LIMITI DI FUNZIONI

$$\lim_{x\to 0} (1+x)^{tgx}$$

$$f(x)^{g(x)} = e^{g(x)\log f(x)}$$

$$\lim_{x\to \frac{\pi}{2}} (1+\underbrace{\cos^2 x}_{\to 0})^{tg^2 x}$$

$$\lim_{x\to 0} \frac{\log(1+2x)}{\sqrt[3]{x^5}}$$

- $$\lim_{x\to 0} (1+x)^{tgx} = \lim_{x\to 0} e^{\overset{\to 0}{tgx}\,\overset{\to 0}{\log(1+x)}} = 1$$

- $y=\cos^2 x$ $$\lim_{x\to \frac{\pi}{2}} (1+\cos^2 x)^{tg^2 x} = \lim_{y\to 0} (1+y)^{\frac{1-y}{y}} \cdots$$

$$tg^2 x = \frac{sen^2 x}{\cos^2 x} = \frac{1-\cos^2 x}{\cos^2 x}$$

$$\cdots = \lim_{y\to 0} e^{\frac{1-y}{\cancel{y}} \overbrace{\frac{\log(1+y)}{y}}^{\to 1} \cdot \cancel{y}} = e$$

per alg. limiti limiti notevoli e continuità di e^x

- $$\lim_{x\to 0} \frac{\log(1+2x)}{x^{5/3}} = \lim_{x\to 0} \underbrace{\frac{\log(1+2x)}{2x}}_{\to 1} \cdot \frac{2\cancel{x}}{x^{5/3-1}} = \lim_{x\to 0} \frac{2}{\sqrt[3]{x^2}} = +\infty$$

$x^{\frac{2}{3}}$

perché $x^2 > 0$

$$\lim_{x\to +\infty} \frac{\log(1+2x)}{x^{5/3}}$$

$$\lim_{x\to 1} \frac{1}{|x-1|} = +\infty$$

$$\lim_{x\to +\infty} \frac{(\sqrt{1+x}-1)(e^x-1)}{x^2(1+2^x)}$$

$$\log\left(2x\left(\frac{1}{2x}+1\right)\right)$$

$$\lim_{x\to+\infty}\frac{\log(1+2x)}{x^{5/3}} = \lim_{x\to+\infty}\frac{\dots}{x^{5/3}} = \lim_{x\to+\infty}\frac{\log 2x}{x^{5/3}} + \frac{\log\left(\frac{1}{2x}+1\right)}{x^{5/3}} \to 0$$

$$= \lim_{x\to+\infty}\left[\frac{\log 2}{x^{5/3}} + \frac{\log x}{x^{5/3}}\right] = 0 \qquad \frac{\log 2}{x^{5/3}} \to 0$$

so $\boxed{\lim_{x\to+\infty}\frac{\log x}{x^b} = 0 \quad b>0}$

$$\log(1+2x) \underset{x\to+\infty}{\sim} \log x \qquad \lim_{x\to+\infty}\frac{\log(1+2x)}{\log x} = 1$$

$b>0$

$$\log x^b = b\log x$$

$$\lim_{x\to+\infty}\frac{(\sqrt{1+x}-1)(e^x-1)}{x^2(1+2^x)} = \lim_{x\to+\infty}\frac{\left(\sqrt{x}\sqrt{\frac{1}{x}+1}-1\right)(e^x-1)}{x^2+x^2 2^x}$$

$$\lim_{x\to+\infty}\frac{e^x\sqrt{x}\left(\sqrt{1+\frac{1}{x}}-\frac{1}{\sqrt{x}}\right)\left(1-\frac{1}{e^x}\right)}{x^2 2^x\left(\frac{1}{2^x}+1\right)}$$

$$\lim_{x\to+\infty}\frac{\left(\frac{e}{2}\right)^x}{x^{2-1/2}}\,\frac{\left(\sqrt{1+\frac{1}{x}}-\frac{1}{\sqrt{x}}\right)\left(1-\frac{1}{e^x}\right)}{\left(\frac{1}{2^x}+1\right)} = +\infty \qquad \left(\sqrt{1+\tfrac{1}{x}} \to 1,\ 1-\tfrac{1}{e^x} \to 1,\ \tfrac{1}{2^x}+1 \to 1\right)$$

$$\lim_{x\to+\infty}\frac{a^x}{x^b} = +\infty \qquad a = e/2 \quad b = 3/2$$

$$\lim_{x\to+\infty}\frac{\log(1+x^2)+x^2}{x^2\log x}$$

$$\lim_{x\to 8}\frac{x-8}{\sqrt[3]{x}-2} = \lim_{x\to 8}\frac{\cancel{(\sqrt[3]{x}-2)}(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}{\cancel{\sqrt[3]{x}-2}} = 4+4+4 = 12$$

$a^3-b^3 = (a-b)(a^2+ab+b^2)$

$$\lim_{x\to\pi}\frac{1-\sin\frac{x}{2}}{\pi-x} = \lim_{\substack{y\to 0\\ y=\pi-x}}\frac{1-\sin\left(\frac{\pi-y}{2}\right)}{y} = \lim_{y\to 0}\frac{1-\sin\left(\frac{\pi}{2}-\frac{y}{2}\right)}{y} = \lim_{y\to 0}\frac{1-\cos\frac{y}{2}}{\left(\frac{y}{2}\right)^2}\cdot\frac{y}{4} = 0 \qquad \frac{1-\cos\frac{y}{2}}{\left(\frac{y}{2}\right)^2} \to \frac{1}{2}$$

$$\lim_{x\to+\infty}\left(\frac{\log(1+x^2)}{x^2\log x}+\frac{1}{\log x}\right) = \lim_{x\to+\infty}\frac{2\log x+\log\left(\frac{1}{x^2}+1\right)}{x^2\log x}+\frac{1}{\log x}$$

$$\lim \dots \quad 2 \quad \log\left(\frac{1}{x^2}+1\right) \dots$$

$$\lim_{x \to +\infty} \frac{\quad}{x^2} + \frac{\quad}{x^2 \log x} + \frac{1}{\log x} = 0$$

$\circledast$

$$\frac{1-\cos\frac{y}{2}}{y} = \frac{1}{\cancel{y}} \cdot \underbrace{\frac{1-\cos\boxed{\frac{y}{2}}}{\left(\frac{y}{2}\right)^2}}_{\frac{1}{2}} \cdot \left(\frac{y}{2^2}\right)^{\cancel{2}} \xrightarrow[y\to 0]{} 0$$

$$\frac{1-\cos x}{x^2} \to \frac{1}{2}$$