Eser 6/12

lunedì 6 dicembre 2021 12:18

INTEGRALI DEFINITI

CALCOLARE L'INTEGRALE

$$\int_0^3 |x-1|\,dx$$

$$\int_1^2 \frac{\log x}{x}\,dx$$

$$\int_0^4 \frac{x-7\sqrt{x}+12}{x\sqrt{x}-6x+9\sqrt{x}}\,dx$$

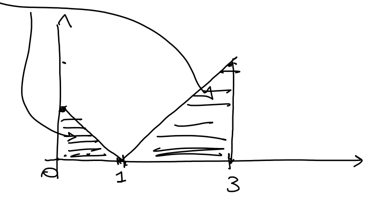
$$\int_0^3 |x-1|\,dx$$

$$|x-1| = \begin{cases} x-1 & \text{se } x \geq 1 \\ 1-x & \text{se } x < 1 \end{cases}$$

$x \in [0,3]$

$$\int_0^1 (-x+1)\,dx + \int_1^3 (-1+x)\,dx = \left[-\frac{x^2}{2}+x\right]_0^1 + \left[-x+\frac{x^2}{2}\right]_1^3 =$$

$$= -\frac{1}{2}+1-3+\frac{9}{2}-\left(-1+\frac{1}{2}\right) = +\frac{8}{2}-2+\frac{1}{2} = +2+\frac{1}{2} = \frac{5}{2}$$



$\log x = f$

$\frac{1}{x} = f'$ $\quad \int f \cdot f'\,dx = \frac{f^2}{2}+c$

$$\int_1^2 \frac{\log x}{x}\,dx = \left[\frac{\log^2 x}{2}\right]_1^2 = \frac{\log^2 2}{2}$$

SOLUZIONE ALTERNATIVA

$\log x = f \qquad f' = \frac{1}{x}$

$\frac{1}{x} = g' \qquad g = \log x$

$$\int_1^2 \frac{\log x}{x}\,dx = \left[\log^2 x\right]_1^2 - \int_1^2 \frac{\log x}{x}\,dx$$

$$2\int_1^2 \frac{\log x}{x}\,dx = \left[\log^2 x\right]_1^2$$

$$\int_1^2 \frac{\log x}{x}\,dx = \frac{1}{2}\left[\log^2 x\right]_1^2 = \frac{\log^2 2}{2}$$

$$\int_0^4 \frac{x-7\sqrt{x}+12}{x\sqrt{x}-6x+9\sqrt{x}}\,dx$$

se $x=0$ allora $\sqrt{0}=0=t$

se $x=4$ allora $\sqrt{4}=2=t$

$$\int_{\varepsilon}^{4} \frac{x - 7\sqrt{x} + 12}{x\sqrt{x} - 6x + 9\sqrt{x}}\, dx \qquad \sqrt{x} = t \quad x = t^2 \qquad dx = 2t\, dt$$

1) 3

$$\int \frac{t^2 - 7t + 12}{t^3 - 6t^2 + 9t} = \int 2\frac{t^{\cancel{3}\,2} - 7t^{\cancel{2}} + 12\cancel{t}}{\cancel{t}(t^2 - 6t + 9)}\, dt = 2\int \frac{t^2 - 7t + 12}{t^2 - 6t + 9}$$

$$= 2\int \frac{(t-4)\cancel{(t-3)}}{(t-3)^{\cancel{2}}}\, dt = 2\int \frac{t-4}{t-3}\, dt = 2\int \frac{t-3-1}{t-3}\, dt = 2\int \left(1 - \frac{1}{t-3}\right) dt$$

$$2\int dt - 2\int \frac{1}{t-3}\, dt = 2t - 2\log|t-3| + c$$

$$\int f(x)\,dx = 2\sqrt{x} - 2\log|\sqrt{x} - 3| + c$$

$$\int_0^4 f(x)\,dx = \lim_{\varepsilon\to 0^+} \Big[2\sqrt{x} - 2\log|\sqrt{x} - 3|\Big]_{\varepsilon}^{4} = \lim_{\varepsilon \to 0^+} \Big[4 - 2\cancel{\log|1|} - \underbrace{2\sqrt{\varepsilon}}_{0} + 2\log|\sqrt{\varepsilon} - 3|\Big]$$

$$= 4 + 2\log 3$$

NON È NECESSARIO TORNARE ALLA VARIABILE x
MA DEVO RICORDARMI DI CAMBIARE GLI ESTREMI D'INTEGRAZIONE

$$\int_0^2 g(t)\, dt = \lim_{\varepsilon\to 0^+}\Big[2t - 2\log|t-3|\Big]_{\varepsilon}^{2} = \lim_{\varepsilon\to 0^+}\Big[4 - 2\cancel{\log|1|} - \underbrace{2\varepsilon}_{0} + 2\log|\varepsilon - 3|\Big]$$

$$4 + 2\log 3$$

INTEGRALI IMPROPRI / GENERAL.

