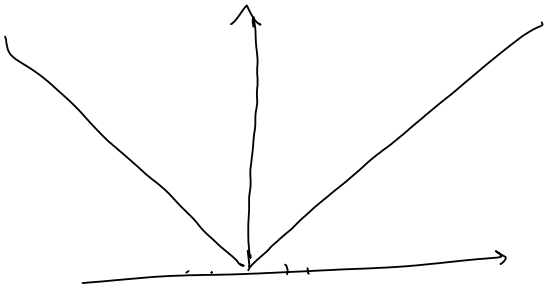
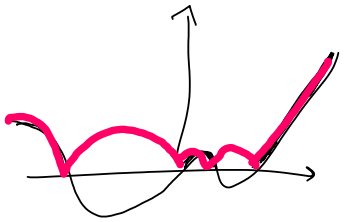
Eserc 22/09

mercoledì 22 settembre 2021 12:30

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$



$$|f(x)| = \begin{cases} f(x) & \text{se } f(x) \geq 0 \\ -f(x) & \text{se } f(x) < 0 \end{cases}$$



$$|x-3| = \begin{cases} x-3 & \text{se } x-3 \geq 0 \\ -x+3 & \text{altrimenti} \end{cases} = \begin{cases} x-3 & \text{se } x \geq 3 \\ -x+3 & \text{se } x < 3 \end{cases}$$

$|x| < r \qquad r > 0$

esempio $\qquad |x| < 3$

$|x| = \sqrt{x^2}$

$\sqrt{(-2)^2} = \sqrt{4} = 2$

$\sqrt{x^2} = \begin{cases} x \\ -x \end{cases}$ $\quad \begin{matrix} x \geq 0 \\ x \leq 0 \end{matrix} = |x|$

$|x| < 3$

$\sqrt{x^2} < 3 \qquad x^2 < 9 \qquad -3 < x < 3$

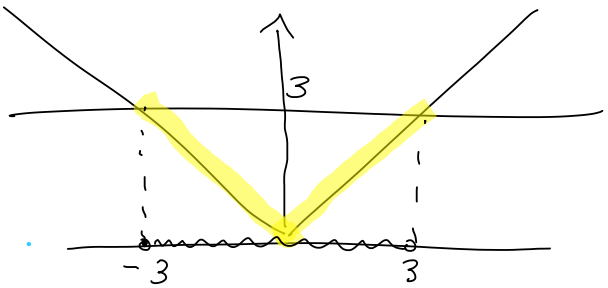
$\begin{cases} x \geq 0 \\ x < 3 \end{cases}$

$0 \leq x < 3$

$\begin{cases} x < 0 \\ -x < 3 \end{cases} \quad \begin{cases} x < 0 \\ x > -3 \end{cases}$

$-3 < x < 0$

$-3 < x < 3$



$|x| < r \qquad r > 0$

$-r < x < r$

$|x| > r \qquad x < -r \qquad x > r$

$|x+2| < x+1$

$\left|\frac{x+1}{x}\right| \geq x$

$|x-2| < |x+3| \Leftrightarrow \frac{|x-2|}{|x+3|} < 1$

$\Leftrightarrow \left|\frac{x-2}{x+3}\right| < 1 \Leftrightarrow -1 < \frac{x-2}{x+3} < 1$

$\begin{cases} \frac{x-2}{x+3} < 1 \\ \frac{x-2}{x+3} > -1 \end{cases}$

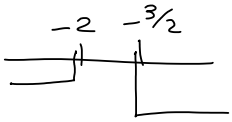
$|x+2| < x+1$

$\begin{cases} x+2 \geq 0 \\ x+2 < x+1 \end{cases}$

$\begin{cases} x \geq -2 \\ 2 < 1 \end{cases} \quad \emptyset$

$\begin{cases} x+2 < 0 \\ -x-2 < x+1 \end{cases}$

$\begin{cases} x < -2 \\ -3 < 2x \end{cases} \quad \begin{cases} x < -2 \\ x > -3/2 \end{cases}$



$\emptyset$

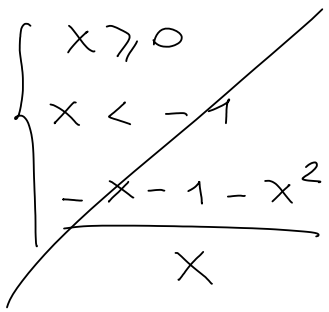
$$\left|\frac{x+1}{x}\right| \geq x.$$

se $x<0$ è soluzione della dis. $\leftarrow$

$$\begin{cases} x \geq 0 \\ \frac{x+1}{x} \geq 0 \\ \frac{x+1}{x} \geq x \end{cases} \qquad \begin{cases} x \geq 0 \\ \frac{x+1}{x} < 0 \\ -\frac{x+1}{x} \geq x \end{cases}$$

$$\begin{cases} x \geq 0 \\ x+1 \geq 0 \\ \frac{x+1}{x} - x \geq 0 \end{cases} \qquad \begin{cases} x \geq 0 \\ x+1 < 0 \\ -\frac{x+1}{x} - x \geq 0 \end{cases}$$

