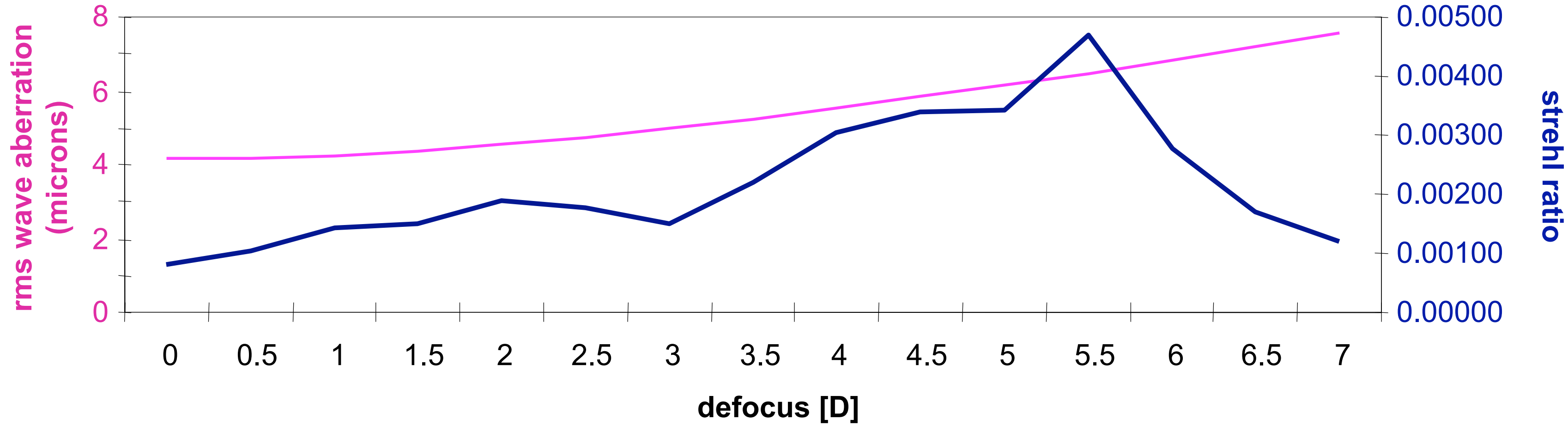
rms = 1.48
strehl ratio = 0.004

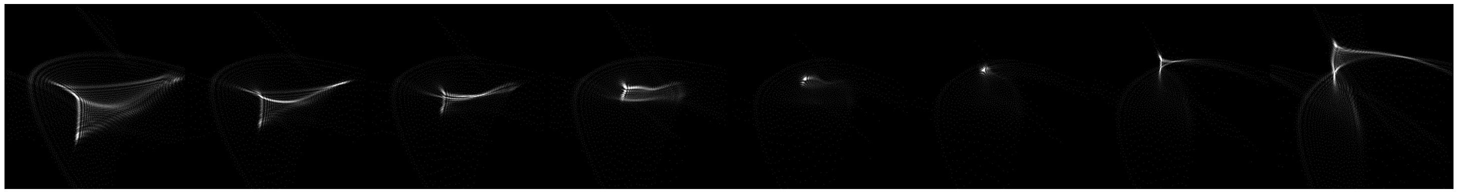
Variazione della mappa corneale al passare del tempo con occhio aperto



W.Charman
Contact Lens & Anterior Eye
28 (2005)
75-92

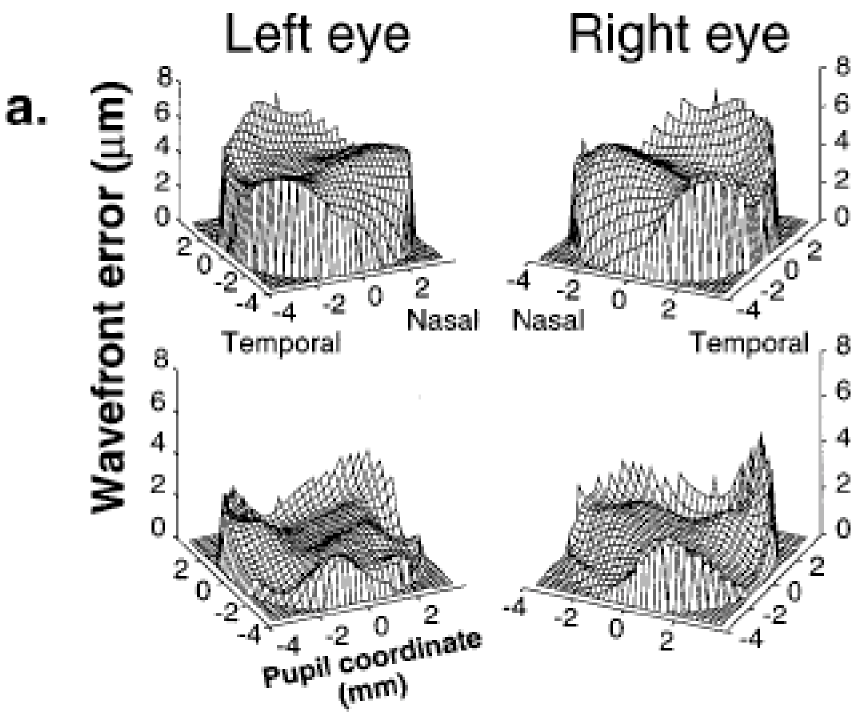
PSF through-focus (5 mm pupil)

