

## Primitive di funzioni elementari

| $f(x)$                                                     | $I$                                                           | $F(x)$ t.c. $F'(x) = f(x), \forall x \in I$ |
|------------------------------------------------------------|---------------------------------------------------------------|---------------------------------------------|
| $x^n, \quad (n \in \mathbb{Z} \setminus \{-1\})$           | $\mathbb{R}_+, \mathbb{R}_-, \mathbb{R}$ (se $n \geq 0$ )     | $\frac{x^{n+1}}{n+1}$                       |
| $x^\alpha, \quad (\alpha \in \mathbb{R} \setminus \{-1\})$ | $\mathbb{R}_+$                                                | $\frac{x^{\alpha+1}}{\alpha+1}$             |
| $e^x$                                                      | $\mathbb{R}$                                                  | $e^x$                                       |
| $a^x$                                                      | $\mathbb{R}$                                                  | $\frac{a^x}{\log a}$                        |
| $\frac{1}{x}$                                              | $\mathbb{R}_+, \mathbb{R}_-$                                  | $\log  x $                                  |
| $\sinh x$                                                  | $\mathbb{R}$                                                  | $\cosh x$                                   |
| $\cosh x$                                                  | $\mathbb{R}$                                                  | $\sinh x$                                   |
| $\frac{1}{\cosh^2 x}$                                      | $\mathbb{R}$                                                  | $\tanh x$                                   |
| $\frac{1}{\sqrt{1+x^2}}$                                   | $\mathbb{R}$                                                  | $\sinh^{-1} x$                              |
| $\frac{1}{\sqrt{x^2-1}}$                                   | $\{x > 1\}$                                                   | $\cosh^{-1} x$                              |
| $\frac{1}{1-x^2}$                                          | $\{ x  < 1\}$                                                 | $\tanh^{-1} x$                              |
| $\sin x$                                                   | $\mathbb{R}$                                                  | $-\cos x$                                   |
| $\cos x$                                                   | $\mathbb{R}$                                                  | $\sin x$                                    |
| $\frac{1}{\cos^2 x}$                                       | $(-\frac{\pi}{2}, \frac{\pi}{2}) + n\pi \ (n \in \mathbb{Z})$ | $\tan x$                                    |
| $-\frac{1}{\sin^2 x}$                                      | $(-\frac{\pi}{2}, \frac{\pi}{2}) + n\pi \ (n \in \mathbb{Z})$ | $\cotan x$                                  |
| $\frac{1}{\sqrt{1-x^2}}$                                   | $\{ x  < 1\}$                                                 | $\operatorname{Arcsen} x$                   |
| $\frac{1}{1+x^2}$                                          | $\mathbb{R}$                                                  | $\operatorname{Arctan} x$                   |

### Alcune formule utili nel calcolo di primitive<sup>1</sup>

$$\sinh^{-1} x = \log(x + \sqrt{x^2 + 1}), \quad \cosh^{-1} x = \log(x + \sqrt{x^2 - 1}) \quad (1)$$

$$\begin{cases} \operatorname{Arccos} x + \operatorname{Arcsen} x = \frac{\pi}{2}, & (\text{rami principali : } \operatorname{Arcsen} 0 = 0, \operatorname{Arccos} 0 = \frac{\pi}{2}) \\ \operatorname{Arccot} x + \operatorname{Arctan} x = \frac{\pi}{2}, & (\text{rami principali : } \operatorname{Arctan} 0 = 0, \operatorname{Arccot} 0 = \frac{\pi}{2}) \end{cases} \quad (2)$$

$$\begin{cases} 2 \operatorname{Arctan}(x + \sqrt{x^2 - 1}) = \pi - \operatorname{Arcsen} \frac{1}{x}, & \forall x \geq 1 \\ \operatorname{Arctan} x = \frac{\pi}{2} - \operatorname{Arctan} \frac{1}{x}, & \forall x > 0 \end{cases} \quad (3)$$

$$t = \tan \frac{x}{2} \implies x = 2 \arctan t, \quad \begin{cases} \sin x = \frac{2t}{1+t^2} \\ \cos x = \frac{1-t^2}{1+t^2} \end{cases}, \quad dx = \frac{2}{1+t^2} dt \quad (4)$$

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<sup>1</sup>Per (1) e (3), si veda [C], rispettivamente, Proposizione 3.11 ed Es 7.6. Le (2) e (4) sono elementari.