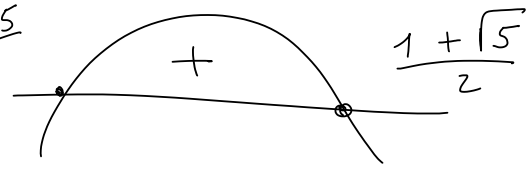
$$\begin{cases} x \geq 0 \\ x \geq -1 \\ \frac{x+1-x^2}{x} \geq 0 \end{cases} \qquad \begin{cases} x \geq 0 \\ x < -1 \\ \frac{-x-1-x^2}{x} \geq 0 \end{cases}$$



$$\begin{cases} x \geq 0 \\ x+1-x^2 \geq 0 \end{cases} \qquad \begin{cases} x \end{cases}$$

$$-x^2+x+1 \geq 0$$

$$\frac{-1 \pm \sqrt{1+4}}{-2} = \frac{+1 \pm \sqrt{5}}{2}$$



$$\begin{cases} x \geq 0 \\ \frac{1-\sqrt{5}}{2} \leq x \leq \frac{1+\sqrt{5}}{2} \end{cases} \rightarrow \quad 0 \leq x \leq \frac{1+\sqrt{5}}{2}$$

$$\left|\frac{x+1}{x}\right| \geq x \quad \Leftrightarrow \quad x \leq \frac{1+\sqrt{5}}{2}$$

$|x-2| \ldots$

$$\left|\frac{\cdot\cdot}{x+3}\right| < 1$$

$$\begin{cases} \dfrac{x-2}{x+3} < 1 \\ \dfrac{x-2}{x+3} > -1 \end{cases} \qquad \begin{cases} \dfrac{\cancel{x}-2-\cancel{x}-3}{x+3} < 0 \\ \dfrac{x-2+x+3}{x+3} > 0 \end{cases} \qquad \begin{cases} \dfrac{-5}{x+3} < 0 \\ \dfrac{2x+1}{x+3} > 0 \end{cases}$$

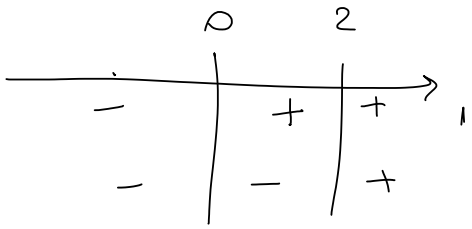
$$\begin{cases} x+3>0 \\ 2x+1>0 \end{cases} \qquad \begin{cases} x > -3 \\ x > -\frac{1}{2} \end{cases} \Rightarrow x > -\frac{1}{2}$$

$$\left|\frac{x^3-3x^2+25}{2+x^4)(x^2+27)}\right| > 0$$

$$|x| < |x-2| + 5$$

segno di x $\qquad x$

segno di $x-2$ $\qquad x-2$



$$\begin{cases} x \leq 0 \\ -\cancel{x} < -\cancel{x} + 2 + 5 \end{cases} \qquad \begin{cases} 0 < x \leq 2 \\ x < -x + 2 + 5 \end{cases} \qquad \begin{cases} x > 2 \\ \cancel{x} < \cancel{x} - 2 + 5 \end{cases}$$

$$\begin{cases} x \leq 0 \\ 0 < 7 \end{cases} \qquad \begin{cases} 0 < x \leq 2 \\ 2x < 7 \end{cases} \qquad \begin{cases} x > 2 \\ 0 < 3 \end{cases}$$

$x \leq 0$ $\qquad 0 < x \leq 2$ $\qquad x > 2$

. $x^2 - |x-3| + 1 - |x-2| = 0$

. $\frac{|3x-1|}{x-1} + \frac{1}{x} > 2$

. $27x^6 + 5 > 0 \qquad \forall x \in \mathbb{R}$

. $4x^4 - 2x^2 - 2 < 0 \qquad x^2 = z \quad 4z^2 - 2z - 2 < 0$

	2	3	
$x-3$	$-$	$-$	$+$
$x-2$	$-$	$+$	$+$

$\begin{cases} x \le 2 \\ x^2 + x - 3 + 1 + x - 2 = 0 \end{cases}$

$x^2 + 2x - 4 = 0$

$-1 \pm \sqrt{1+4} = -1 \pm \sqrt{5}$

accettabili

$\begin{cases} 2 < x \le 3 \\ x^2 + x - 3 + 1 - x + 2 = 0 \end{cases}$

$x = 0$

$\begin{cases} x > 3 \\ x^2 - x + 3 + 1 - x + 2 = 0 \end{cases}$

$x^2 - 2x + 6 = 0$

$1 \pm \sqrt{1-6}$

$\frac{|3x-1|}{x-1} + \frac{1}{x} > 2$

$\begin{cases} 3x - 1 \ge 0 \\ \frac{3x-1}{x-1} + \frac{1}{x} - 2 > 0 \end{cases}$

