Eserc 29/11

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$$ax^2+bx+c=p(x)$$

$$\int \frac{q(x)}{p(x)}\,dx$$

$$\Delta \geq 0$$

$$ax^2+bx+c=a(x-x_1)(x-x_2)$$

$$\int \frac{x+3}{x^2-6x}\,dx = \int \frac{x+3}{x(x-6)}\,dx$$

$$\frac{x+3}{x(x-6)} = \frac{A}{x} + \frac{B}{x-6} = \frac{Ax-6A+Bx}{x(x-6)}$$

$$x+3 = (A+B)x - 6A$$

PRINCIPIO D'IDENTITÀ DEI POLINOMI

$$\begin{cases} 1 = A+B \\ 3 = -6A \end{cases}$$

$$\begin{cases} 1+\frac{1}{2} = +B \\ A = -\frac{1}{2} \end{cases} \qquad \begin{cases} B = +\frac{3}{2} \\ A = -\frac{1}{2} \end{cases}$$

VERIFICA

$$\left[-\frac{1}{2x} + \frac{3}{2(x-6)} = \frac{-x+6+3x}{2x(x-6)} = \frac{2x+6}{2x(x-6)} = \frac{x+3}{x(x-6)} \right]$$

$$\int \frac{x+3}{x^2-6x}\,dx = -\frac{1}{2}\int \frac{1}{x}\,dx + \frac{3}{2}\int \frac{1}{x-6}\,dx =$$

$$= -\frac{1}{2}\log|x| + \frac{3}{2}\log|x-6| + c$$

$$= \log\sqrt{\frac{|x-6|^3}{|x|}} + c$$

$$\frac{x+3}{x(x-6)} = \frac{A}{x} + \frac{B}{x-6} \qquad x \neq 0 \quad x \neq 6$$

MOLTIPLICO AMBO I MEMBRI PER x

$$\frac{x+3}{x-6} = A + \frac{Bx}{x-6}$$

PASSO AL LIMITE PER $x \to 0$

PER $x-6$

$$\frac{x+3}{x} = \frac{A(x-6)}{x} + B$$

PASSO AL LIM. $x \to 6$

$3 \cdot$ _ R

$$-\frac{1}{2} = -\frac{3}{6} = A$$

$2\cancel{A}$

$$\int \frac{x}{(x-3)^2}\,dx$$

$$\frac{x}{(x-3)^2} = \frac{A}{(x-3)} + \frac{B}{(x-3)^2}$$

$$\frac{x}{(x-3)^2} = \frac{Ax-3A+B}{(x-3)^2}$$

$$\begin{cases} A=1 \\ -3A+B=0 \end{cases} \qquad \begin{cases} A=1 \\ B=3 \end{cases}$$

$$\int \frac{1}{x-3}\,dx + 3\int \frac{1}{(x-3)^2}\,dx \qquad \int f(x)^p \cdot f'(x)\,dx = \frac{f^{p+1}}{p+1}+c$$

$$= \log|x-3| + 3\int (x-3)^{-2}\,dx \qquad x-3=f \quad f'=1 \quad p=-2$$

$$= \log|x-3| + 3\,\frac{(x-3)^{-2+1}}{-1} + c = \log|x-3| - \frac{3}{(x-3)} + c$$

$$\int \frac{3x-1}{x^2-x-6}\,dx$$

$$= \frac{7}{5}\log|x+2| + \frac{8}{5}\log|x-3| + c$$

$$\int \frac{x^4-10x^3+20x^2+11x-20}{x^2-10x+21}\,dx$$

$$= \frac{x^3}{3} - x - \log|x-3| + 2\log|x-7| + c$$

$$\int \frac{3x+1}{x^2-6x+5}\,dx$$

$$= -\log|x-1| + 4\log|x-5| + c$$

$$\int \frac{3x-1}{x^2-x-6}\,dx \qquad x^2-x-6=0 \qquad x^2-x-6=(x-3)(x+2)$$

$$\frac{3x-1}{x^2-x-6} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$A = \frac{8}{5} \qquad B = \frac{+7}{+5}$$

$$\frac{\ }{5}\int\frac{\ }{x-3}dx+\frac{\ }{5}\int\frac{\ }{x+2}dx=\frac{8}{5}\log|x-3|+\frac{7}{5}\log|x+2|+c$$

