Eserc 29/10

venerdì 29 ottobre 2021 12:22

GRAFICI DI FUNZIONI

TRACCIARE IL GRAFICO DELLE SEGUENTI FUNZIONI

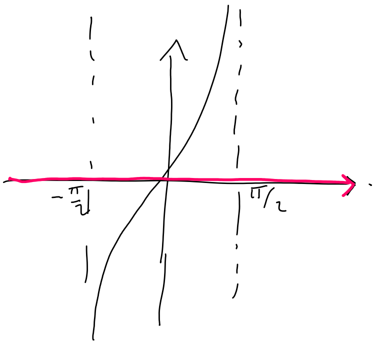
$f(x) = \operatorname{arctg} x + \operatorname{arctg} \frac{1}{x}$

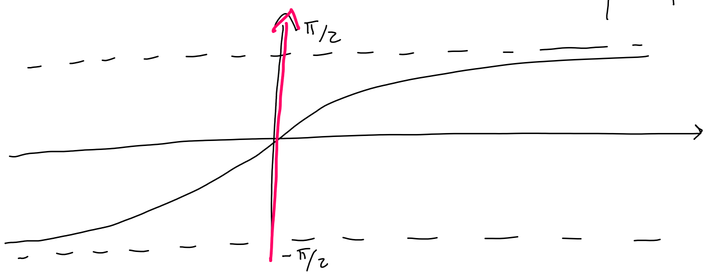
$f(x) = \arcsin x + \arccos x$

276. $f(x) = (x-2)e^{\frac{x}{x-1}}$

277. $f(x) = \ln \frac{x^2}{x-1}$

271. $f(x) = x + 2\cos x$





$f(x) = \operatorname{arctg} x + \operatorname{arctg} \frac{1}{x}$

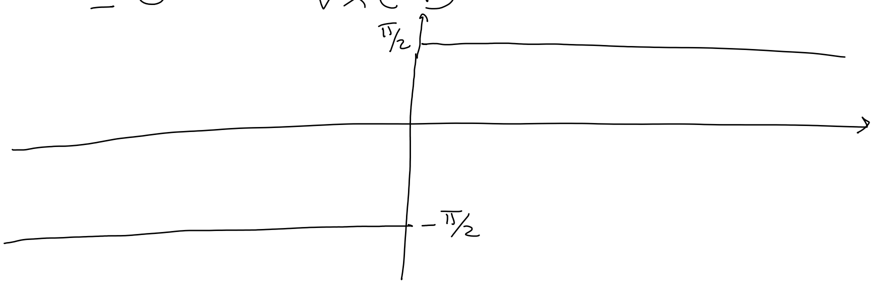
$D = (-\infty, 0) \cup (0, +\infty)$

$\lim_{x \to -\infty} f(x) = -\frac{\pi}{2}$ $\lim_{x \to +\infty} f(x) = \frac{\pi}{2}$

$\lim_{x \to 0^-} f(x) = -\frac{\pi}{2}$ $\lim_{x \to 0^+} f(x) = \frac{\pi}{2}$

$$f'(x) = \frac{1}{1+x^2} + \frac{1}{1+\left(\frac{1}{x}\right)^2}\left(-\frac{1}{x^2}\right) = \frac{1}{1+x^2} + \frac{x^2}{x^2+1}\left(-\frac{1}{x^2}\right)$$

$= 0 \qquad \forall x \in D$



$\operatorname{arctg} x + \operatorname{arctg} \frac{1}{x} = \frac{\pi}{2} \qquad x > 0$

$\operatorname{arctg} x + \operatorname{arctg} \frac{1}{x} = -\frac{\pi}{2} \qquad x < 0$

$D \operatorname{arctg} x$

$y = \operatorname{arctg} x$

$\operatorname{tg} y = x$

1

1 1

$$D\,\text{arctg}\,x = \frac{\quad}{D\,\text{tg}\,y\Big|_{y=\text{arctg}\,x}} - \frac{\quad}{1+\text{tg}^2 y\Big|_{y=\text{arctg}\,x}} - \frac{\quad}{1+x^2}$$

$$D\,\text{tg}\,y = \frac{1}{\cos^2 y} = \frac{\text{sen}^2 y + \cos^2 y}{\cos^2 y} = \text{tg}^2 y + 1$$

$$(\text{tg}(\text{arctg}\,x))^2$$

$$D\,\text{arcsen}\,x = \frac{1}{\sqrt{1-x^2}}$$

$$y = \text{arc sen}\,x \qquad y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\text{sen}\,y = x$$

$$D\,\text{arcsen}\,x = \frac{1}{D\,\text{sen}\,y\Big|_{y=\text{arc sen}\,x}} = \frac{1}{\cos y\Big|_{y=\text{arc sin}\,x}} = \frac{1}{\sqrt{1-\text{sen}^2 y}\Big|_{y=\text{arc sen}\,x}}$$

