Eserc 27/10
mercoledì 27 ottobre 2021 12:19

# GRAFICI DI FUNZIONI

STUDIAMO IL GRAFICO DI

1. $f(x) = (4x+3)e^{1/x}$

2. $f(x) = \dfrac{e^x+1}{e^x-1}$

3. $f(x) = \dfrac{x e^{-x}}{x-1}$

4. $f(x) = \log(1-2\,\mathrm{sen}^2 x)$

DOMINIO
SEGNO (OVE POSSIBILE)
SIMMETRIE (SE NECESSARIO)
ASINTOTI (OR/VERT/OBL)
MAX/MIN
CONC./CONV.

AS. OR.

$$\lim_{x\to\mp\infty} f(x) = \ell \in \mathbb{R}$$

$\mp\infty$ NON C'E' ASINT. OSSIA LA FUNZ NON PARTE (ARRIVA) COME UNA RETTA ORIZZ. —

AS. OBL.

$f(x) \sim mx+q \longrightarrow f(x) - mx \sim q$

$$\boxed{\lim_{x\to\pm\infty} \frac{f(x)}{x}} = \lim_{x\to\pm\infty} \frac{mx+q}{x} = m \in \mathbb{R}$$

$$\boxed{\lim_{x\to\pm\infty} f(x) - mx = q \in \mathbb{R}}$$

$f(x) = (4x+3)e^{1/x}$

DOMINIO $x \neq 0$ $\quad (-\infty, 0) \cup (0, +\infty)$

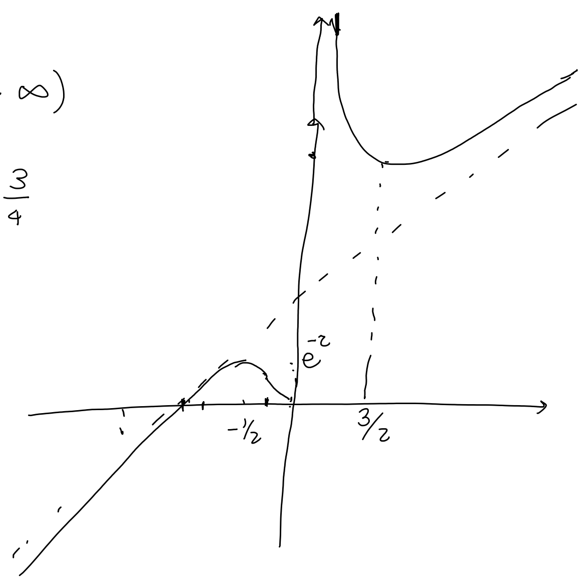
SEGNO $f(x) \geq 0 \Leftrightarrow 4x+3 \geq 0 \Leftrightarrow x \geq -\frac{3}{4}$

ASINTOTI

$$\lim_{x\to\mp\infty} (4x+3)e^{1/x} = \mp\infty$$

$$\lim_{x\to\mp\infty} \frac{4x+3}{x} e^{1/x} = 4 = m$$

$$\lim_{x\to+\infty} (4x+3)e^{1/x} - 4x =$$




$\lim 4x(e^{1/x}-1) + 3e^{1/x} =$

$$= \lim_{x\to+\infty} 4x e^{\dots} + 3e^{\dots} - 4x = \lim_{x\to+\infty} 4x(e^{\dots}$$

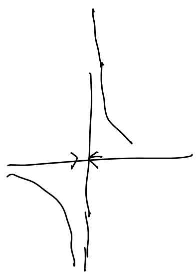
$$\lim_{x\to+\infty} 4\,\frac{e^{1/x}-1}{\frac{1}{x}} + 3e^{1/x} = \lim_{y\to0^+} 4\,\frac{e^{y}-1}{y} + 3e^{y} = 7 = 9$$

$$\boxed{y = 4x+7}$$

$$\lim_{x\to-\infty} (4x+3)e^{1/x} - 4x = \lim_{y\to0^-} 4\,\frac{e^{y}-1}{y} + 3e^{y} = 7$$

$$\lim_{x\to0^-} (4x+3)e^{1/x} = 0 \qquad e^{1/x} \to 0$$

$$\frac{1}{x} \xrightarrow[x\to0^+]{} +\infty$$



$$\lim_{x\to0^+} (4x+3)e^{1/x} = +\infty \qquad e^{1/x} \to +\infty$$

$$D\frac{1}{x} = Dx^{-1} = -\frac{1}{x^2}$$

MAX / MIN

$$f'(x) = 4e^{1/x} + (4x+3)e^{1/x}\left(-\frac{1}{x^2}\right)$$

$$= e^{1/x}\left(4 - \frac{4}{x} - \frac{3}{x^2}\right) = \underbrace{e^{1/x}\,\frac{4x^2-4x-3}{x^2}}\geq 0 \iff 4x^2-4x-3 \geq 0$$

$$\frac{2 \pm \sqrt{4+12}}{4} = \frac{2\pm4}{4} \begin{cases} \frac{3}{2} \\ -\frac{1}{2} \end{cases}$$