$$\int\frac{x^4-10x^3+20x^2+11x-20}{x^2-10x+21}dx$$

$$\begin{array}{rrrrr|l}
x^4 & -10x^3 & +20x^2 & +11x & -20 & x^2-10x+21 \\
\hline
-x^4 & +10x^3 & -21x^2 & + & & x^2-1 \\
\hline
// & // & -x^2 & +11x & -20 & \\
 & & x^2 & -10x & +21 & \\
\hline
 & & // & x & +1 &
\end{array}$$

$$\int(x^2-1)dx+\int\frac{x+1}{x^2-10x+21}dx$$

$$\frac{x+1}{x^2-10x+21}=\frac{A}{(x-7)}+\frac{B}{(x-3)}=\frac{Ax-3A+Bx-7B}{(x-7)(x-3)}$$

$$\begin{cases}1=A+B\\ 1=-3A-7B\end{cases}\quad\begin{cases}A=1-B\\ 1=-3(1-B)-7B\end{cases}\quad\begin{cases}A=2\\ B=-1\end{cases}$$

$$1=-3+3B-7B$$

$$4=-4B$$

$$=\frac{x^3}{3}-x+2\int\frac{1}{x-7}dx-\int\frac{1}{x-3}dx=\frac{x^3}{3}-x+2\log|x-7|-$$

$$-\log|x-3|+c$$

$$\int\frac{3x+1}{x^2-6x+5}dx$$

$$\frac{3x+1}{x^2-6x+5}=\frac{A}{(x-5)}+\frac{B}{(x-1)}=\frac{Ax-A+Bx-5B}{(x-5)(x-1)}$$

$$\begin{cases}A+B=3\\ -A-5B=1\end{cases}\quad\begin{cases}A=3-B\\ B-3-5B=1\end{cases}\quad\begin{cases}A=4\\ B=-1\end{cases}$$

$$-4B=4$$

$$\int\frac{4}{x-5}dx-\int\frac{1}{x-1}dx=4\log|x-5|-\log|x-1|+c$$

$$\int \frac{P(x)}{Q(x)}\,dx$$

Sia $Q(x)$ un polinomio di grado n e con coefficiente di termine di grado x^n uguale a 1

CASO 1 RADICI TUTTE SEMPLICI

$$Q(x) = (x-x_1)(x-x_2)\cdots(x-x_r)(x^2+p_1x+q_1)(x^2+p_2x+q_2)\cdots(x^2+p_sx+q_s)$$

ESEMPIO $Q(x) = x^3-1 = (x-1)(x^2+x+1)$

$$\frac{P(x)}{Q(x)} = \frac{A_1}{(x-x_1)} + \frac{A_2}{(x-x_2)} + \cdots + \frac{A_r}{(x-x_r)} + \frac{B_1x+C_1}{x^2+p_1x+q_1} + \cdots + \frac{B_sx+C_s}{x^2+p_sx+q_s}$$

$$\int \frac{1}{x^3-1}\,dx \qquad \frac{1}{x^3-1} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

$$\frac{1}{x^3-1} = \frac{Ax^2+Ax+A+Bx^2-Bx+Cx-C}{x^3-1}$$

$$\begin{cases} A+B=0 \\ A-B+C=0 \\ A-C=1 \end{cases} \qquad \begin{cases} A=-B \\ A+A+A-1=0 \\ C=A-1 \end{cases} \qquad \begin{aligned} B &= -1/3 \\ A &= 1/3 \\ C &= -2/3 \end{aligned}$$

continuo...

$$\frac{1}{3}\int \frac{1}{x-1}\,dx + \frac{1}{3}\int \frac{-x-2}{x^2+x+1}\,dx$$

2 CASO $Q(x)$ ha radici reali multiple e radici complesse semplici

$$\frac{P(x)}{Q(x)} = \frac{A_1}{x-x_1} + \frac{A_2}{(x-x_1)^2} + \cdots + \frac{A_{m_1}}{(x-x_1)^{m_1}} + \cdots + \frac{B_1}{(x-x_r)} + \cdots$$

$$\cdots + \frac{B_{m_r}}{(x-x_r)^{m_r}} + \frac{C_1x+D_1}{x^2+p_1x+q_1} + \cdots + \frac{C_sx+D_s}{x^2+p_sx+q_s}$$

