Eser 22/10
venerdì 22 ottobre 2021 12:25

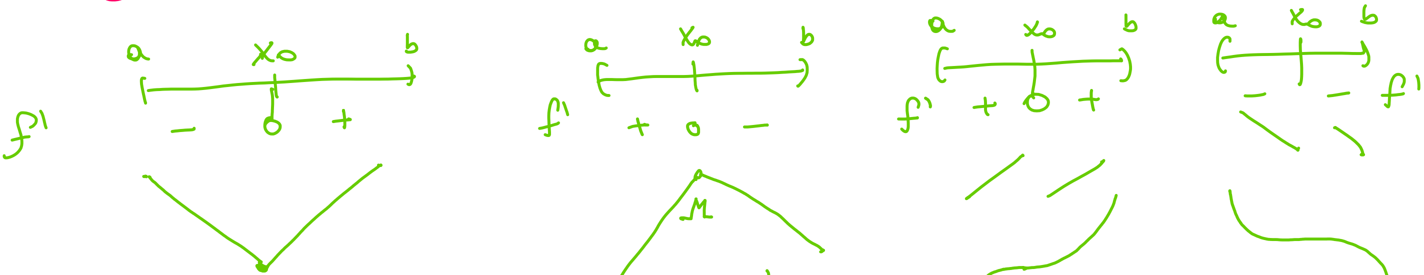
DERIVATE / CALCOLO DI MAX E MIN

Trovare, se esistono, il massimo M e il minimo m delle seguenti funzioni negli intervalli indicati

51. $\sin x - \cos x$ $\quad [0, 2\pi]$
52. $\dfrac{x}{1+x^2}$ $\quad [-2, 3]$
53. $x \ln x$ $\quad [\frac{1}{2}, 2]$
54. $x(x-2)^2$ $\quad [0, 3]$
55. $\dfrac{e^x}{x}$ $\quad [\frac{1}{2}, 2]$
57. $ax^2 + \dfrac{b}{x}$ $\quad (0, +\infty)$

PER TROVARE MAX E/O MIN DI $f(x)$ DERIVABILE IN (a,b) CERCO LA $f'(x)$ E CERCO i PUNTI $x_0 \in (a,b)$ / $f'(x_0) = 0$ PERCHÉ CANDIDATI AD ESSERE DI MAX O DI MINIMO.

STUDIO IL SEGNO DELLA DERIVATA PRIMA

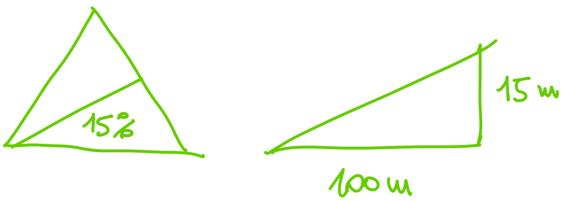


x_0 punto di min. $f(x_0)$ m

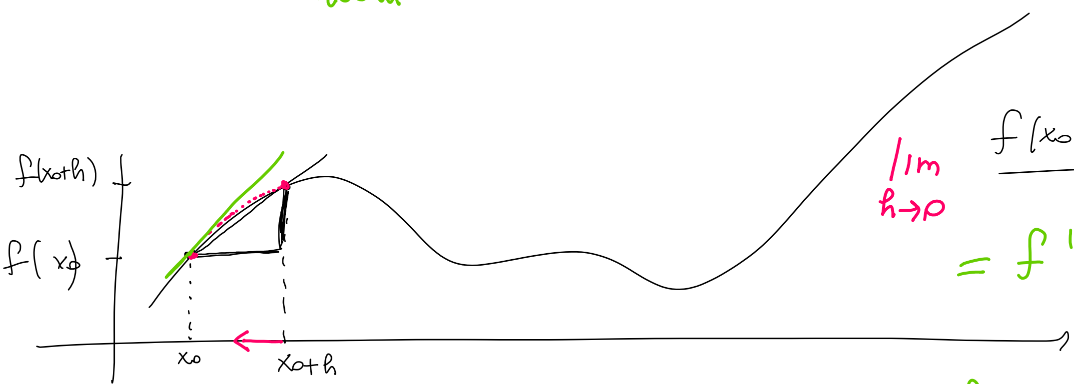
x_0 punto di max $f(x_0)$ M

flesso a Tg orizz.

flesso a Tg orizz.



