Eserc 25/10

lunedì 25 ottobre 2021 12:22

GRAFICI DI FUNZIONI

261 $\sqrt{x+1} - \sqrt{x}$

262 $(x^2+2x)e^x$

263 $-\ln(\sin x)$

264 $\sqrt{\dfrac{x^3}{x+3}}$

265 $\dfrac{1-x}{x^2+3}$

DOMINIO

SEGNO (OVE POSSIBILE)

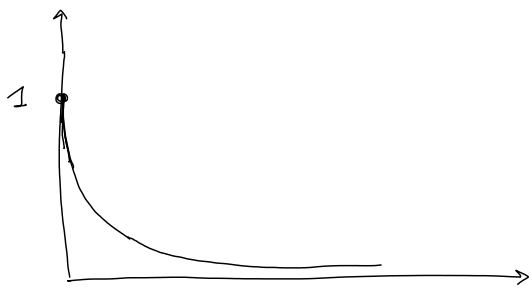
ASINTOTI OR./VERT.

MAX/MIN

CONC./CONV.

261 $\sqrt{x+1} - \sqrt{x}$

DOMINIO $\begin{cases} x+1 \geq 0 \\ x \geq 0 \end{cases} \Rightarrow x \geq 0 \qquad [0, +\infty) = D$



SEGNO $\sqrt{x+1} - \sqrt{x} \geq 0$ in D

$\sqrt{x+1} \geq \sqrt{x}$

$x+1 \geq x \quad \forall x \in D$

ASINTOTI $\lim_{x \to 0^-} f(x) = f(0) = 1$ $\qquad \lim_{x \to +\infty} \dfrac{x+1-x}{\sqrt{x+1}+\sqrt{x}} = 0$ AS. OR. $y = 0$

MAX/MIN

$$f'(x) = \frac{1}{2\sqrt{x+1}} - \frac{1}{2\sqrt{x}} = \frac{\sqrt{x} - \sqrt{x+1}}{2\sqrt{x}\cdot\sqrt{x+1}} = \frac{x - x - 1}{2\sqrt{x}\sqrt{x+1}\left(\sqrt{x}+\sqrt{x+1}\right)} < 0$$

f DECRESCE IN D

SI OSSERVI CHE IN $x=0$ f' NON ESISTE

$$f'(x) = \frac{1}{2}(x+1)^{-1/2} - \frac{1}{2}x^{-1/2}$$

$$f''(x) = -\frac{1}{4}(x+1)^{-3/2} + \frac{1}{4}x^{-3/2} = -\frac{1}{4\sqrt{(x+1)^3}} + \frac{1}{4\sqrt{x^3}}$$

$$= -\frac{\sqrt{x^3} + \sqrt{(x+1)^3}}{4\sqrt{(x+1)^3}\sqrt{x^3}} = \frac{(x+1)^3 - x^3}{4\sqrt{(x+1)^3}\sqrt{x^3}\left(\sqrt{(x+1)^3}+\sqrt{x^3}\right)} \geq 0$$

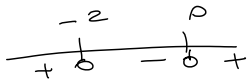
$\Leftrightarrow \quad x^3+3x^2+3x+1-x^3 \geq 0 \qquad \forall x \in D$

$-3 \pm \sqrt{9-12}$ $\qquad f$ CONVESSA $\cup$

262 $(x^2+2x)e^x$

DOMINIO $\quad \mathbb{R} = (-\infty, +\infty)$

SEGNO $\quad f \geq 0 \Leftrightarrow x^2+2x \geq 0 \quad x(x+2) \geq 0$



ASINTOTI $\quad \lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{x^2+2x}{e^{-x}} = \lim_{y \to +\infty} \frac{y^2-2y}{e^y} = 0$