$\begin{cases} x \ge 1/3 \\ \frac{(3x-1)x + (x-1) - 2x(x-1)}{x(x-1)} > 0 \end{cases}$

$\begin{cases} x \ge 1/3 \end{cases}$

$\begin{cases} 3x - 1 < 0 \\ \frac{1-3x}{x-1} + \frac{1}{x} - 2 > 0 \end{cases}$

$\begin{cases} x < 1/3 \\ \frac{(1-3x)x + x - 1 - 2x(x-1)}{x(x-1)} > 0 \end{cases}$

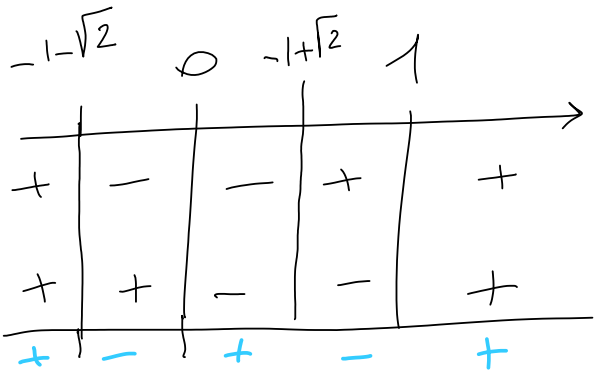
$\begin{cases} x < 1/3 \end{cases}$

$$\begin{cases} \dfrac{3x^2 - x + x - 1 - 2x^2 + 2x}{x(x-1)} > 0 \end{cases}$$

$$\begin{cases} \dfrac{x - 3x^2 + x - 1 - 2x^2 + 2x}{x(x-1)} > 0 \end{cases}$$

$$\begin{cases} x \geq 1/3 \\ \dfrac{x^2 + 2x - 1}{x(x-1)} > 0 \end{cases}$$

$$\begin{cases} x < 1/3 \\ \dfrac{-5x^2 + 4x - 1}{x(x-1)} > 0 \end{cases}$$

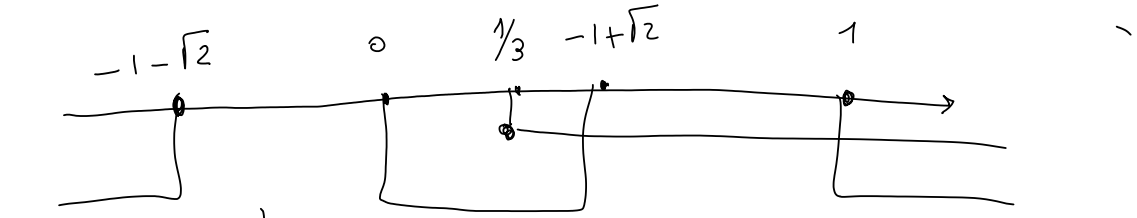


D

$$\begin{cases} x \geq 1/3 \\ x < -1-\sqrt{2} \;/\; 0 < x < -1+\sqrt{2} \;/ \\ x > 1 \end{cases}$$

$x^2 + 2x - 1 = 0$

$-1 \pm \sqrt{1+1} = -1 \pm \sqrt{2}$



$-1+\sqrt{2} < \frac{1}{3}$

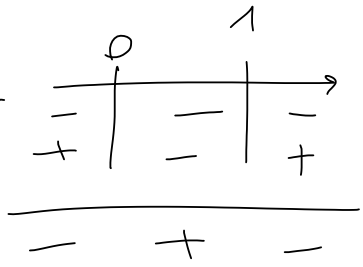
$\sqrt{2} < \frac{1}{3} + 1 = \frac{4}{3}$ $\qquad 2 < \frac{16}{9}$ $\qquad 18 < 16$ NO

$$\frac{1}{3} \leq x < -1+\sqrt{2} \qquad x > 1$$

$-5x^2 + 4x - 1$

$-2 \pm \sqrt{4-5}$

N



$$\begin{cases} x < 1/3 \\ 0 < x < 1 \end{cases} \Rightarrow \underline{0 < x < \frac{1}{3}}$$

$0 < x < -1+\sqrt{2}$ $\qquad$ $x > 1$