DIRE QUALI FUNZIONI SONO INVERTIBILI E IN CASO DI RISPOSTA POSITIVA TROVARNE L'INVERSA

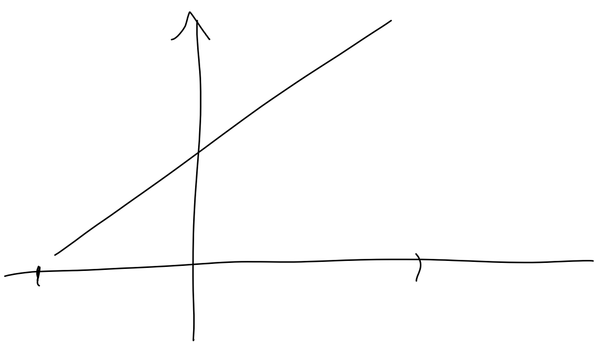
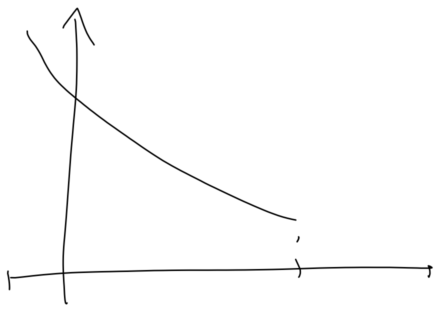
$$y = \arccos \frac{x-3}{x+1}$$

$$y = \sqrt{\operatorname{arctg} \frac{x+2}{x}}$$

$$y = \ln \arcsin(x^2-3)$$

STUDIARE GRAFICO QUALITATIVO DI $y = \ln(1+2\sin^2 x)$

STRETT. MONOTONA IN UN INTERVALLO

$$y = \arccos \frac{x-3}{x+1}$$

DOMINIO $-1 \leq \frac{x-3}{x+1} \leq 1$

$$\begin{cases} \frac{x-3}{x+1} - 1 \leq 0 \\ \frac{x-3}{x+1} + 1 \geq 0 \end{cases} \quad \begin{cases} \frac{\cancel{x}-3-\cancel{x}-1}{x+1} \leq 0 \\ \frac{x-3+x+1}{x+1} \geq 0 \end{cases} \quad \begin{cases} x+1 > 0 \\ 2x \geq 2 \end{cases} \quad \begin{cases} x > -1 \\ x \geq 1 \end{cases}$$

$[1, +\infty)$

$$y' = -\frac{1}{\sqrt{1-\left(\frac{x-3}{x+1}\right)^2}} \cdot \frac{\cancel{x}+1-\cancel{x}+3}{(x+1)^2} < 0$$