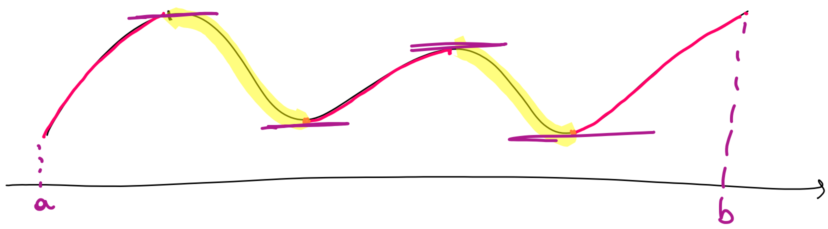
$$\frac{\Delta y}{\Delta x} = m$$



$$\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} = f'(x_0)$$

è il coeff angolare della retta

tg al grafico



53 $\quad x \cdot \ln x \quad \left[\frac{1}{2}, 2\right]$

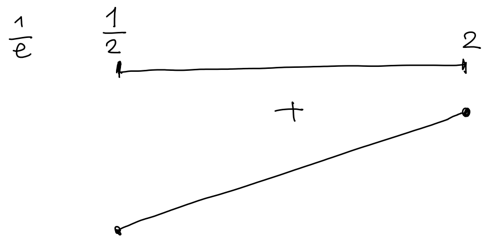
$$f' = \ln x + x \cdot \frac{1}{x} = \ln x + 1$$

Qual è il segno di f' in $\left[\frac{1}{2}, 2\right]$

$$\ln x + 1 > 0$$

$$\ln x > -1$$

$$x > e^{-1} = \frac{1}{e}$$

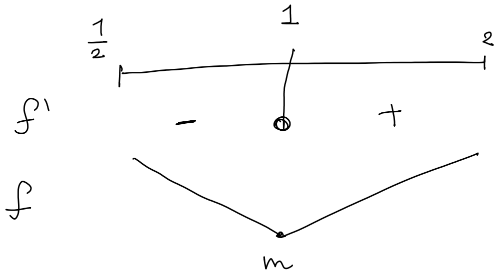


$$f\left(\tfrac{1}{2}\right) = \frac{1}{2}\ln\frac{1}{2} = \frac{1}{2}(-\ln 2) = -\ln\sqrt{2} \quad \text{min.}$$

$$f(2) = 2\ln 2 = \ln 4 \quad M$$

55. $f(x) = \dfrac{e^x}{x} \quad \left[\frac{1}{2}, 2\right]$

$$f'(x) = \frac{e^x \cdot x - e^x}{\underset{+}{x^2}} \geq 0 \Leftrightarrow \underset{+}{e^x}(x-1) \geq 0 \qquad x \geq 1$$



$f(1) = e$

$f\left(\frac{1}{2}\right) = 2\sqrt{e}$

$f(2) = \dfrac{e^2}{2}$

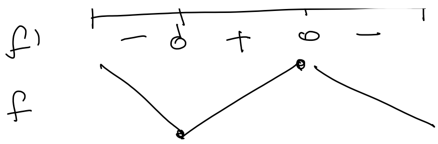
52. $f(x) = \dfrac{x}{1+x^2} \quad$ in $[2, 3]$ $\qquad \left(\dfrac{f}{g}\right)' = \dfrac{f'g - g'f}{g^2}$

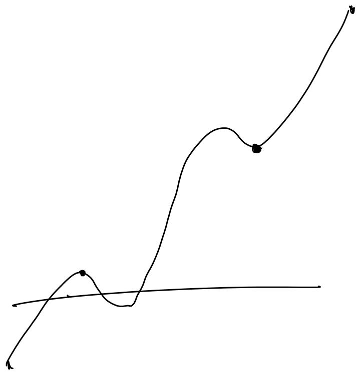
$$f' = \frac{1+x^2-2x^2}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2} \geq 0$$

$1 - x^2 \geq 0$

$1 - x^2 = 0$

$x = \pm 1$





m $f(-1) = \frac{-1}{2}$ m. ass.

