INTEGRALI FUNZIONI TRIGONOMETRICHE

$\int f(\sin x, \cos x)\,dx$ f razionale

$\int \frac{\sin x}{1+\cos^2 x}\,dx$

ESERCIZI

$\int \frac{1}{\cos x - 1}\,dx$

$\int \frac{1}{1+\sin x - \cos x}\,dx$

$t = \operatorname{tg}\frac{x}{2}$ $\quad 2\arctan t = x$ $\quad \frac{2}{1+t^2}\,dt = dx$

$\sin x = \frac{2t}{1+t^2}$ $\quad \cos x = \frac{1-t^2}{1+t^2}$

$\int f\left(\frac{2t}{1+t^2}, \frac{1-t^2}{1+t^2}\right)\frac{2}{1+t^2}\,dt$

$$\int \frac{1}{\cos x - 1}\,dx = \int \frac{1}{\frac{1-t^2}{1+t^2} - 1}\,\frac{2}{1+t^2}\,dt = \int \frac{1}{\frac{1-t^2-1-t^2}{1+t^2}}\cdot\frac{2}{1+t^2}\,dt$$

$$= \int \frac{2}{-2t^2}\,dt = -\int t^{-2}\,dt = \frac{1}{t} + c = \frac{1}{\operatorname{tg}\frac{x}{2}} + c$$

$$\int \frac{1}{1+\sin x - \cos x}\,dx = \int \frac{1}{1+\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2}}\,\frac{2}{1+t^2}\,dt =$$

$$= \int \frac{1}{\frac{1+t^2+2t-1+t^2}{1+t^2}}\cdot\frac{2}{1+t^2}\,dt = \int \frac{2}{2t^2+2t}\,dt = \int \frac{1}{t(t+1)}\,dt$$

$$\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1}$$

$$\frac{1}{t+1} = A + \frac{Bt}{t+1} \Rightarrow A = 1$$

$$\frac{1}{t} = \frac{A(t+1)}{t} + B \rightarrow B = -1$$

$$\int \frac{dt}{t} - \int \frac{1}{t+1}\,dt = \log|t| - \log|t+1| + c$$

$$= \log\left|\operatorname{tg}\frac{x}{2}\right| - \log\left|\operatorname{tg}\frac{x}{2}+1\right| + c$$

$2\operatorname{tg}\frac{x}{2}$ $\quad 2\cdot\frac{\sin x/2}{\cos x/2}$ $\quad \sin x/2$ ...

$$\frac{2t}{1+t^2} = \frac{2\,tg\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{2\,\frac{\operatorname{sen}\frac{x}{2}}{\cos\frac{x}{2}}}{1+\frac{\operatorname{sen}^2\frac{x}{2}}{\cos^2\frac{x}{2}}} = \frac{2\,\frac{\operatorname{sen}\frac{x}{2}}{\cos\frac{x}{2}}}{\frac{\cos^2\frac{x}{2}+\operatorname{sen}^2\frac{x}{2}}{\cos^2\frac{x}{2}}}$$

$$= 2\cos\frac{x}{2}\operatorname{sen}\frac{x}{2} = \operatorname{sen}2\frac{x}{2} = \operatorname{sen}x$$

$$\operatorname{sen}2\alpha = 2\operatorname{sen}\alpha\cos\alpha$$

$$\int f\left(\operatorname{sen}^2x, \cos^2x, tgx\right)dx$$

EJERCICIO

$$\int \frac{tgx}{1+\operatorname{sen}^2x}\,dx$$

$$tgx = t \qquad x = \operatorname{arctg}t \qquad dx = \frac{1}{1+t^2}\,dt$$

$$\operatorname{sen}^2x = \frac{t^2}{1+t^2} \qquad \cos^2x = \frac{1}{1+t^2}$$

$$\int f\left(\frac{t^2}{1+t^2}, \frac{1}{1+t^2}, t\right)\frac{1}{1+t^2}\,dt$$

$$\int \frac{\operatorname{sen}x}{1+\cos^2x}\,dx$$

$$t = tg\frac{x}{2} \qquad \operatorname{sen}x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2}$$

$$2\operatorname{arctg}t = \frac{x}{2} \qquad \frac{2}{1+t^2}\,dt = dx$$

$$\int \frac{2t}{(1+t^2)} \cdot \frac{1}{1+\frac{(1-t^2)^2}{(1+t^2)^2}} \cdot \frac{2}{(1+t^2)}\,dt = \int \frac{4t}{(1+t^2)^2\left(\frac{(1+t^2)^2+(1-t^2)^2}{(1+t^2)^2}\right)}\,dt$$

$$\int \frac{4t}{1+t^4+2t^2+1+t^4-2t^2}\,dt = \int \frac{2\cdot 2t}{2+2t^4}\,dt = \int \frac{2t}{1+t^4}\,dt$$

$$f = t^2 \qquad f' = 2t$$

$$\int \frac{1}{1+f^2}f'\,dt = \operatorname{arctg}t^2 + c$$

$$= \left(tg\frac{x}{2}\right)^2$$

$= \operatorname{arctg}(\bar{v}\,\bar{i}) + C$

$$\int \frac{\operatorname{tg} x}{1+\operatorname{sen}^2 x}\,dx$$

$t = \operatorname{tg} x \qquad \operatorname{arctg} t = x \qquad \frac{1}{1+t^2}\,dt = dx$

$$\operatorname{sen}^2 x = \frac{t^2}{1+t^2}$$

$$\int \frac{t}{1+\frac{t^2}{1+t^2}} \cdot \frac{1}{1+t^2}\,dt = \int \frac{t}{\frac{1+t^2+t^2}{(1+t^2)} \cdot (1+t^2)}\,dt = \frac{1}{4}\int \frac{4t}{1+2t^2}\,dt$$

$f = 1+2t^2 \qquad f' = 4t$

$$= \frac{1}{4}\log(1+2t^2) + C = \frac{1}{4}\log(1+2\operatorname{tg}^2 x) + C$$

$$\int \operatorname{sen}^2 x\,dx \qquad \int \cos^2 x\,dx$$

$$\operatorname{sen}^2 x = \frac{1-\cos 2x}{2} \qquad \cos^2 x = \frac{1+\cos 2x}{2}$$

$$\int \operatorname{sen}^2 x\,dx \int \left(\frac{1}{2} - \frac{\cos 2x}{2}\right) dx = \frac{1}{2}x - \frac{1}{4}\int 2\cdot\cos 2x\,dx$$

$$= \frac{1}{2}x - \frac{1}{4}\operatorname{sen} 2x + C$$

$$\int \cos^2 x\,dx = \frac{1}{2}x + \frac{1}{4}\operatorname{sen} 2x + C$$

$$\int \operatorname{sen}^4 x\,dx \qquad \int \operatorname{sen}^3 x\,dx$$

$$\int \operatorname{sen}^2 x \cdot \operatorname{sen} x\,dx =$$

$$= \int (1-\cos^2 x)\operatorname{sen} x\,dx$$

$$= \int \operatorname{sen} x\,dx - \int \cos^2 x \operatorname{sen} x\,dx$$

$$= -\cos x + \cos^3$$