$-1/2 \quad 0 \quad 3/2$

$+ \quad - \quad - \quad +$ $\quad f'$

M $\quad$ m

$$M = f(-1/2) = \left(4\left(-\tfrac{1}{2}\right)+3\right)e^{-2} = e^{-2}$$

$$m = f(3/2) = \left(4\cdot\tfrac{3}{2}+3\right)e^{2/3}$$

$$f'' = \left(-\frac{1}{x^2}\right)e^{1/x}\,\frac{4x^2-4x-3}{x^2} + e^{1/x}\,\frac{(8x-4)x^2 - 2x(4x^2-4x-3)}{x^4}$$

$$= e^{1/x}\cdot\frac{1}{x^4}\left(-\cancel{4x^2} + 4x + 3 + \cancel{8x^3} - \cancel{4x^2} - \cancel{8x^3} + \cancel{8x^2} + 6x\right)$$

$$= e^{1/x}\cdot\frac{1}{x^4}\,(10x+3) \geq 0 \iff 10x+3 \geq 0 \iff x \geq -\frac{3}{10}$$

$-3/10 \quad 0$

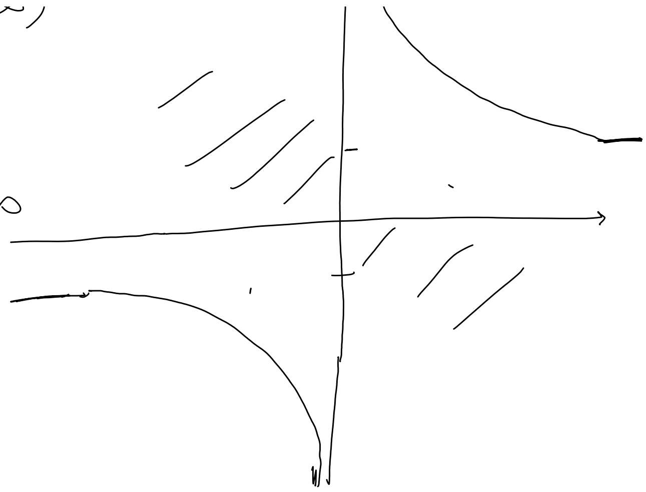
$- \quad + \quad +$

$\frown \quad \smile \quad \smile$

$$f(x) = \frac{e^x+1}{e^x-1}$$

DOMINIO $\quad e^x - 1 \neq 0 \quad e^x \neq 1 \quad x \neq 0$

$D = (-\infty, 0) \cup (0, +\infty)$



SEGNO

$e^x + 1 > 0$ SEMPRE

$e^x - 1 > 0$ SE $e^x > 1$ SE $x > 0$

ASINT.

$$\lim_{x \to -\infty} \frac{e^x+1}{e^x-1} = -1 \qquad y = -1$$

AS. OR.

$$\lim_{x \to +\infty} \frac{e^x+1}{e^x-1} = 1 \qquad y = 1$$

$$\lim_{x \to 0^-} \frac{e^x+1}{e^x-1} = -\infty \qquad x = 0 \text{ AS VERT.}$$

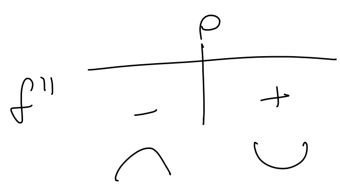
$$\lim_{x \to 0^+} \frac{e^x+1}{e^x-1} = +\infty$$

$$f'(x) = \frac{e^x(e^x-1) - e^x(e^x+1)}{(e^x-1)^2} = \frac{e^x(\cancel{e^x} - 1 - \cancel{e^x} - 1)}{(e^x-1)^2} = \frac{-2e^x}{(e^x-1)^2}$$