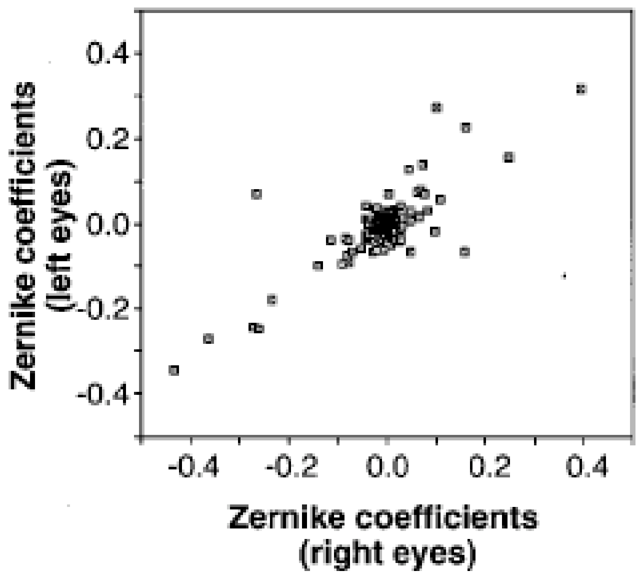




The highest strehl ratio does not correlate with rms when aberrations are high

Simmetria tra i due occhi

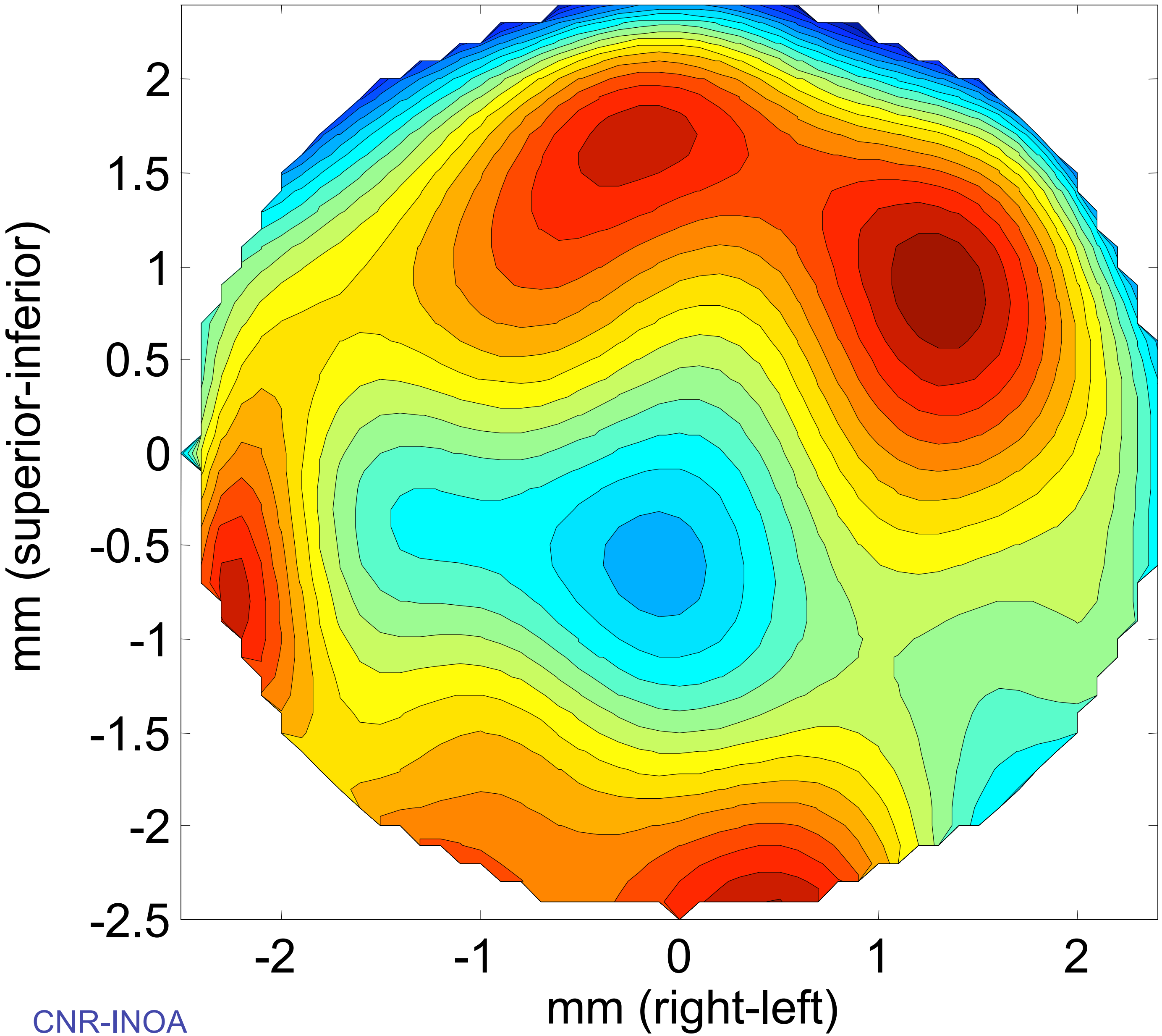




Junzhong Liang and
David R. Williams

Metrics to Define Image Quality

Wave Aberration Contour Map



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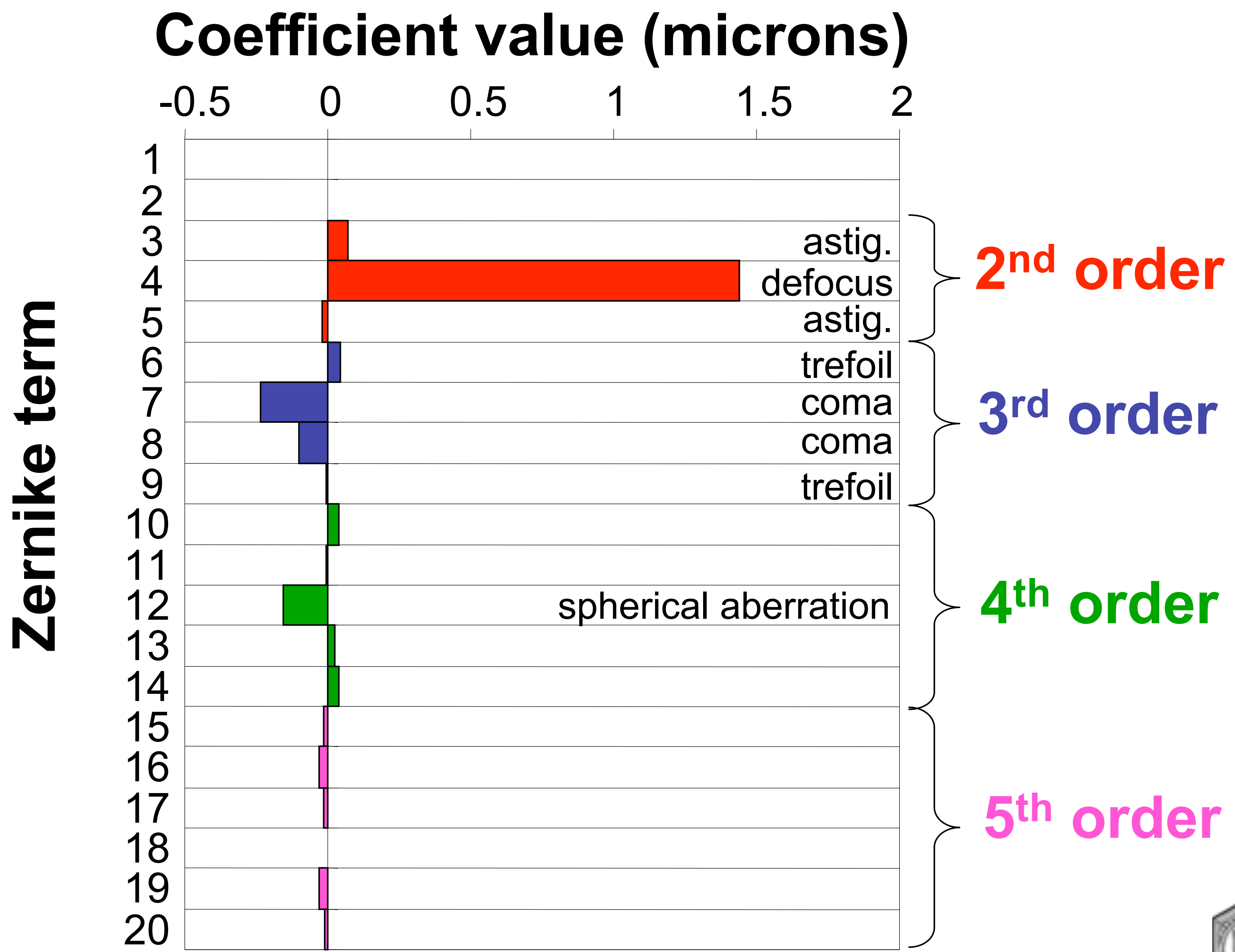
Breakdown of Zernike Terms



CNR-INOA

Root Mean Square

$$RMS = \sqrt{\frac{1}{A}\iint \left(W(x,y) - \overline{W(x,y)} \right)^2 dxdy}$$

$A -$ pupil area

$W(x,y) -$ wave aberration

$\overline{W(x,y)} -$ average wave aberration

Root Mean Square

$$RMS = \sqrt{\left(Z_2^{-2}\right)^2 + \left(Z_2^{0}\right)^2 + \left(Z_2^{2}\right)^2 + \left(Z_3^{-1}\right)^2 \ldots\ldots}$$

