Eserc 24/11

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PRIMITIVE

$p \neq -1$ $\quad \int x^p \, dx = \frac{x^{p+1}}{p+1} + c$

$\int \frac{1}{x} \, dx = \log|x| + c$

$\int \sin x \, dx = -\cos x + c$

$\int \cos x \, dx = \sin x + c$

$\int e^x \, dx = e^x + c$

$\int \frac{1}{1+x^2} \, dx = \operatorname{arctg} x + c$

$\int f^p(x) \cdot f'(x) \, dx = \frac{f^{p+1}(x)}{p+1} + c$

$\int \frac{1}{f(x)} f'(x) \, dx = \int \frac{f'(x)}{f(x)} \, dx = \log|f(x)| + c$

$\int [\sin f(x)] f'(x) \, dx = -\cos f(x) + c$

$\int [\cos f(x)] \cdot f'(x) \, dx = \sin f(x) + c$

$\int e^{f(x)} \cdot f'(x) \, dx = e^{f(x)} + c$

$\int \frac{1}{1+f^2(x)} f'(x) \, dx = \operatorname{arctg} f(x) + c$

ESEMPI

$\int 4 \cos x \, dx = 4 \int \cos x \, dx = 4 \sin x + c$

$\int \left(5x^5 + \frac{x^3}{\sqrt{x}} - 7x^5 \sqrt[3]{x} \right) dx =$

$5 \int x^5 \, dx + \int \frac{x^3}{\sqrt{x}} \, dx - 7 \int x^5 \sqrt[3]{x} \, dx =$

$5 \frac{x^6}{6} + \int x^{3-1/2} \, dx - 7 \int x^{5+\frac{1}{3}} \, dx = \frac{5}{6} x^6 + \frac{x^{\frac{5}{2}+1}}{\frac{7}{2}} - 7 \frac{x^{\frac{16}{3}+1}}{\frac{19}{3}} + c$

$\frac{5}{6} x^6 + \frac{2}{7} x^{7/2} - \frac{21}{19} x^{\frac{19}{3}} + c$

$\int \sin^4 x \cdot \cos x \, dx \qquad f = \sin x \quad f' = \cos x \qquad \frac{\sin^5 x}{5} + c$

(2) $\int \frac{6}{6} \sin(6x+3) \, dx \qquad f = 6x+3 \quad f' = 6 \qquad \frac{1}{6} \int 6 \cdot \sin(6x+3) \, dx = -\frac{1}{6} \cos(6x+3) + c$

(3) $\int \frac{2}{2} x \cdot e^{x^2} \, dx \qquad f = x^2 \quad f' = 2x \qquad \frac{1}{2} \int 2x \cdot e^{x^2} \, dx = \frac{1}{2} e^{x^2} + c$

(4) $\int \frac{\cos(\log x)}{x} \, dx \qquad f = \log x \quad f' = \frac{1}{} \qquad \int \frac{1}{x} \cdot \cos(\log x) \, dx = \operatorname{sen}(\log x) + c$

5 $\int \frac{\sin x}{\cos x}\,dx$

$f = \cos x$
$f' = -\sin x$

$-\int \frac{-\sin x}{\cos x}\,dx = -\log|\cos x| + c$

$\int$ funzioni razionali $\int \frac{p(x)}{q(x)}\,dx$

$$\int (3x^4 + 4x^2 + 2x + 1)\,dx =$$

$$3\frac{x^5}{5} + 4\frac{x^3}{3} + \cancel{2}\frac{x^2}{\cancel{2}} + x + c$$

se $p(x)$ ha grado maggiore del grado di $q(x)$
eseguo la divisione fra polinomi

$$\frac{p(x)}{q(x)} = f(x) + \frac{r(x)}{q(x)}$$

grado di $r(x)$
$<$ grado di $q(x)$

$$\int \frac{p(x)}{q(x)}\,dx = \int f(x)\,dx + \int \frac{r(x)}{q(x)}\,dx$$

$\int \frac{3}{2+x}\,dx$

$f = 2+x$
$f' = 1$

$$3\int \frac{1}{2+x}\,dx = 3\log|x+2| + c$$

$$\int \frac{3+x}{5+x}\,dx = \int \frac{x+3+2-2}{5+x}\,dx = \int \frac{x+5}{x+5} - \frac{2}{x+5}\,dx = \int 1 - \frac{2}{x+5}\,dx$$

$$x - 2\log|x+5| + c$$

$\int \frac{}{\text{grado}2}$

$\Delta < 0$

$\int \frac{1}{1+f^2} f'\,dx = \operatorname{arctg} f + c$

$$\int \frac{1}{x^2+9}\,dx$$

$$\int \frac{1}{9\left(1+\frac{x^2}{9}\right)}dx = \frac{1}{9}\int \frac{1}{1+\frac{x^2}{9}}dx = \frac{1}{3}\int \frac{1}{1+\left(\frac{x}{3}\right)^2}\frac{1}{3}\,dx = \frac{1}{3}\operatorname{arctg}\frac{x}{3}+c$$

$$f^2 = \frac{x^2}{9} \qquad f = \frac{x}{3} \qquad f' = \frac{1}{3}$$

$\boxed{a>0}$

$$\int \frac{1}{x^2+a}dx = \frac{1}{\sqrt{a}}\operatorname{arctg}\frac{x}{\sqrt{a}}+c$$

