EXERCISES

1. Comparing coefficients in the Laurent developments of $\cot \pi z$ and of its expression as a sum of partial fractions, find the values of

$$\sum_1^\infty \frac{1}{n^2}, \qquad \sum_1^\infty \frac{1}{n^4}, \qquad \sum_1^\infty \frac{1}{n^6}.$$

Give a complete justification of the steps that are needed.

2. Express

$$\sum_{-\infty}^{\infty} \frac{1}{z^2 - n^2}$$

in closed form.

3. Use (13) to find the partial fraction development of $1/\cos \pi z$, and show that it leads to $\pi/4 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$.

4. What is the value of

$$\sum_{-\infty}^{\infty} \frac{1}{(z+n)^2 + a^2}?$$

5. Using the same method as in Ex. 1, show that

$$\sum_1^\infty \frac{1}{n^{2k}} = 2^{2k-1} \frac{B_k}{(2k)!} \pi^{2k}.$$

(See Sec. 1.3, Ex. 4, for the definition of B_k.)