Eserc 18/10

lunedì 18 ottobre 2021 12:22

SUP E INF

TROVARE ESTREMO SUPERIORE ED INFERIORE DIRE SE SI ~~TRATTA~~ DI MASSIMO O MINIMO.

1. $\{x \in \mathbb{R} : x^3 < 8\}$
2. $\{x \in \mathbb{R} : x+1 \geq 3x-2\}$
3. $\{x \in \mathbb{R} : x+5 \leq 3x+4\}$
4. $\{x \in \mathbb{R} : x^2 \leq 2\}$
5. $(0,1] \cup [2,3]$
6. $(-\infty, 4] \cap (1,5)$
7. $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$
8. $\left\{\frac{n-1}{n} : n \in \mathbb{N}\right\}$

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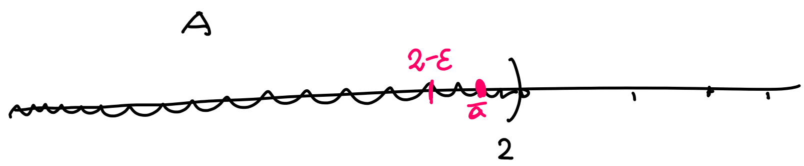
1. $\{x \in \mathbb{R} : x^3 < 8\} = (-\infty, 2) = A$

$\inf A = -\infty$ $\qquad$ $\sup A = 2$

non è massimo.



$$\bar{s} = \sup A = \begin{cases} s \geq a & \forall a \in A \\ \forall \varepsilon > 0 \ \exists \bar{a} & \bar{s} - \varepsilon < \bar{a} \end{cases}$$

2. $\{x \in \mathbb{R} : x+1 \geq 3x-2\} = \left(-\infty, \frac{3}{2}\right] = B$

$3 \geq 2x$

$\frac{3}{2} \geq x$

$\inf B = -\infty$

$\sup B = \frac{3}{2} = \max B$

3. $\{x \in \mathbb{R} : x+5 \leq 3x+4\} = \left[\frac{1}{2}, +\infty\right) = C$

$1 \leq 2x$

$\frac{1}{2} \leq x$

$\min C = \frac{1}{2} = \inf C$

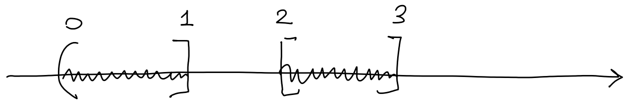
$\sup C = +\infty$

4. $\{x \in \mathbb{Q} : x^2 \leq 2\} = \mathbb{Q} \cap [-\sqrt{2}, \sqrt{2}] = \mathbb{Q} \cap (-\sqrt{2}, \sqrt{2}) = D$

$\inf D = -\sqrt{2} \qquad \sup D = \sqrt{2}$

$\frac{m}{n} \in \mathbb{Q} \qquad m \in \mathbb{Z} \quad n \in \mathbb{N}$

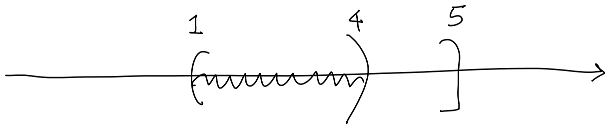
5. $E = (0, 1] \cup [2, 3]$



$\sup E = 3 = \max E$

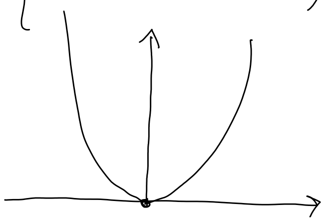
$\inf E = 0$ non è minimo

6. $(-\infty, 4] \cap (1, 5) = (1, 4] = F$



$\inf F = 1 \qquad \sup F = \max F = 4$

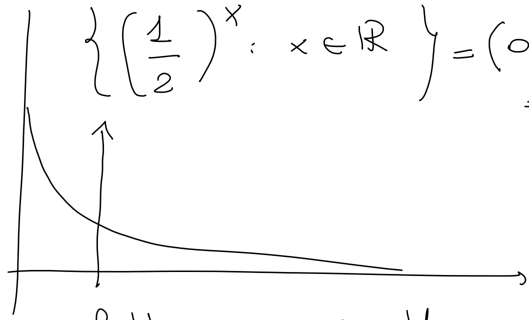
$\{x^2 : x \in \mathbb{R}\} = [0, +\infty) = G$



$\min G = 0$

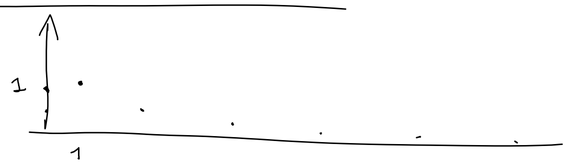
$\sup G = +\infty$

$\left\{\left(\frac{1}{2}\right)^x : x \in \mathbb{R}\right\} = (0, +\infty) = H$



$\inf H = 0 \qquad \sup H = +\infty$

$\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$



$0 < \frac{1}{n} \leq 1$

$\frac{1}{n} > \frac{1}{n+1} \quad \forall n \in \mathbb{N}$

decrescente

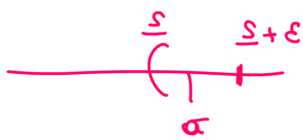
1 è maggiorante

1 è massimo $=$ sup.

0 è minorante ma non è minimo

0 è il massimo dei minoranti.