STRETT. DECRESCENTE IN $[1, +\infty)$
$\Rightarrow$ INVERTIBILE

$$\cos y = \frac{x-3}{x+1}$$

$$(\cos y)(x+1) = x-3$$

$$-x\cos y + x = +\cos y + 3$$

$$x(1-\cos y) = \cos y + 3$$

$$x = \frac{\cos y + 3}{1-\cos y}$$

- $f(x) = \arccos \frac{x-3}{x+1}$
- $f^{-1}(x) = \frac{\cos x + 3}{1-\cos x}$

$$y = \sqrt{\operatorname{arctg} \frac{x+2}{x}}$$

DOMINIO

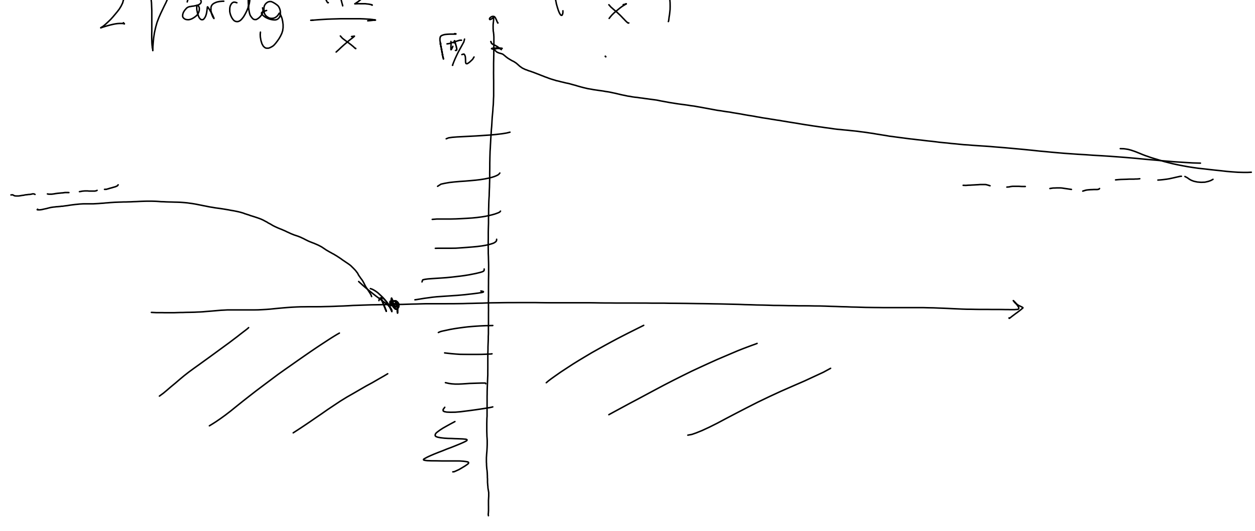
$$\begin{cases} \operatorname{arctg} \frac{x+2}{x} \geq 0 \\ x \neq 0 \end{cases}$$

$$\frac{x+2}{x} \geq 0$$

	-2		0	
$-$		$+$		$+$
$-$		$-$		$+$
$+$		$-$		$+$

$$(-\infty, -2] \cup (0, +\infty)$$

$$y' = \frac{1}{2\sqrt{\operatorname{arctg} \frac{x+2}{x}}} \cdot \frac{1}{1+\left(\frac{x+2}{x}\right)^2} \quad \frac{\cancel{x}-\cancel{x}-2}{x^2} < 0$$



$$\lim_{x \to \mp\infty} \sqrt{\operatorname{arctg} \frac{x+2}{x}} = \sqrt{\frac{\pi}{4}}$$

$$\lim_{x \to 0^+} \sqrt{\operatorname{arctg} \frac{x+2}{x}} = \sqrt{\frac{\pi}{2}}$$

$\sqrt{\quad - \quad x+2}$

$$y = \sqrt{\operatorname{arctg} \frac{\quad}{x}}$$

$$y^2 = \operatorname{arctg} \frac{x+2}{x}$$

$$\operatorname{tg} y^2 = \frac{x+2}{x}$$

$$x \operatorname{tg} y^2 = x + 2$$

$$x(\operatorname{tg} y^2 - 1) = 2$$

$$x = \frac{2}{\operatorname{tg} y^2 - 1}$$

$$f(x) = \sqrt{\operatorname{arctg} \frac{x+2}{x}}$$

$$f^{-1}(x) = \frac{2}{\operatorname{tg} x^2 - 1}$$

$$y = \ln \arcsin (x^2 - 3)$$

DOMINIO $\begin{cases} \arcsin (x^2-3) > 0 \\ -1 \leq x^2 - 3 \leq 1 \end{cases}$

$$\begin{cases} x^2 - 3 > 0 \\ -1 \leq x^2 - 3 \leq 1 \end{cases} \qquad 0 < x^2 - 3 \leq 1 \qquad \begin{cases} x^2 > 3 \\ x^2 \leq 4 \end{cases} \qquad \begin{cases} x < -\sqrt{3} \quad x > \sqrt{3} \\ -2 \leq x \leq 2 \end{cases}$$