astigmatism term

defocus term

astigmatism term

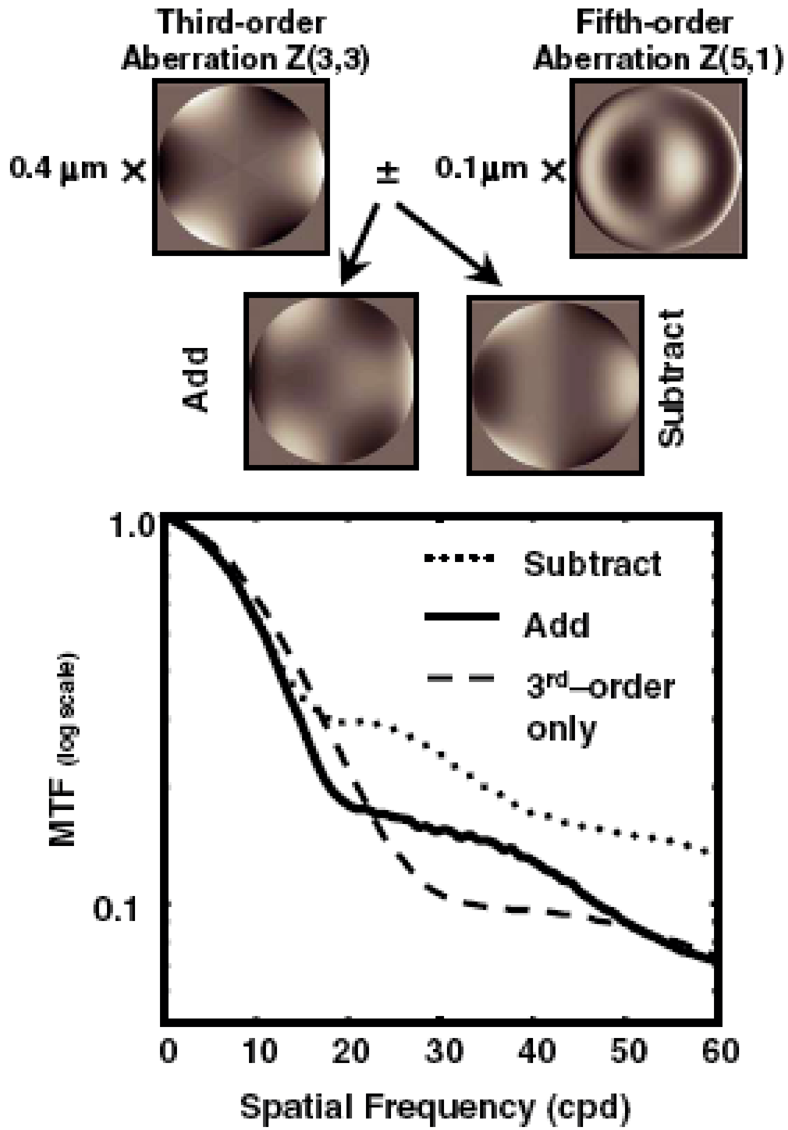
trefoil term

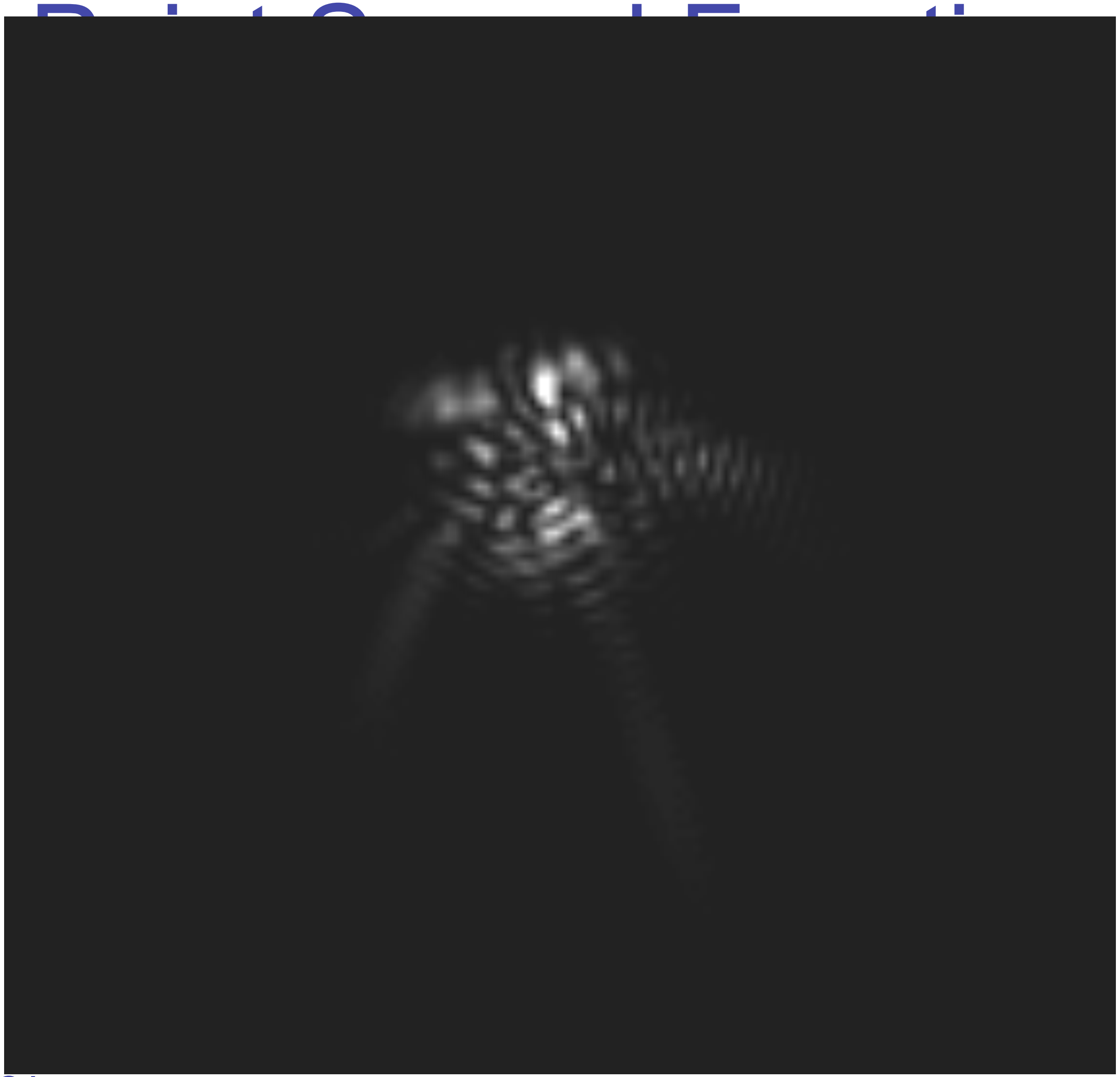
……

Include the terms for which you want to determine their impact (eg defocus and astigmatism only, third order terms or high order terms etc.)