$$f = \frac{x}{\sqrt{a}} \qquad f' = \frac{1}{\sqrt{a}}$$

$$\int \frac{1}{a\left(1+\left(\frac{x}{\sqrt{a}}\right)^2\right)}dx \qquad \frac{1}{\sqrt{a}}\int \frac{1}{1+\left(\frac{x}{\sqrt{a}}\right)^2}\cdot\frac{1}{\sqrt{a}}\,dx$$

$$\int \frac{1}{x^2+x+1}dx \qquad \Delta = 1-4<0$$

$$x^2+x+1 = \left(x+\frac{1}{2}\right)^2-\frac{1}{4}+1 = \underline{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$\int \frac{1}{x^2+x+1}dx = \int \frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}dx = \int \frac{1}{y^2+\frac{3}{4}}dy =$$

$$\boxed{x+\frac{1}{2}=y}$$

$$dx = dy$$

$$\int \frac{1}{\frac{3}{4}\left(y^2\frac{4}{3}+1\right)}dy = \frac{4}{3}\int \frac{1}{\frac{4}{3}y^2+1}dy = \frac{2}{\sqrt{3}}\int \frac{1}{\frac{4}{3}y^2+1}\cdot\frac{2}{\sqrt{3}}\,dy = \frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2}{\sqrt{3}}y+c$$

$$f = \frac{2}{\sqrt{3}}y \qquad f' = 2/\sqrt{3}$$

$$= \frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)+c$$

$$\int \frac{3x+2}{x^2+1} = \int \frac{3x}{x^2+1} + \int \frac{2}{x^2+1} = \frac{3}{2}\int \frac{2x}{x^2+1} + 2\int \frac{1}{x^2+1}dx$$

$$= \frac{3}{2}\log\left(x^2+1\right) + 2\operatorname{arctg} x$$

$$f = x^2+1$$

$$f' = 2x$$

$$\int \frac{x^3-1}{x^2+2}\,dx$$

$$\begin{array}{l|l} x^3 - 1 & x^2+2 \\ \hline -x^3-2x & x \\ \hline /\ -2x-1 & \end{array}$$

$$\int x\,dx + \int \frac{-2x-1}{x^2+2}\,dx$$

$$\frac{x^2}{2} - \int \frac{2x+1}{x^2+2}\,dx = \frac{x^2}{2} - \int \frac{2x\,dx}{x^2+2} - \int \frac{1}{x^2+2}\,dx =$$

$$\frac{x^2}{2} - \log(x^2+2) - \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{2}\left(\frac{x^2}{2}+1\right)}\,dx = \frac{x^2}{2} - \log(x^2+2) - \frac{1}{\sqrt{2}}\,\mathrm{arctg}\,\frac{x}{\sqrt{2}} + c$$

$$\int \frac{\ln^4 x}{x}\,dx \qquad \int \frac{x^3}{x^4-3}\,dx$$

$$\int \sin x\sqrt{1+\cos x}\,dx \qquad \int \frac{x^3-2x+1}{x^2+3}\,dx$$

$$\frac{1}{4}\int 4e^{4x-2}\,dx = \frac{1}{4}e^{4x-2}+c \qquad \int \frac{e^{\arcsin x}}{\sqrt{1-x^2}}\,dx = e^{\arcsin x}+c$$

$$\int \frac{\ln^4 x}{x}\,dx = \int f^4(x)\cdot f'(x)\,dx = \frac{\ln^5 x}{5}+c$$

$f(x) = \ln x$

$$\int \frac{\ln^4 x}{x}\,dx = \int t^4\,dt = \frac{t^5}{5}+c = \frac{\ln^5 x}{5}+c$$

$t = \ln x$

$dt = 1/x\,dx$

$$\int \frac{x^3}{x^4-3}\,dx = \frac{1}{4}\int \frac{4x^3}{x^4-3}\,dx = \frac{1}{4}\log|x^4-3|+c$$

$f = x^4-3$

$f' = 4x^3$

$$\boxed{\begin{array}{l} t = x^4-3 \\ dt = 4x^3\,dx \\ \frac{dt}{4} = x^3\,dx \end{array}}$$ ALTERNATIVA

$$\int \sin x \sqrt{1+\cos x}\, dx = \int \sin x \, (1+\cos x)^{1/2} \, dx$$

$f = 1+\cos x$

$f' = -\sin x$

$p = 1/2$

$$= -\int (-\sin x)\,(1+\cos x)^{1/2}\, dx$$

$$= -\frac{2(1+\cos x)^{3/2}}{3} + c$$

$$\int f^p f' \, dx = \frac{f^{p+1}}{p+1} + c$$

$$\int \sin x \sqrt{1+\cos x}\, dx$$

$1+\cos x = t$

$-\sin x \, dx = dt$

$\sin x \, dx = -dt$

$$-\int \sqrt{t}\, dt = -\frac{2t^{3/2}}{3} + c$$

$$\begin{array}{l|l} x^3 - 2x + 1 & x^2+3 \\ \hline -x^3 - 3x & x \\ \hline \quad\;\; -5x+1 & \end{array}$$

$$\int x\, dx + \int \frac{-5x+1}{x^2+3}\, dx = \frac{x^2}{2} - \frac{5}{2}\int \frac{2x}{x^2+3}\, dx + \int \frac{1}{x^2+3}\, dx$$

$$\frac{x^2}{2} - \frac{5}{2}\log|x^2+3| + \frac{1}{\sqrt{3}} \operatorname{arctg}\frac{x}{\sqrt{3}} + c$$