$f(3) = \frac{3}{10}$

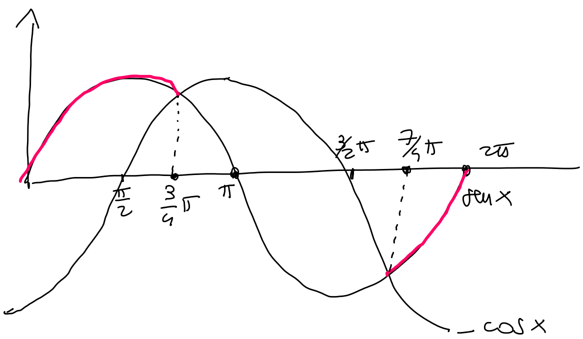
M $f(-2) = -\frac{2}{5}$

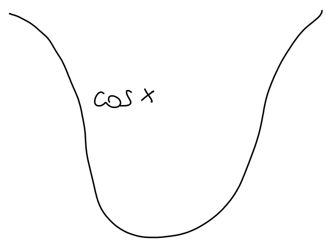
$f(1) = \frac{1}{2}$ M ass.

51. $\sin x - \cos x \quad [0, 2\pi]$

$f' = \cos x + \sin x \quad [0, 2\pi]$

$f' > 0$ se $\sin x > -\cos x$ in $[0, 2\pi]$



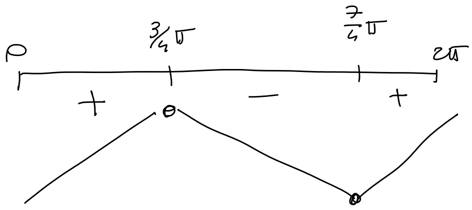


$\sin x = -\cos x$

divido per $\cos x$

$\operatorname{tg} x = -1 \quad x \in [0, 2\pi]$

$x = \pi - \pi/4 = \frac{3}{4}\pi \qquad 2\pi - \frac{\pi}{4} = \frac{7}{4}\pi$

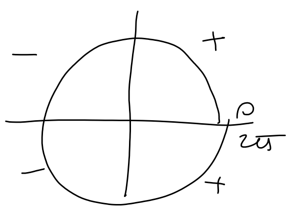


$f(\frac{3}{4}\pi) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}_{ass.}$ $\qquad f(2\pi) = -1_{rel.}$ M

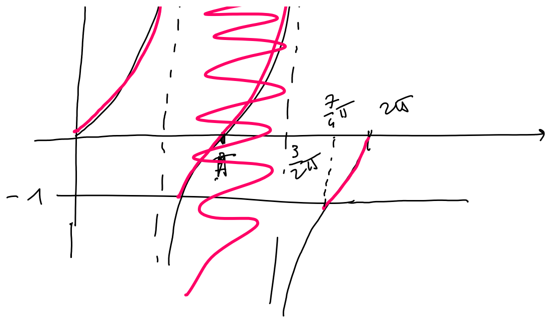
$f(0) = -1_{rel.}$ $\qquad f(\frac{7}{4}\pi) = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}_{ass.}$ m.

