Eserc 15/11

lunedì 15 novembre 2021 12:18

SERIE

CRITERIO DEL CONFRONTO

Se $0 \leq a_n \leq b_n \quad \forall n \geq n_0$

se $\sum_{n=1}^{\infty} b_n$ converge allora $\sum_{n=1}^{\infty} a_n$ converge

se $\sum_{n=1}^{\infty} a_n$ diverge allora $\sum_{n=1}^{\infty} b_n$ diverge

CRITERIO CONFRONTO ASINTOTICO

$0 \leq a_n; b_n$

$(a_n \sim b_n)$

Se esiste finito diverso da zero $\boxed{\lim_{n \to +\infty} \frac{a_n}{b_n} = \ell}$

allora $\sum_{n=1}^{\infty} a_n$ e $\sum_{n=1}^{\infty} b_n$ hanno lo stesso comportamento.

Se $\boxed{\lim_{n \to \infty} \frac{a_n}{b_n} = +\infty}$

$\frac{a_n}{b_n} > M$ definitiv.

$a_n > M b_n$

$\boxed{\text{se } \sum_{n=1}^{\infty} b_n \text{ diverge} \Rightarrow \sum_{n=1}^{\infty} a_n \text{ diverge}}$

Se $\boxed{\lim_{n \to \infty} \frac{a_n}{b_n} = 0}$

$\frac{a_n}{b_n} < \varepsilon$ definit.

$0 < a_n < \varepsilon b_n$

$\boxed{\sum_{n=1}^{\infty} b_n \text{ converge} \Rightarrow \sum_{n=1}^{\infty} a_n \text{ converge}}$

$\sum_{n=1}^{\infty} \frac{1}{n} = +\infty$

$\sum_{n=1}^{\infty} \frac{1}{n^2} < +\infty$

$\sum_{n=1}^{\infty} a_n < +\infty$ È NECESSARIO CHE $\lim_{n \to +\infty} a_n = 0$.

$$\sum_{n=1}^{\infty} n = +\infty \quad \text{perché} \quad \lim_{n\to+\infty} n = +\infty$$

$$\boxed{\sum_{n=1}^{\infty} \frac{1}{n} = +\infty}$$

$\lim_{n\to+\infty} \frac{1}{n} = 0$ ma non è sufficiente a far convergere la serie ma $\frac{1}{n}$ tende a 0 lentamente

$$\sum_{n=1}^{\infty} \frac{1}{n^\alpha} \begin{cases} \alpha \leq 1 & \text{DIVERGE} \\ \alpha > 1 & \text{CONVERGE} \end{cases}$$

SE $\alpha < 0$ $\quad \frac{1}{n^\alpha} = n^{-\alpha} \xrightarrow[n\to\infty]{} +\infty$ LA SERIE DIVERGE

$\alpha = 0$ $\quad \frac{1}{n^\alpha} = 1 \xrightarrow[n\to\infty]{} 1$ " " " PERCHE' NON E' SODDISFATTA LA COND NECESSARIA

$0 < \alpha \leq 1$ $\quad \frac{1}{n^\alpha} > \frac{1}{n}$ e $\sum_{n=1}^{\infty} \frac{1}{n} = +\infty$ QUINDI PER CONFRONTO $\sum_{n=1}^{\infty} \frac{1}{n^\alpha}$ DIVERGE

$\alpha \geq 2$ $\quad \frac{1}{n^\alpha} < \frac{1}{n^2}$ e $\sum_{n=1}^{\infty} \frac{1}{n^2} < +\infty$ QUINDI PER CONFRONTO $\sum_{n=1}^{\infty} \frac{1}{n^\alpha}$ CONVERGE

$\alpha > 1$ SI DIM. CHE CONVERGE

ESERCIZI

$$\sum_{n=0}^{\infty} \frac{n^2+3n}{n^4+2}$$

$a_n \sim b_n$

$$a_n = \frac{n^2+3n}{n^4+2} \geq 0 \qquad \lim_{n\to+\infty} \frac{n^2+3n}{n^4+2} = 0 \qquad \frac{n^2+3n}{n^4+2} = \frac{\cancel{n^2}\left(1+\frac{3}{n}\right)}{n^{\cancel{4}2}\left(1+\frac{2}{n^4}\right)} \sim \frac{1}{n^2}$$

$$\text{ossia } \lim_{n\to\infty} \cancel{n^2} \cdot \frac{\left(1+\frac{3}{n}\right)}{\cancel{n^2}\left(1+\frac{2}{n^4}\right)} = 1$$

LA SERIE PROPOSTA SI COMPORTA COME $\sum_{n=1}^{\infty} \frac{1}{n^2}$ CHE CONVERGE QUINDI " " " CONVERGE

$$\sum_{n=1}^{\infty} \frac{n+1}{2n+1} \qquad \sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n}\right) \qquad \sum_{n=1}^{\infty} \operatorname{sen} \frac{1}{n^2}$$

$$\sum_{n=1}^{\infty} \frac{3n-2}{n^2+1} \qquad \sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{n^2}\right) \qquad \sum_{n=1}^{\infty} \operatorname{arcsen} \frac{1}{\sqrt{n}}$$