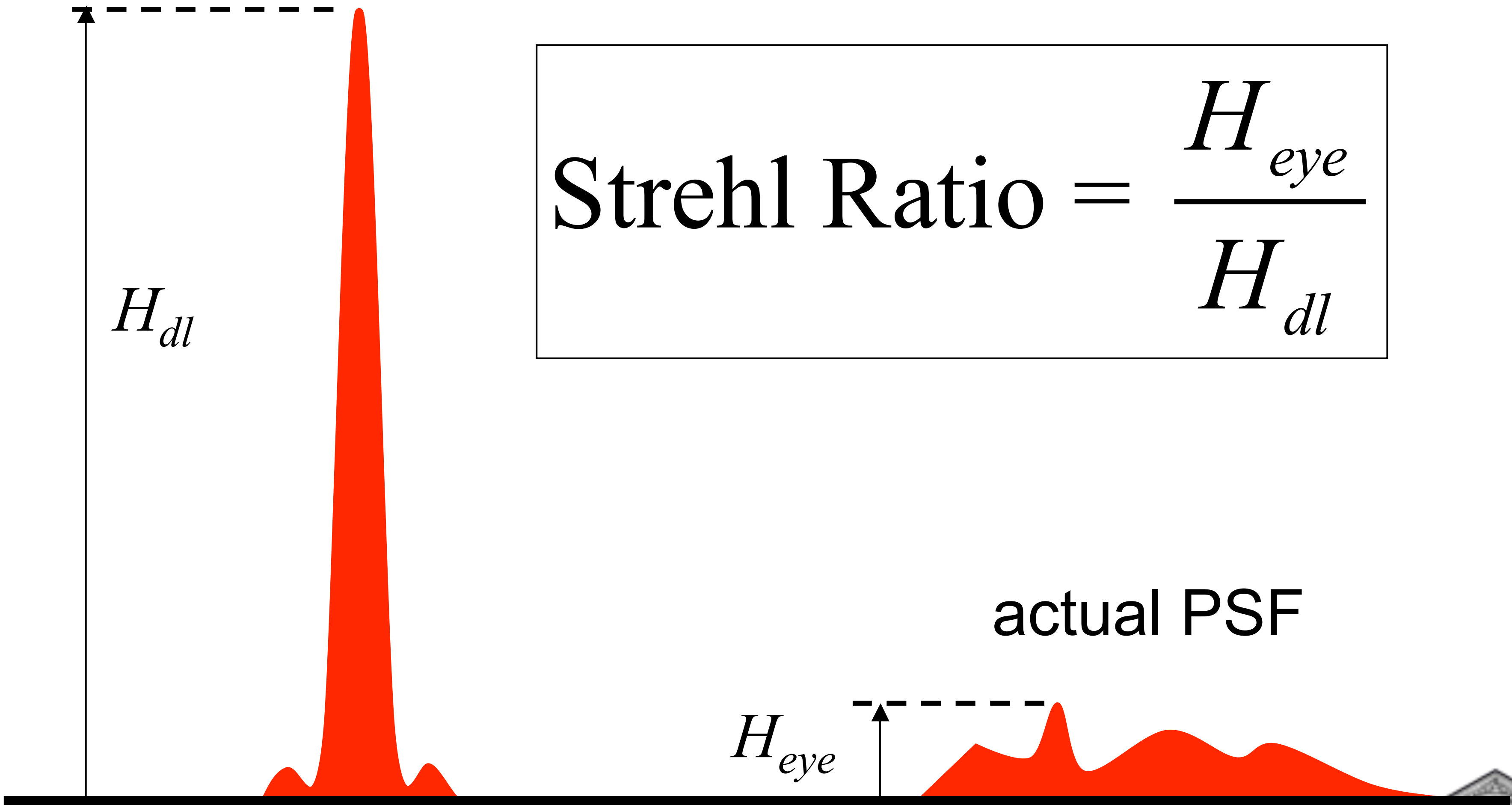
Problemi nel RMS

J.S. McLellan et al.
Vis. Res. 46 (2006)
3009-3016





Strehl Ratio



$$\text{Strehl Ratio} = \frac{H_{eye}}{H_{dl}}$$

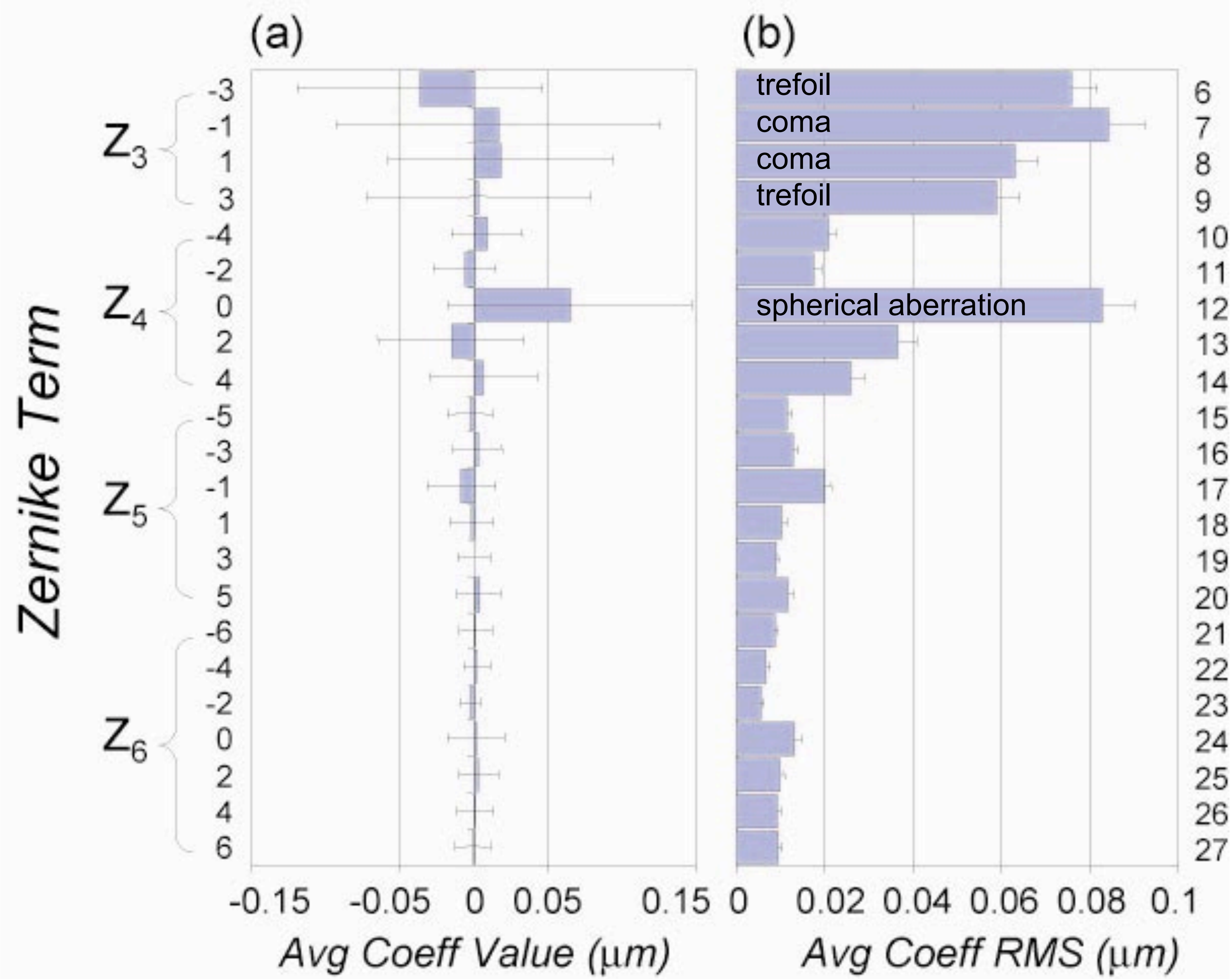
Typical Values for Wave Aberration