ESEMPIO

$$(x-2)^4(x-1)^6(x+7)^9(x^2+x+1)$$

$$\frac{A_1}{\quad} + \frac{A_2}{\quad} + \frac{A_3}{\quad} + \frac{A_4}{\quad} + \frac{B_1}{\quad} \cdots \frac{B_6}{\quad} \cdots$$

$$x-2 \quad , \quad (x-2)^2 \quad , \quad (x-2)^3 \quad , \quad (x-2)^4 \quad , \quad (x-1) + \dots \quad (x-1)^6 + \dots$$

$$\dots + \frac{C_1 x + D_1}{x^2+x+1}$$

3 CASO $\quad Q(x)$ ha radici reali e complesse multiple

$$Q(x) = (x-x_1)^{m_1} \dots (x-x_r)^{m_r} (x^2+p_1x+q_1)^{m_1} \dots (x^2+p_sx+q_s)^{m_s}$$

ESEMPIO $\quad Q(x) = (x-1)^2 (x^2+x+1)^3$

$$\frac{P(x)}{Q(x)} = \frac{A_{1,1}}{x-x_1} + \frac{A_{1,2}}{(x-x_1)^2} + \dots + \frac{A_{1,m_1}}{(x-x_1)^{m_1}} + \dots + \frac{A_{r,1}}{(x-x_r)} + \dots \frac{A_{r,m_r}}{(x-x_r)^{m_r}}$$

$$+ \frac{C_{1,1}x+D_{1,1}}{x^2+p_1x+q_1} + \frac{C_{1,2}x+D_{1,2}}{(x^2+p_1x+q_1)^2} + \dots + \frac{C_{1,m_1}x+D_{1,m_1}}{(x^2+p_1x+q_1)^{m_1}} + \dots +$$

$$\frac{C_{s,1}x+D_{s,1}}{x^2+p_sx+q_s} + \frac{C_{s,2}x+D_{s,2}}{(x^2+p_sx+q_s)^2} + \dots + \frac{C_{s,m_s}x+D_{s,m_s}}{(x^2+p_sx+q_s)^{m_s}}$$

$$\int \frac{1}{x^2(x^2+1)^2}\, ds$$

$$\frac{1}{x^2(x^2+1)^2} = \frac{A_1}{x} + \frac{A_2}{x^2} + \frac{C_1x+D_1}{x^2+1} + \frac{C_2x+D_2}{(x^2+1)^2}$$

mettendo a sistema si trova

$$A_1 = C_1 = C_2 = 0$$
$$A_2 = 1$$
$$D_1 = D_2 = -1$$

$$\int \frac{1}{x^2}\,dx - \int \frac{1}{x^2+1}\,dx - \boxed{\int \frac{1}{(x^2+1)^2}\,dx} = -\frac{1}{x} - \operatorname{arctg} x - \frac{1}{2}\left[\operatorname{arctg} x + \frac{x}{1+x^2}\right] + c$$

VERIFICARE CHE PER OGNI $m > 1$ SI HA,

POSTO $\quad I_m = \int \frac{1}{(x^2+1)^m}\,dx \qquad I_1 = \operatorname{arctg} x$

$$I_m = \frac{1}{2(m-1)}\left[(2m-3)\, I_{n-1} + \frac{\quad}{(1+x^2)^{m-1}}\right]$$

$$\int \frac{1}{(x^2+1)^2} = I_2 = \frac{1}{2}\left[\quad I_1 + \frac{x}{(1+x^2)}\right]$$

$$= \frac{1}{2}\left[\operatorname{arctg} x + \frac{x}{1+x^2}\right] + c$$

DIMOSTRIAMO LA FORMULA

$$\int f g' = fg - \int f' g$$

$$I_{m-1} = \int \frac{1}{(1+x^2)^{m-1}}\, dx =$$

$$f = \frac{1}{(1+x^2)^{m-1}} = (1+x^2)^{1-m} \qquad f' = (1-m)(1+x^2)^{-m} \cdot 2x = (1-n)\frac{2x}{(1+x^2)^n}$$

$$g' = 1 \qquad g = x$$

$$= \frac{x}{(1+x^2)^{m-1}} - 2(1-m)\int \frac{x^2+1-1}{(1+x^2)^m}\, dx$$

$$I_{n-1} = \frac{x}{(1+x^2)^{m-1}} - 2(1-m)\underbrace{\int \frac{1}{(1+x^2)^{m-1}}\, dx}_{I_{m-1}} + 2(1-m)\underbrace{\int \frac{1}{(1+x^2)^n}\, dx}_{I_n}$$