OSSERVANDO CHE È UNA SERIE GEOMETRICA DIRE PER QUALI $x>0$

$$\sum_{n=1}^{\infty} \left(\log \frac{x+1}{x}\right)^n$$ CONVERGE

$$\sum_{n=1}^{\infty} \frac{n+1}{2n+1} \qquad \lim_{n\to\infty} \frac{n+1}{2n+1} = \frac{1}{2} \neq 0$$

NON È SODD. IL CRIT. NECESSARIO, $\frac{n+1}{2n+1} > 0 \Rightarrow$

$$\sum_{n=1}^{\infty} \frac{n+1}{2n+1} = +\infty$$

$$\sum_{n=1}^{\infty} \frac{3n-2}{n^2+1} \qquad \lim_{n\to\infty} \frac{3n-2}{n^2+1} = 0 \qquad \frac{3n-2}{n^2+1} > 0 \quad \forall n$$

$$\frac{3n-2}{n^2+1} \sim \frac{3}{n} \qquad 3\sum_{n=1}^{\infty} \frac{1}{n} \text{ diverge } \Rightarrow \sum_{n=1}^{\infty} \frac{3n-2}{n^2+1} = +\infty$$

$$\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n}\right) \qquad \lim_{n\to\infty} \ln\left(1+\frac{1}{n}\right) = 0 \qquad \ln\left(1+\frac{1}{n}\right) > 0$$

$$\ln\left(1+\frac{1}{n}\right) \sim \frac{1}{n} \quad e \quad \sum_{n=1}^{\infty} \frac{1}{n} = +\infty \underset{C.C.A.}{\Rightarrow}$$

$$\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n}\right) = +\infty$$

$$\sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{n^2}\right) \qquad \ln\left(\frac{n^2+1}{n^2}\right) \xrightarrow[n\to\infty]{} 0 \qquad \ln\left(1+\frac{1}{n^2}\right) > 0$$

$$\ln\left(1+\frac{1}{n^2}\right) \sim \frac{1}{n^2} \quad \text{dato che } \sum_{n=1}^{\infty} \frac{1}{n^2} < \infty$$

$$\Rightarrow \sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n^2}\right) \text{ converge}$$

$$\sum_{n=1}^{\infty} \operatorname{sen} \frac{1}{n^2}$$

$$\lim_{n\to\infty} \operatorname{sen} \frac{1}{n^2} = 0 \qquad \operatorname{sen}\frac{1}{n^2} > 0 \text{ perche } 0 < \frac{1}{n^2} < \frac{\pi}{2}$$

$$\operatorname{sen}\frac{1}{n^2} \overset{n\to\infty}{\sim} \frac{1}{n^2} \qquad \text{dato che } \sum_{n=1}^{\infty} \frac{1}{n^2} < +\infty$$

$$\Rightarrow \sum_{n=1}^{\infty} \operatorname{sen}\frac{1}{n^2} < +\infty$$

$$\sum_{n=1}^{\infty} \operatorname{arcsen} \frac{1}{\sqrt{n}}$$

$$\lim_{n\to\infty} \operatorname{arc\,sen} \frac{1}{\sqrt{n}} = 0 \qquad \operatorname{arc\,sen}\frac{1}{\sqrt{n}} > 0 \text{ perché } \frac{1}{\sqrt{n}} > 0$$

$$\operatorname{arcsen}\frac{1}{\sqrt{n}} \sim \frac{1}{\sqrt{n}} \qquad \text{dato che } \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = +\infty$$

$$\underset{\text{C.C.A}}{\Rightarrow} \sum_{n=1}^{\infty} \operatorname{arcsen}\frac{1}{\sqrt{n}} = +\infty$$

$x > 0$

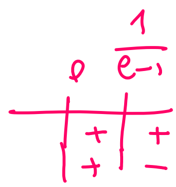
$$\sum_{n=1}^{\infty} \left(\log \frac{x+1}{x}\right)^n$$

$$\left|\log \frac{x+1}{x}\right| < 1$$

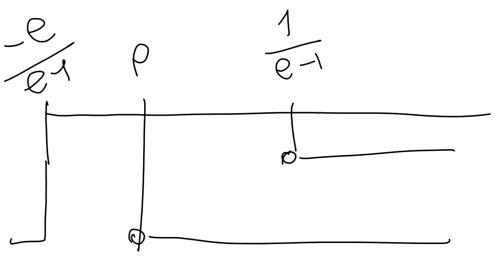
$$-1 < \log\frac{x+1}{x} < 1$$

$$\frac{1}{e} = e^{-1} < \frac{x+1}{x} < e$$

$$\begin{cases} x(1-e) > -1 \\ x(e-1) < 1 \\ x < \frac{1}{e-1} \end{cases}$$



$$\begin{cases} \frac{x+1}{x} - e < 0 \\ \frac{x+1}{x} - \frac{1}{e} > 0 \end{cases} \qquad \begin{cases} x+1-ex < 0 \\ e(x+1)-x > 0 \end{cases} \qquad \begin{cases} x > \frac{1}{e-1} \\ ex+e-x > 0 \\ x(e-1) > -e \end{cases}$$



$$\begin{cases} x > \frac{1}{e-1} \\ x > -\frac{e}{e-1} \end{cases}$$

$$x > \frac{1}{e-1}$$