Strehl Ratio

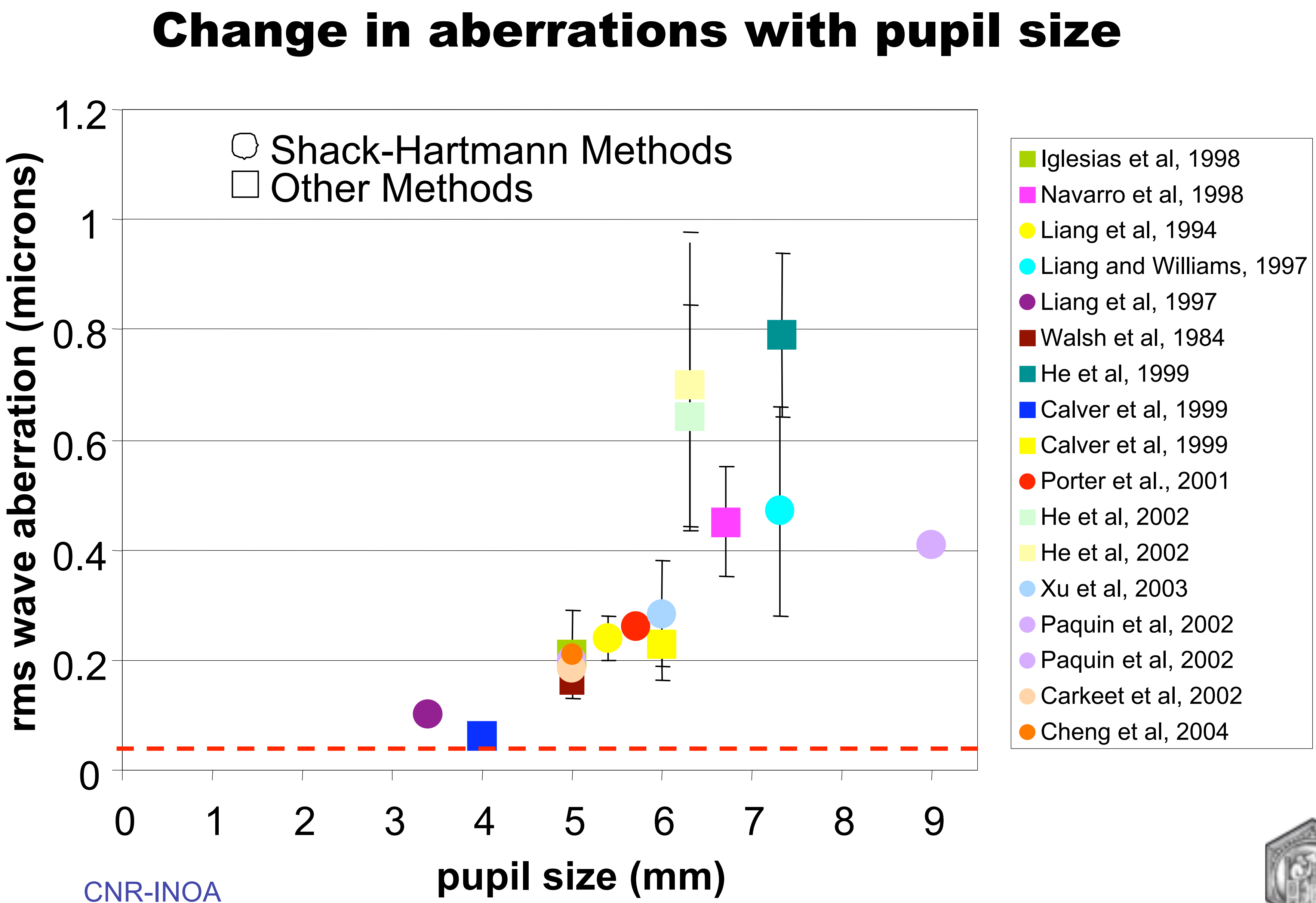
- Strehl ratios are about 5% for a 5 mm pupil that has been corrected for defocus and astigmatism.
- Strehl ratios for small (~ 1 mm) pupils approach 1, but the image quality is poor due to diffraction.