$< 0 \quad \forall x \in D$

$$f''(x) = \frac{-2e^x(e^x-1)^{\cancel{2}} + 2e^x \, 2\cancel{(e^x-1)}e^x}{(e^x-1)^{\cancel{4}3}} = \frac{2e^x(-e^x+1+2e^x)}{(e^x-1)^3}$$

$$= \frac{2e^x(e^x+1)}{(e^x-1)^3} \geq 0 \iff e^x - 1 > 0 \iff x > 0$$

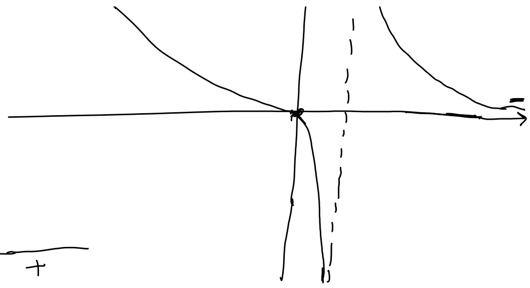



$$f(x) = \frac{e^x+1}{e^x-1}$$

$$f(-x) = \frac{e^{-x}+1}{e^{-x}-1} = \frac{\frac{1}{e^x}+1}{\frac{1}{e^x}-1} = \frac{\frac{1+e^x}{\cancel{e^x}}}{\frac{1-e^x}{\cancel{e^x}}} = \frac{1+e^x}{1-e^x} = -\frac{1+e^x}{e^x-1} = -f(x)$$

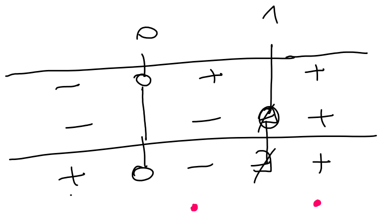
AVREI POTUTO STUDIARE $f(x)$ IN $(0, +\infty)$
E RIPORTARE IL GRAFICO PER SIMMETRIA

$$f(x)=\frac{xe^{-x}}{x-1}$$



DOMINIO $\quad x \neq 1 \quad (-\infty,1)\cup(1,+\infty)$

SEGNO



ASINTOTI

$$\lim_{x\to-\infty} \boxed{\frac{xe^{-x}}{x-1}} = +\infty$$

$$\lim_{x\to+\infty} \frac{xe^{-x}}{x-1} = \lim_{x\to+\infty} \frac{x}{e^{x}(x-1)} = 0 \qquad y=0 \text{ AS ORIZ.}$$

$$\lim_{x\to1^+} f(x) = +\infty$$

$$\lim_{x\to1^-} f(x) = -\infty$$

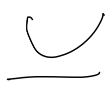
MAX/MIN

$$f'(x)=\frac{(e^{-x}-xe^{-x})(x-1)-xe^{-x}}{(x-1)^2}=\frac{e^{-x}[(1-x)(x-1)-x]}{(x-1)^2}$$

$$=\frac{e^{-x}}{(x-1)^2}(-1-x^2+2x-x)=\frac{e^{-x}}{(x-1)^2}(-1-x^2+x)\geq 0 \iff -x^2+x-1\geq 0$$

$$\iff x^2-x+1\leq 0 \qquad \text{MAI} \Rightarrow f'(x)<0 \quad \forall x\in D$$

$$1\pm\sqrt{1-} \qquad \Delta<0$$



$f$ DECRESCE

$$f'=\frac{(-1-x^2+x)}{(x-1)^2e^x}$$

$$f''=\frac{(-2x+1)(x-1)^2e^x+(1+x^2-x)(2(x-1)\cancel{e^x}+\cancel{e^x}(x-1)^2)e^x}{(x-1)^4e^{2x}}$$

$$=\frac{\cancel{e^x}[(x-1)^2(-2x+1)+(1+x^2-x)(2+x-1)(x-1)]}{(x-1)^4e^{\cancel{2x}}}$$

$$=\frac{(x-1)(-2x+1)+(1+x^2-x)(1+x)}{(x-1)^3e^x}=\frac{-2x^2+x+2x\cancel{-1}+\cancel{1}+\cancel{x}+\cancel{x^2}+x^3\cancel{-x}\cancel{-x^2}}{e^x(x-1)^3}$$

$$(x^2 \quad 2x+3)$$

$$= \frac{x^3-2x^2+3x}{e^x(x-1)^3} = \frac{x(x^2-2x+3)}{e^x(x-1)^3} > 0 \iff \frac{x}{x-1} > 0$$

$x^2-2x+3=0$ $\quad$ $x^2-2x+3>0$ δσ μρνσ

$1 \pm \sqrt{\cancel{1-}}$

| | | 0 | | 1 | |
|---|---|---|---|---|---|
| | $-$ | | $+$ | | $+$ |
| | $-$ | | $-$ | | $+$ |
| | $+$ | | $-$ | | $+$ |
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