Eserc 17/11

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SERIE A TERMINI POSITIVI

$$\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n^3}\right) \qquad \sum_{n=1}^{\infty} \frac{1}{n(\ln n)^2} \qquad \sum_{n=1}^{\infty} \frac{n^n}{\binom{2n}{n}}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{1}{n+1} \qquad \sum_{n=1}^{\infty} \frac{n!}{n^n} \qquad \sum_{n=1}^{\infty} \frac{1}{\binom{3n}{n}}$$

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^3} \qquad \sum_{n=0}^{\infty} \frac{n+1}{n!} \qquad \sum_{n=0}^{\infty} 2^{-\sqrt{n}}$$

$$\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n^3}\right)$$

$$a_n = \ln\left(1+\frac{1}{n^2}\right) > 0 \qquad \lim_{n\to\infty} a_n = 0$$

$$\ln\left(1+\frac{1}{n^3}\right) \sim \frac{1}{n^3}$$

C.C.A. converge

$$\sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{1}{n+1}\right)$$

$$\lim_{n\to\infty} \frac{1}{n} \sin\left(\frac{1}{n+1}\right) = 0 \qquad \frac{1}{n} \sin\left(\frac{1}{n+1}\right) \sim \frac{1}{n(n+1)} \sim \frac{1}{n^2}$$

C.C.A converge

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$$

$\frac{\ln n}{n} < 1$ definitivamente

$$\frac{\ln n}{n^3} < \frac{1}{n^2}$$

per confronto converge

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \qquad a_n = \frac{1}{n(\ln n)^2}$$

Considero $\displaystyle\sum_{n=2}^{\infty} 2^n a_{2^n}$

$$\sum_{n=2}^{\infty} \cancel{2^n} \frac{1}{\cancel{2^n}(\ln 2^n)^2} = \sum_{n=2}^{\infty} \frac{1}{n^2 (\ln 2)^2}$$

converge

$$\sum_{n=1}^{\infty} \frac{n!}{n^n} \qquad \alpha \; a_n = \frac{n!}{n^n} \xrightarrow[n\to\infty]{} 0$$

$$\frac{a_{n+1}}{a_n} = \frac{\cancel{(n+1)!}}{(n+1)^{\cancel{n+1}}} \cdot \frac{n^n}{\cancel{n!}} = \left(\frac{n}{n+1}\right)^n = \left(\frac{1}{1+\frac{1}{n}}\right)^n \xrightarrow[n\to\infty]{} \frac{1}{e} < 1$$

la serie converge per C. Rapporto

$$\sum_{n=0}^{\infty} \frac{n+1}{n!} \qquad \frac{n+1}{n!} > 0$$

$$\frac{n+2}{(n+1)\cancel{!}} \cdot \frac{\cancel{n!}}{(n+1)} \xrightarrow[n\to\infty]{} 0 < 1$$

la serie converge per criterio rapporto

$$\sum_{n=0}^{\infty} \frac{\cos \pi n}{n+2} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+2}$$

SEGNI ALTERNI

$$\sum_{n=1}^{\infty} \frac{n^n}{\binom{2n}{n}} \qquad \binom{2n}{n} = \frac{(2n)!}{n!\,n!}$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$\sum_{n=1}^{\infty} \frac{n^n \cdot n! \cdot n!}{(2n)!}$$

$$\frac{(n+1)^{n+1} (n+1)! (n+1)!}{\cancel{(2n+2)!}} \cdot \frac{\cancel{(2n)!}}{n^n\, \cancel{n!}\, \cancel{n!}} =$$

$$(2n+2)(2n+1)$$

$$\left(\frac{n+1}{n}\right)^n \frac{\cancel{(n+1)}((n+1))}{2(\cancel{n+1})(2n+1)} \cdot (n+1) \xrightarrow[n\to+\infty]{} +\infty \qquad \text{diverge}$$

$1/2$

$$\sum_{n=1}^{\infty} \frac{1}{\binom{3n}{n}} = \sum_{n=1}^{\infty} \frac{n!\,(2n)!}{(3n)!}$$

$$\lim_{n\to\infty} \frac{(n+1)\cancel{!}\,(2n+2)!}{(3n+3)!} \cdot \frac{(3n)!}{\cancel{n!}\,(2n)!} =$$

$$\lim_{n\to\infty} \frac{(n+1)(2n+2)(2n+1)}{(3n+3)(3n+2)(3n+1)} = \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{27} < 1$$

CONVERGE

$$\sum^{\infty} 2^{-\sqrt{n}}$$

$n=0$

$$\frac{1}{2^{\sqrt{n}}} < \frac{1}{n^2}$$ per confronto converge

$$n^2 < 2^{\sqrt{n}} = e^{\sqrt{n}\ln 2}$$

$$\sum_{n=0}^{\infty} \frac{n^{\sqrt{n}}}{2^n}$$

$$\frac{n^{\sqrt{n}}}{2^n} > 0$$

$$\sqrt[n]{\frac{n^{\sqrt{n}}}{2^n}} = \frac{n^{\frac{\sqrt{n}}{n}}}{2} \xrightarrow[n\to\infty]{} \frac{1}{2} < 1$$ CONV. CRITERIO RADICE

$$n^{\frac{1}{\sqrt{n}}} = e^{\ln n^{1/\sqrt{n}}} = e^{\frac{\ln n}{\sqrt{n}}} \xrightarrow[n\to\infty]{} 1$$