Typical Values for Wave Aberration

Population Statistics

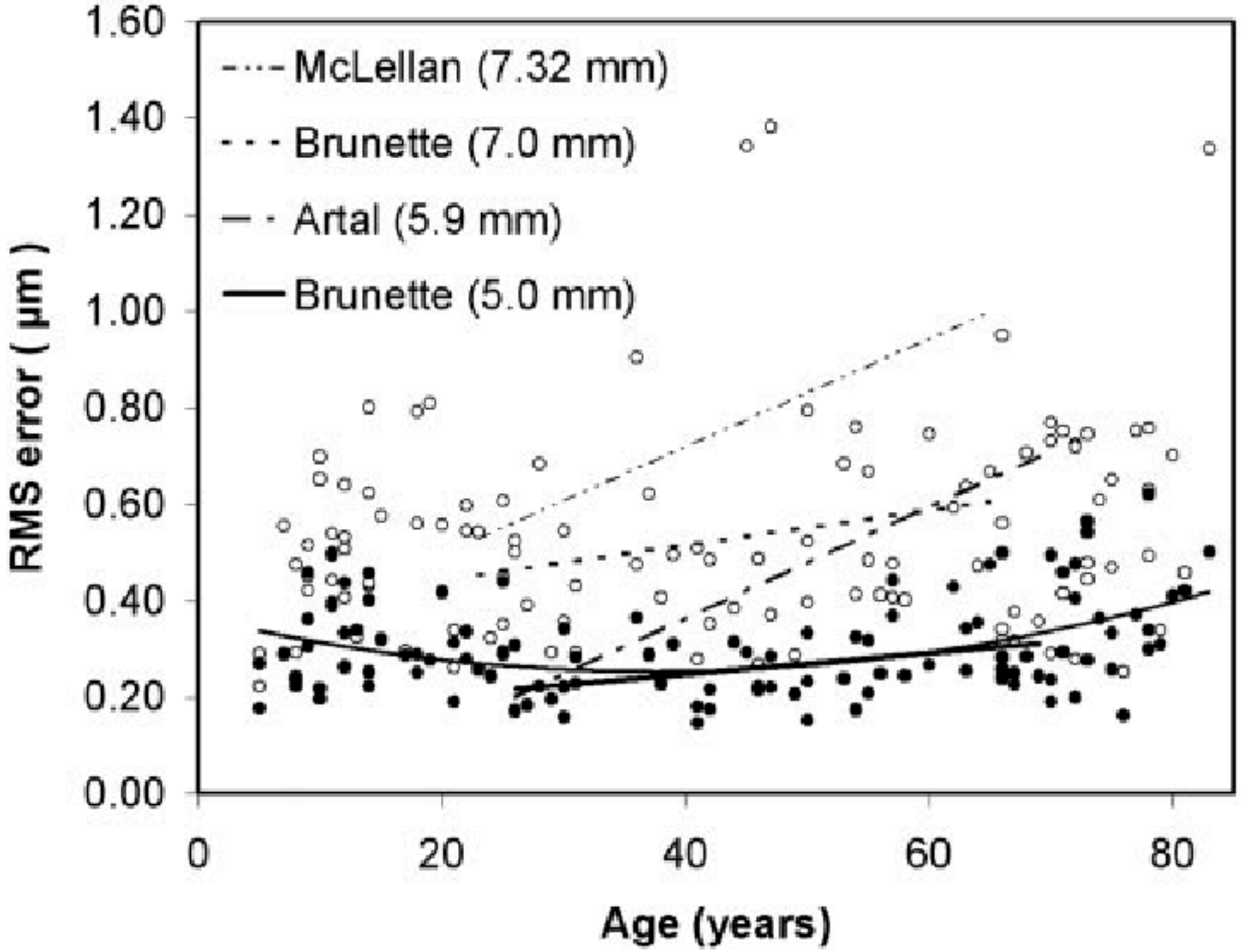


Typical Values for Wave Aberration



Typical Values for Wave Aberration

Change in aberrations with age



Monochromatic Aberrations as a Function of Age, from Childhood to Advanced Age

Isabelle Brunette,[1] Juan M. Bueno,[2] Mireille Parent,[1,3] Habib Hamam,[3] and Pierre Simonet[3]

