

## PART 1: The set of real numbers and its main subsets

- Sets, relations and functions.
- Axioms of real numbers.
- Elementary properties of ordered fields.
- Symmetric sets and functions. Absolute value and distance.
- Natural numbers. Subtraction in  $\mathbb{N}$ ; principle of well-ordering and its consequences.
- Sequences and recursion theorem (optional proof). Recursive definition of sums, products and powers.
- $N^{\text{th}}$  powers, geometric sum and formula for  $a^n - b^n$ . Newton's binomial formula.
- Finite and infinite sets.
- Rational numbers. The rationals are countable. Gauss lemma.
- Least upper bound and greatest lower bound. Elementary consequences of the completeness axiom on integers.
- Roots. Powers with rational exponent.
- Monotone functions.

## PART 2: Theory of limits

- The extended real system  $\mathbb{R}^*$ . Intervals and neighbourhoods.
- Internal, isolated, accumulation points. General definition of limit. Uniqueness of the limit.
- Sign permanence theorem. Comparison theorems.
- Side limits and monotone functions.
- Algebra of finite limits. Extended limit algebra.
- Some notable limits of sequences.
- The number of Nepero.
- Bridge theorem and characterisation of the  $\sup / \inf$  by sequences.
- Continuity: general considerations; theorem of existence of zeros. Intermediate value theorem.
- Classification of discontinuities.
- Limits for compound functions.
- Limits for inverse functions.
- A continuous and strictly monotone function on an interval admits a continuous inverse.
- Logarithms.

- Notable limits (exponential and logarithms).

### PART 3: Series

- Numerical series: Elementary properties of series.  
Comparison criteria.
- Decimal expansions.
- Convergence criteria for series with positive terms.
- Criteria for series with real terms (Abel-Dirichlet, Leibniz).
- Exponential series. Irrationality of  $e$ . Speed of divergence of the harmonic series.
- Properties of trigonometric functions (in particular proof of the cosine addition theorem).
- Periodic functions. Monotonic properties of trigonometric functions.
- Inverse trigonometric functions.