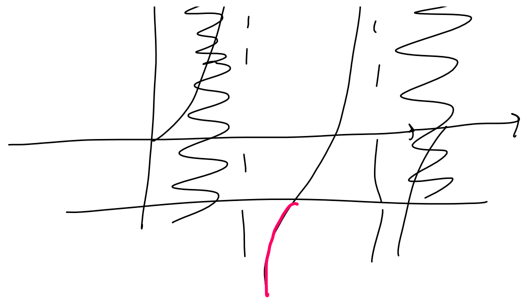
METODO ALTERNATIVO

$\sin x > -\cos x$



$\begin{cases} 0 < x < \frac{\pi}{2} \lor \frac{3}{2}\pi < x < 2\pi \\ \operatorname{tg} x > -1 \end{cases} \qquad \begin{cases} \frac{\pi}{2} < x < \frac{3}{2}\pi \\ \operatorname{tg} x < -1 \end{cases}$

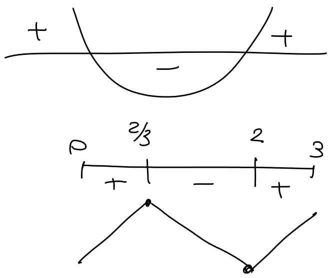




54 $\quad x(x-2)^2 \qquad [0,3]$

$$f' = (x-2)^2 + x\,2(x-2)\cdot 1 = x^2+4-4x+2x^2-4x$$
$$= 3x^2-8x+4$$

$$\frac{4\pm\sqrt{16-12}}{3} = \frac{4\pm 2}{3} \begin{cases} 2 \\ \frac{2}{3} \end{cases}$$



M $\quad f\left(\frac{2}{3}\right) = .. \qquad f(3)=3$

m $\quad f(0)=0 \qquad f(2)=0$

$(0,+\infty)$

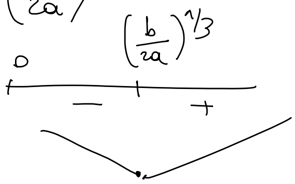
57 $\quad ax^2+\frac{b}{x}=f \qquad f' = 2ax - \frac{b}{x^2} = \frac{2ax^3-b}{x^2}$ +

$(x^{-1})' = -x^{-2}$

$2ax^3-b>0$

$\cancel{2a}x^3 > \frac{b}{2a} \qquad x > \left(\frac{b}{2a}\right)^{1/3} \qquad a>0$

$b>0$

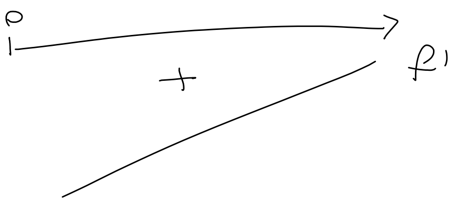


$a>0$
$b>0$

m $\quad f\left(\left(\frac{b}{2a}\right)^{1/3}\right) = --$

$\lim_{x\to 0^+} f(x) \qquad \lim_{x\to+\infty} f(x)$

$a>0$
$b<0 \quad \left(\frac{b}{2a}\right)^{1/3}$



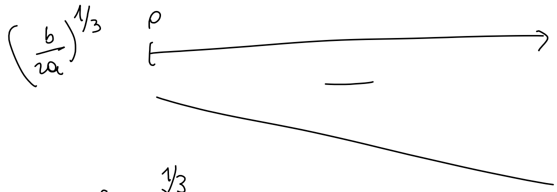
$a<0 \qquad 2ax^3-b>0$

$2ax^3>b$

$$x^3 < \frac{b}{2a}$$

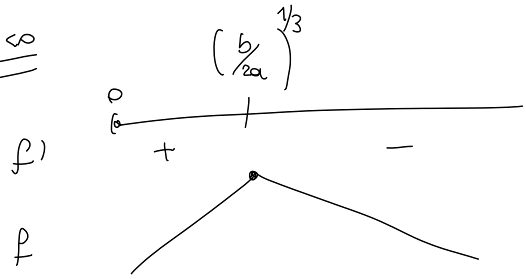
$$x < \left(\frac{b}{2a}\right)^{1/3}$$

se $b>0$



$ax^2 + \cancel{b/x}$

se $b<0$



$$M = f\left(\frac{b}{2a}\right)^{1/3}$$

inf $\lim_{x\to 0^+} f = -\infty$ $\qquad \lim_{x\to +\infty} f$