Eserc 22/11

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$$\sum_{n=1}^{\infty} \frac{1}{2^{\log n}}$$

$$\boxed{\text{SUGG: } a^{\log b} = b^{\log a}}$$

$$e^{\log a \cdot \log b} = e^{\log b \cdot \log a}$$
$$e^{\log b \cdot \log a} = e^{\log a \cdot \log b}$$

$$\sum_{n=1}^{\infty} \frac{1}{2^{\log n!}}$$

$$\sum_{n=1}^{\infty} \frac{3^{n^2}}{(n!)^n}$$

$$\sum_{n=0}^{\infty} \frac{n^2 x^n}{5^n} \qquad x \in \mathbb{R}$$

$$\sum_{n=1}^{\infty} \frac{x^n(1-x)}{n}$$

$$\sum_{n=1}^{\infty} \frac{1}{(\log x)^{\log n}} \qquad x > 1$$

$$\sum_{n=1}^{\infty} \frac{1}{2^{\log n}} = \sum_{n=1}^{\infty} \frac{1}{n^{\log 2}} \qquad \log 2 < \log e = 1 \qquad \sum_{n=1}^{\infty} \frac{1}{n^p} \quad p \le 1$$

DIVERGE

$$\sum_{n=1}^{\infty} \frac{1}{2^{\log n!}} = \sum_{n=1}^{\infty} \frac{1}{(n!)^{\log 2}} \qquad n! > n^2 \quad n \geq 4 \qquad \frac{1}{n!} < \frac{1}{n^2}$$

$$< \sum_{n=1}^{3} \frac{1}{(n!)^{\log 2}} + \sum_{n=4}^{\infty} \frac{1}{n^{2\log 2}} \qquad 2\log 2 > 1$$

CONFRONTO

$$2^{\log n!} > 0$$

CONVERGE

$$\sum_{n=1}^{\infty} \frac{3^{n^2}}{(n!)^n} \qquad a_n = \frac{3^{n^2}}{(n!)^n} > 0$$

$$\lim_{n\to\infty} \sqrt[n]{\frac{3^{n^2}}{(n!)^n}} = \lim_{n\to\infty} \frac{3^n}{n!} = 0 < 1$$

la serie converge

$$\sum_{n=0}^{\infty} \frac{n^2 x^n}{5^n} \qquad x \in \mathbb{R}$$

$$\lim_{n\to+\infty} \frac{n^2 x^n}{5^n} = \lim_{n\to\infty} n^2 \left(\frac{x}{5}\right)^n = \begin{cases} 0 & |x| < 5 \\ +\infty & x \geq 5 \\ \text{non esiste} & x \leq -5 \end{cases}$$

la serie non converge se $|x| \geq 5$

Sia $|x| < 5$

$$\lim \sqrt[n]{\frac{n^2 |x|^n}{5^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n^2}\,|x|}{5} = \frac{|x|}{5} < 1$$

CONVERGE

$n \to \infty$

$$\sum_{n=0}^{\infty} \frac{n^2 x^n}{5^n} = \begin{cases} < +\infty & \text{se } |x| < 5 \\ +\infty & \text{se } x \geq 5 \\ \text{NON CONV.} & \text{se } x \leq -5 \end{cases}$$

$$\sum_{n=1}^{\infty} \frac{x^n (1-x)}{n}$$

$$\lim_{n \to +\infty} \frac{x^n (1-x)}{n} = \begin{cases} 0 & |x| \leq 1 \\ -\infty & x > 1 \\ \nexists & x < -1 \end{cases}$$

se $|x| > 1$ la serie non converge

Sia $|x| \leq 1$

$$\lim_{n \to +\infty} \sqrt[n]{\frac{|x|^n (1-x)}{n}} = \begin{cases} |x| & x \neq 1 \\ 0 & x = 1 \end{cases}$$

QUINDI se $|x| < 1$ per il criterio radice la serie CONVERGE

" se $x = 1$ " " " " " "

" se $x = -1$ COSA FA LA SERIE?

$$\sum_{n=1}^{\infty} \frac{(-1)^n 2}{n} = 2 \sum_{n=1} \frac{(-1)^n}{n}$$ CONVERGE PER LEIBNITZ

$$\sum_{n=1}^{\infty} \frac{x^n (1-x)}{n} \begin{cases} < +\infty & |x| \leq 1 \\ \text{NON CONV.} & |x| > 1 \end{cases}$$

$$\sum_{n=1}^{\infty} \frac{1}{(\log x)^{\log n}}$$ $x > 1$ QUINDI LA SERIE È A TERMINI POSITIVI.

$$\lim_{n \to \infty} \frac{1}{(\log x)^{\log n}} = \begin{cases} 0 & \text{se } x > e \\ 1 & \text{se } x = e \\ +\infty & \text{se } 1 < x < e \end{cases}$$ la serie diverge a $+\infty$

CONDENS. $\frac{1}{(\log x)^{\log n}} > 0$ E DECRESCENTE

Sia $x > e$

$$\sum^{\infty} \frac{1}{(\log x)^{\log n}} \sim \sum_{n=1}^{\infty} 2^n \frac{1}{(\log x)^{\log 2^n}} = \sum_{n=1}^{\infty} \frac{2^n}{(\log x)^{n \cdot \log 2}}$$

$$= \sum_{n=1}^{\infty} \left[\frac{2}{(\log x)^{\log 2}}\right]^n \qquad \text{GEOM.} \begin{cases} < \infty & x > e^{2^{\frac{1}{\log 2}}} \\ +\infty & x \leq e^{2^{\frac{1}{\log 2}}} \end{cases}$$

LA SERIE CONV. $x > e^{2^{\frac{1}{\log 2}}}$

// // DIV. $1 < x \leq e^{2^{\frac{1}{\log 2}}}$

$x > 1$

SFRUTTANDO $a^{\log b} = b^{\log a}$

$$\dots \sum_{n=1}^{\infty} \frac{1}{(\log x)^{\log n}} = \sum_{n=1}^{\infty} \frac{1}{n^{\log(\log x)}}$$

$$\log(\log x) > 1$$

$$x > e^e$$

OSSERVO CHE $\boxed{2^{\frac{1}{\log 2}}} = e^{\log 2^{\frac{1}{\log 2}}} = e^{\frac{1}{\log 2} \cdot \log 2} = \boxed{e}$