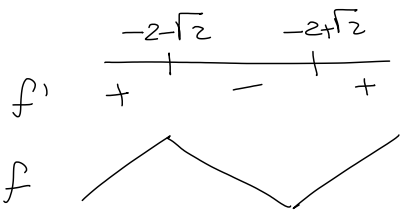
$y=0$ AS. OR. A $-\infty$

$\lim_{x \to +\infty} f(x) = +\infty$

MAX/MIN $\quad f'(x) = (2x+2)e^x + (x^2+2x)e^x$

$= e^x(x^2+4x+2) \geq 0 \Leftrightarrow x^2+4x+2 \geq 0$

$-2 \pm \sqrt{4-2} = -2 \pm \sqrt{2}$



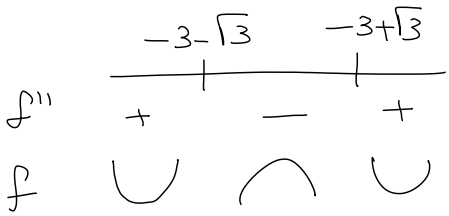
$M = f(-2-\sqrt{2})$

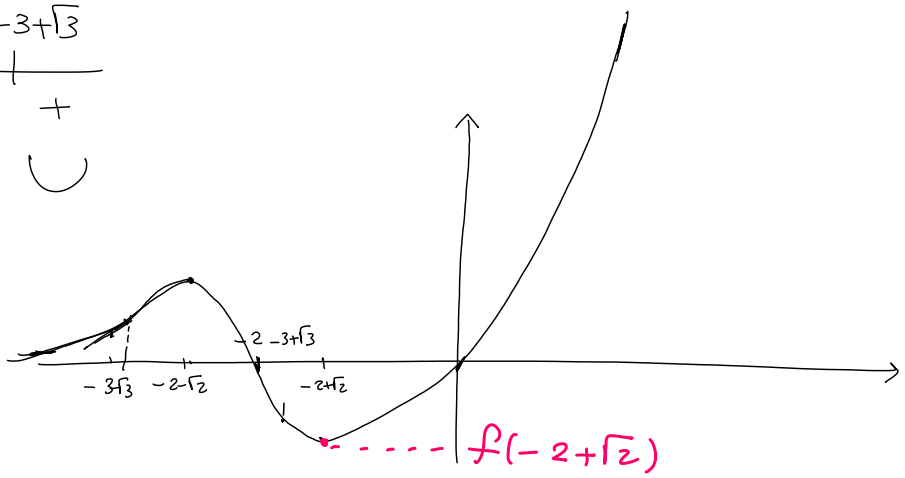
$m = f(-2+\sqrt{2})$

CONC/CONV.

$f''(x) = e^x(x^2+4x+2) + e^x(2x+4) = e^x(x^2+6x+6) \geq 0$

$\Leftrightarrow \quad x^2+6x+6 \geq 0 \qquad -3 \pm \sqrt{9-6} = -3 \pm \sqrt{3}$





263 $-\ln(\sin x)$

D $\sin x > 0$ $\quad 2k\pi < x < \pi + 2k\pi = (2k+1)\pi$

periodica di periodo 2π

LA STUDIO $(0, \pi)$

SEGNO

$-\ln(\sin x) \geq 0$

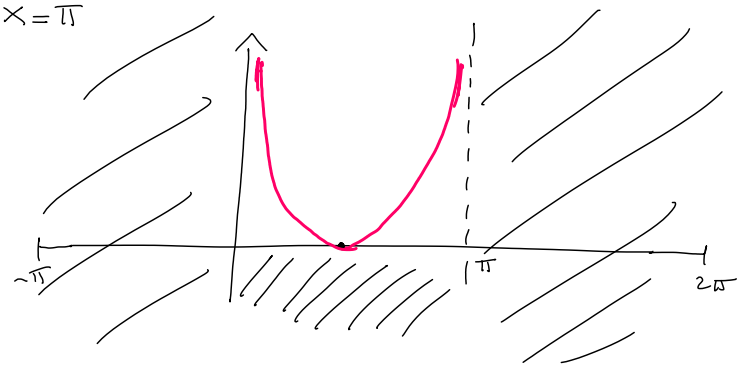
$\ln(\sin x) \leq 0$

$\sin x \leq 1 \quad \forall x \in D$

ASINTOTI

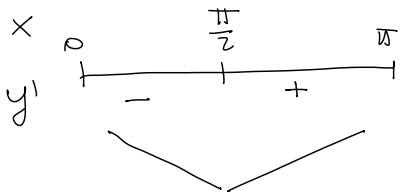
$\lim_{x \to 0^+} f(x) = +\infty$ $\quad x = 0$ AS. VERTICALE

$\lim_{x \to \pi^-} f(x) = +\infty$ $\quad x = \pi$



MAX / MIN

$y' = -\frac{\cos x}{\sin x}$



$m = f\left(\frac{\pi}{2}\right) = -\ln \sin\frac{\pi}{2} = 0$

CONC / CONV.

$y'' = \frac{\sin x - \sin x + \cos x \cdot \cos x}{\sin^2 x} = \frac{1}{\sin^2 x} > 0$ $\quad$ CONVESSA