$\underline{s} = \inf A = \begin{cases} \underline{s} \leq a & \forall a \in A \\ \forall \varepsilon > 0 & \bar{a} < \underline{s} + \varepsilon \end{cases}$



$\underline{s} = 0$ $\begin{cases} 0 < \frac{1}{n} \\ \forall \varepsilon > 0 \ \exists \bar{n} : \frac{1}{\bar{n}} < \varepsilon \end{cases}$ vera perché $\lim_{n \to \infty} \frac{1}{n} = 0$

$\frac{1}{n} < \varepsilon$ defin.

$$L = \left\{ \frac{n-1}{n} : n \in \mathbb{N} \right\}$$

$$\frac{n-1}{n} = 1 - \frac{1}{n}$$

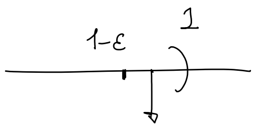
so che

$0 < \frac{1}{n} \leq 1 \Rightarrow -1 \leq -\frac{1}{n} < 0 \Rightarrow$

$0 \leq 1 - \frac{1}{n} < 1$

se $n = 1$ $\frac{n-1}{n} = 0$ $\min L = \inf L = 0$

1 è maggiorante. È il sup? sì



se provo che $\forall \varepsilon > 0$ $1 - \varepsilon < 1 - \frac{1}{n}$

$+\varepsilon > \frac{1}{n}$ questa è defin. vera. quindi 1 è il sup

$$A: \left\{ \frac{2n}{n^2+1} : n \in \mathbb{Z} \right\}$$

$$C := \left\{ \frac{(-1)^n}{n} \quad n \in \mathbb{N} \right\}$$

$$G := \left\{ \sin\left(\frac{\pi}{2} \cdot \frac{n^4}{n^4+1} \right) : n \in \mathbb{N} \right\}$$

$n \in \mathbb{Z}$

$\frac{2n}{2}$. $(n+1)^2 = n^2 + 1 + 2n \geq 0$

$n+1$

$$1 = \frac{n^2+1}{n^2+1} \geq -\frac{2n}{n^2+1}$$

$$(n-1)^2 = n^2+1-2n \geq 0$$

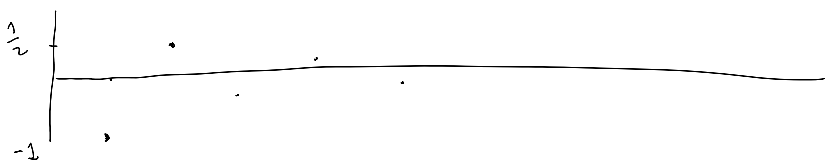
$$1 = \frac{n^2+1}{n^2+1} \geq \frac{2n}{n^2+1}$$

$$-1 \leq \frac{2n}{n^2+1} \leq 1$$

se $n=-1$ $\quad \frac{-2}{2} = -1$ | se $n=1$ $\quad \frac{2}{2} = 1$

$\min A = -1 = \inf A \qquad \max A = 1 = \sup A$

$$C = \left\{ \frac{(-1)^n}{n} ; n \in \mathbb{N} \right\}$$



se n è pari

$$0 < \frac{(-1)^n}{n} = \frac{1}{n} \leq \frac{1}{2}$$

decresc.

se n è dispari

$$-1 \leq \frac{(-1)^n}{n} = \frac{-1}{n} < 0$$

è crescente

$$\sup C = \max C = \frac{1}{2}$$

$$\inf C = \min C = -1$$

$$G = \left\{ \sin\left(\frac{\pi}{2} \frac{n^4}{n^4+1} \right) . \ n \in \mathbb{N} \right\}$$

$\sin\left(\frac{\pi}{2} \frac{n^4}{n^4+1} \right)$ $\quad -1$ è minorante

1 è maggiorante

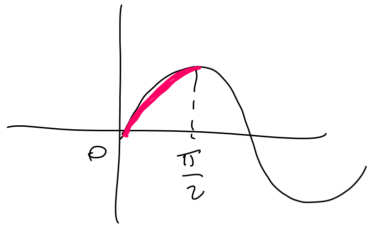
$$\frac{n^4}{n^4+1} = \frac{n^4+1-1}{n^4+1} = 1 - \frac{1}{n^4+1}$$

$m' + 1$ $\quad m + 1 \quad$ $\underbrace{m' + 1}$ crescente

$\frac{1}{n}$ decr. $\quad -\frac{1}{n}$ cresce

$$0 < \frac{m^4}{m^4+1} < 1$$

$$0 < \frac{m^4}{m^4+1} \cdot \frac{\pi}{2} < \frac{\pi}{2}$$



essendo $\sin y$ crescente in $y \in (0, \frac{\pi}{2})$

min G si ottiene per $m = 1$ $\quad \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$$\sin\left(\frac{m^4}{m^4+1} \cdot \frac{\pi}{2}\right) < \sin \frac{\pi}{2} = 1$$

1 è magg. ma non è massimo. Voglio mostrare che $1 = \sup G$. Devo far vedere che

$\forall \varepsilon > 0$ $\exists \bar{n}$: $\quad 1 - \varepsilon < \sin\left(\frac{m^4}{m^4+1} \cdot \frac{\pi}{2}\right) \quad \forall n > \bar{n}$

Ma io so che

$$\lim_{n \to \infty} \sin \frac{n^4}{n^4+1} \cdot \frac{\pi}{2} = 1$$

$\forall \varepsilon > 0 \quad \exists \bar{n}: \quad -\varepsilon < \sin\left(\frac{m^4}{m^4+1} \cdot \frac{\pi}{2}\right) - 1 < \varepsilon$

quindi è verificato