$$-2 \leq x < -\sqrt{3} \quad \vee \quad \sqrt{3} < x \leq 2$$

ESSENDO UNA FUNZIONE PARI $f(x) = f(-x)$

NON E' INIETTIVA QUINDI NON E' INVERTIBILE

$$y = \ln (1 + 2 \sin^2 x)$$

DOMINIO $\quad 1 + 2\sin^2 x > 0 \quad \forall x \in \mathbb{R}$

$$D = (-\infty, +\infty)$$

SEGNO $\quad y \geq 0 \iff 1 + 2\sin^2 x \geq 1 \quad \forall x \in \mathbb{R}$

$$y = 0 \iff 2\sin^2 x = 0 \iff \sin x = 0 \iff x = k\pi$$

$$k \in \mathbb{Z}$$

FUNZIONE PERIODICA DI PERIODO 2π, LA STUDIO IN $[-\pi, \pi]$

ED E' PARI LA STUDIO IN $[0, \pi]$

$f(0)=0 \qquad f(\pi)=0$

$$y' = \frac{1}{1+2\sin^2 x} 4\sin x \cdot \cos x \geq 0 \iff \qquad \sin x \cos x \geq 0$$

$\sin x = 0 \quad \vee \quad \cos x = 0 \qquad x \in [0, \pi]$ $\qquad x=0 \quad x=\pi/2 \quad x=\pi$

	0		$\pi/2$		π
$\sin x$	$\cdot$	$+$		$+$	$\cdot$
$\cos x$		$+$	$\cdot$	$-$	
		$+$	0	$-$	

$M = f(\pi/2) = \ln 3$

$$y'' = \frac{(4\sin x \cdot \cos x)'(1+2\sin^2 x) - (1+2\sin^2 x)'(4\sin x \cos x)}{(1+2\sin^2 x)^2} =$$

$$y'' = \frac{(4\cos^2 x - 4\sin^2 x)(1+2\sin^2 x) - (4\sin x \cdot \cos x)^2}{(1+2\sin^2 x)^2} \geq 0$$

$4\cos^2 x + 8\cos^2 x \sin^2 x - 4\sin^2 x - 8\sin^4 x - 16\sin^2 x \cos^2 x \geq 0$

$4\cos^2 x - 8\cos^2 x \sin^2 x - 4\sin^2 x - 8\sin^4 x \geq 0$

$\cos^2 x = 1 - \text{sen}^2 x$

$4 - 4\,\text{sen}^2 x - 8\sin^2 x + \cancel{8\sin^4} x - 4\sin^2 x - 8\,\cancel{\sin^4} x \geq 0$

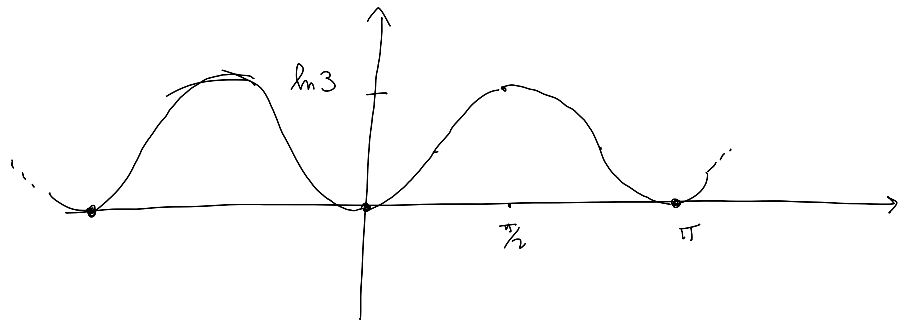
$4 - 16\,\text{sen}^2 x \geq 0$

$1 - 4\,\text{sen}^2 x \geq 0 \qquad\qquad 1-4y^2=0 \qquad \frac{1}{4}=y^2 \qquad y=\pm\frac{1}{2}$

$1-4y^2 \geq 0 \ \cap$

$-\frac{1}{2} \leq \text{sen}\, x \leq \frac{1}{2}$

	0		$\pi/6$		$5\pi/6$		π
f''		$+$		$-$		$+$	
		$\cup$		$\cap$		$\cup$	


ln3
π/2
π