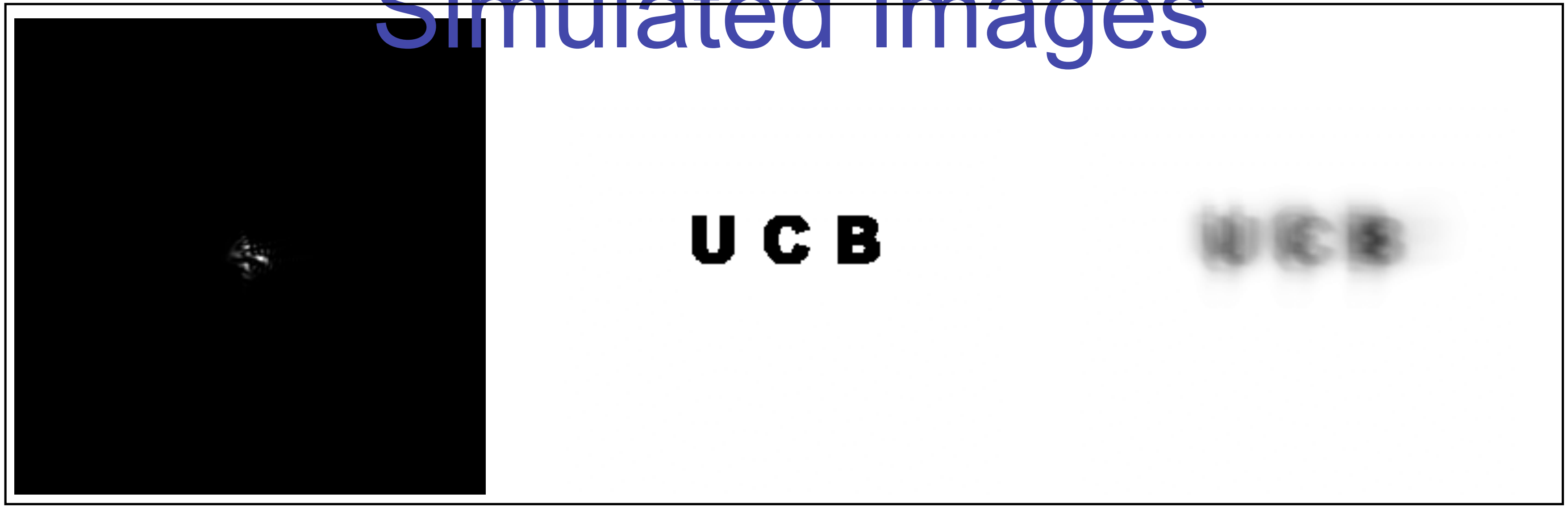
Convolution

Convolution

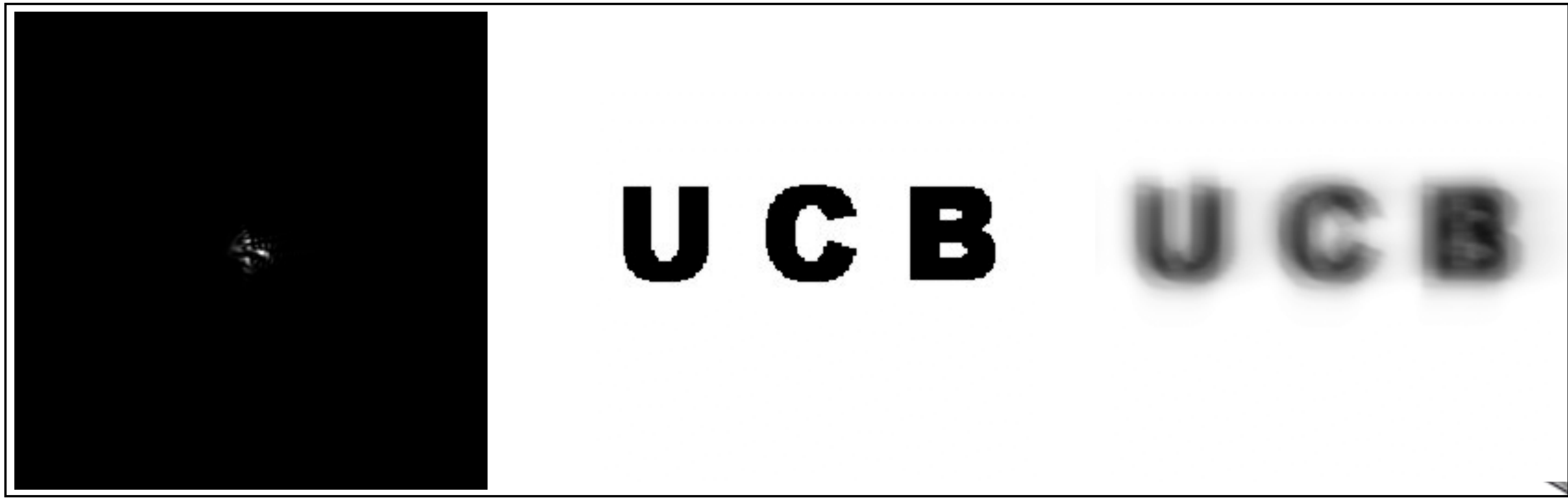
$$PSF(x,y) \otimes O(x,y) = I(x,y)$$

Simulated Images

20/20 letters



20/40 letters



"*I have never experienced any inconvenience from this imperfection, nor did I ever discover it till I made these experiments; and I believe I can examine minute objects with as much accuracy as most of those whose eyes are differently formed*"

Thomas Young (1801) on his own aberrations.

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