$\in [-\pi/2, \pi/2]$

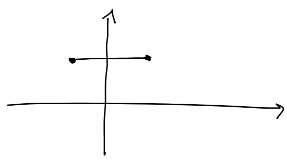
$\cos y > 0$

$$= \frac{1}{\sqrt{1-x^2}}$$

$$f(x) = \text{arc}\cos x + \text{arc sen}\,x$$

$$D = [-1, 1]$$

$$f'(x) = -\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}} = 0$$



$$f(0) = \text{arc sen}\,0 + \text{arc}\cos 0 = \frac{\pi}{2}$$

$$f(x) = (x-2)e^{\frac{x}{x-1}}$$

$$D = (-\infty, 1) \cup (1, +\infty)$$

SEGNO $f(x) \geq 0 \iff x-2 \geq 0 \iff x \geq 2$

$$f(0) = -2$$

ASINTOTI

$$\lim_{x \to \mp\infty} (x-2)e^{\frac{x}{x-1}} = \mp\infty$$

$$\lim_{x \to 1^+} (x-2)e^{\frac{x}{x-1}} = -\infty \qquad \text{AS VERT A DESTRA} \quad x=1$$

$$\lim_{x \to 1^-} (x-2)e^{\frac{x}{x-1}} = 0$$

AS OBLIQUO

$$\lim \frac{x-2}{\quad} e^{\frac{x}{x-1}} = e \qquad \lim (x-2)e^{\frac{x}{x-1}} - ex =$$

$$= \lim_{x \to \pm\infty} x e^{\frac{x}{x-1}} - 2e^{\frac{x}{x-1}} - ex = \lim_{x \to \pm\infty} xe\left(e^{\frac{x}{x-1}-1} - 1\right) - 2e^{\frac{x}{x-1}}$$

$$= \lim_{x \to \pm\infty} xe\left(e^{\frac{\cancel{x}-\cancel{x}+1}{x-1}} - 1\right) - 2e^{\frac{x}{x-1}} =$$

$$= \lim_{x \to \pm\infty} x\, e \cdot \frac{1}{x-1} - 2e^{\frac{x}{x-1}} = e - 2e = -e$$

$$y = ex - e = e(x-1)$$

$$f(x) = (x-2)e^{\frac{x}{x-1}}$$

$$f'(x) = e^{\frac{x}{x-1}} + (x-2)e^{\frac{x}{x-1}}\left(\frac{\cancel{x}-1-\cancel{x}}{(x-1)^2}\right)$$

$$= e^{\frac{x}{x-1}}\left(1 - \frac{x-2}{(x-1)^2}\right) = e^{x/x-1}\left(\frac{x^2+1-2x-x+2}{(x-1)^2}\right) \geq 0$$

$$x^2 - 3x + 3 \geq 0 \qquad \Delta = 9 - 12 < 0$$

SEMPRE $\forall x \in D$

f CRESCENTE

$$\left(\frac{x^2-3x+3}{(x-1)^2}\right)$$

$$f''(x) = e^{\frac{x}{x-1}}\left(-\frac{1}{(x-1)^2}\right) + e^{x/x-1}\,\frac{(2x-3)(x-1)^{\cancel{4}} - 2\cancel{(x-1)}(x^2-3x+3)}{(x-1)^{\cancel{4}\,3}}$$

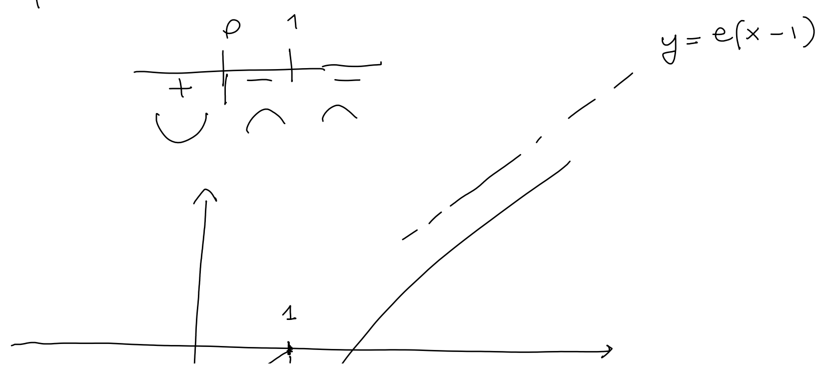
$$= e^{\frac{x}{x-1}}\left(\frac{-x^2+3x-3}{(x-1)^4} + \frac{\cancel{2x^2} - 2x - 3x + 3 - \cancel{2x^2} + 6x - 6}{(x-1)^3}\right)$$

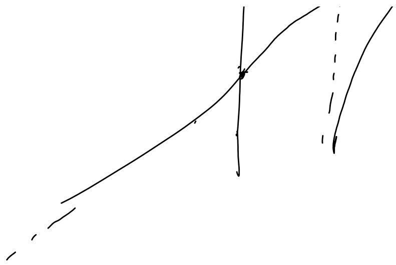
$$= e^{\frac{x}{x-1}}\left(\frac{-x^2+3x-3}{(x-1)^4} + \frac{x-3}{(x-1)^3}\right)$$

$$= e^{\frac{x}{x-1}}\,\frac{1}{(x-1)^4}\left(-\cancel{x^2} + 3x - \cancel{3} + \cancel{x^2} - 4x + \cancel{3}\right) \geq 0$$

$$\Leftrightarrow \quad -x > 0$$

$$x < 0$$





$$f(x) = \ln \frac{x^2}{x-1}$$

DOMINIO $\quad \frac{x^2}{x-1} > 0 \iff \begin{cases} x \neq 0 \\ x > 1 \end{cases} \qquad (1, +\infty) = D$

SEGNO
$f(x) > 0$
$x \in D$

$\frac{x^2}{x-1} > 1 \qquad \Big| \qquad \frac{x^2 - x + 1}{x-1} > 0 \qquad x \in D$

$\frac{x^2}{x-1} - 1 > 0 \qquad \Rightarrow \qquad x^2 - x + 1 > 0$

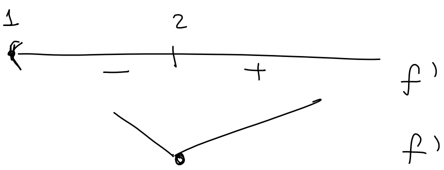
$1 \pm \cancel{\sqrt{1-}} \qquad \forall x \in D$

ASINTOTI

$$\lim_{x \to 1^+} \ln \frac{x^2}{x-1} = +\infty \qquad \lim_{x \to +\infty} f(x) = +\infty$$

$$f'(x) = \frac{1}{\frac{x^2}{\cancel{x-1}}} \cdot \frac{2x(x-1) - x^2}{(x-1)^2} = \frac{2x^2 - 2x - x^2}{x^2(x-1)} = \boxed{\frac{x^2 - 2x}{\underset{x^3 - x^2}{x^2(x-1)}}}$$

$$= \frac{x(x-2)}{x^2(x-1)}$$



$m = f(2) = \ln 4$

$$f'' = \frac{(2x-2)x^2(x-1) + (-x^2 + 2x)(3x^2 - 2x)}{x^4 (x-1)^2} \geq 0$$

$$\iff (2x-2)(x^3 - x^2) - 3x^4 + 2x^3 + 6x^3 - 4x^2 \geq 0$$

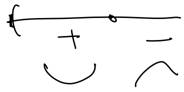
$$2x^4 - 2x^3 - \cancel{2x^3} + 2x^2 - 3x^4 + \cancel{2x^3} + 6x^3 - 4x^2 \geq 0$$

$$-x^4 + 4x^3 - 2x^2 \geq 0 \qquad x^2(-x^2 + 4x - 2) \geq 0$$

$$\iff \quad x^2 - 4x + 2 \leq 0 \qquad 2 \pm \sqrt{4-2} = 2 \pm \sqrt{2}$$

$$x = 2 + \sqrt{2}$$

$1 \qquad 2+\sqrt{2}$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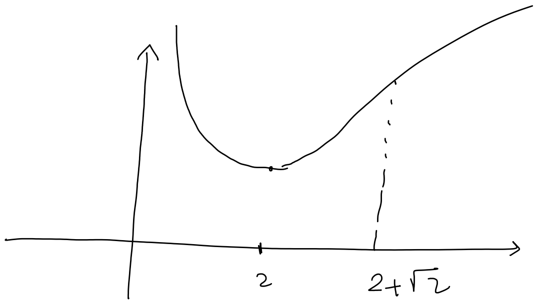
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