CALCOLARE

$$\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}\, dx \qquad\qquad \int_2^3 \frac{x(x+1)}{\sqrt{9-x^2}}\, dx$$

USARE $x = 3\,\text{sen}\,t$

$$\int_0^9 \frac{1}{\sqrt[3]{(x-1)^2}}\, dx$$

$$\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}\, dx$$

Dominio di $\frac{1}{\sqrt{1-x^2}}$ $\quad x \neq \pm 1 \quad \lim_{x\to\pm 1} \frac{1}{\sqrt{1-x^2}} = +\infty$

$$\int_{-1}^{0} \frac{1}{\sqrt{1-x^2}}\, dx + \int_0^1 \frac{1}{\sqrt{1-x^2}}\, dx$$

$$\lim_{a\to -1^+} \int_a^0 \frac{1}{\sqrt{1-x^2}} + \lim_{b\to 1^-} \int_0^b \frac{1}{\sqrt{1-x^2}}\,dx$$

$$\lim_{a\to -1^+} \left[\arcsin x\right]_a^0 + \lim_{b\to 1^-} \left[\arcsin x\right]_0^b =$$

$$\lim_{a\to -1^+} -\arcsin a + \lim_{b\to 1^-} \arcsin b = \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

$$-\arcsin(-1) = +\frac{\pi}{2}$$

$$\int_0^9 \frac{1}{\sqrt[3]{(x-1)^2}}\,dx$$

Dominio di $(x-1)^{-2/3}$ è $x \neq 1$

$$\int_0^1 (x-1)^{-2/3}\,dx + \int_1^9 (x-1)^{-2/3}\,dx =$$

$$\lim_{a\to 1^-} \int_0^a (x-1)^{-2/3}\,dx + \lim_{b\to 1^+} \int_b^9 (x-1)^{-2/3}\,dx =$$

$$\lim_{a\to 1^-} \left[\frac{(x-1)^{1/3}}{\frac{1}{3}}\right]_0^a + \lim_{b\to 1^+} \left[3(x-1)^{1/3}\right]_b^9 =$$

$$\lim_{a\to 1^-} \left[3(a-1)^{1/3} + 3\right] + \lim_{b\to 1^+} \left[3(8)^{1/3} - 3(b-1)^{1/3}\right]$$

$$3 + 6 = 9$$

$$\int_2^3 \frac{x(x+1)}{\sqrt{9-x^2}}\,dx = 3 + \frac{9}{4}\pi$$

Dominio $x \neq \pm 3$

$$\lim_{a\to 3^-} \int_2^a \frac{x(x+1)}{\sqrt{9-x^2}}\,dx =$$

$x = 3\sin t$

$dx = 3\cos t\,dt$

$$\int \frac{3\sin t(3\sin t + 1)}{\sqrt{9 - 9\sin^2 t}} \cdot 3\cos t\,dt =$$

$\frac{x}{3} = \text{sen}\, t$

$\text{arc sen}\, \frac{x}{3} = t$

$$\int \frac{\cancel{9}^{3} \text{sen}\,t\,(3\,\text{sen}\,t + 1)}{\cancel{3}\,\cancel{\cos t}} \cdot \cancel{\cos t}\, dt =$$

$$\int 9\,\text{sen}^2 t\, dt + \int 3\,\text{sen}\,t\, dt =$$

$$\text{sen}^2 t = \frac{1 - \cos 2t}{2}$$

$$\int \frac{9}{2}(1 - \cos 2t)\, dt - 3\cos t =$$

$$\frac{9}{2}t - \frac{9}{4}\,\text{sen}\,2t - 3\cos t + C$$

se $x \in [2, 3]$

$x = 3\,\text{sen}\,t$

$\frac{x}{3} = \text{sen}\,t$

$\text{arc sen}\, \frac{x}{3} = t$

$t \in \left[\text{arc sen}\,\frac{2}{3}, \frac{\pi}{2}\right]$

$$\frac{9}{2}t - \frac{9}{\cancel{4}_{2}}\cancel{2}\,\text{sen}\,t \cdot \cos t - 3\cos t + C$$

$\cos t = \sqrt{1 - \text{sen}^2 t}$

$$\cos\left(\text{arc sen}\,\frac{2}{3}\right) = \sqrt{1 - \left(\frac{2}{3}\right)^2} = \sqrt{1 - \frac{4}{9}} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$$

$$\lim_{a \to \frac{\pi}{2}} \int_{\text{arc sen}\,2/3}^{a} g(t)\, dt = \lim_{a \to \frac{\pi}{2}} \left[\frac{9}{2}t - \frac{9}{2}\,\text{sen}\,t \cos t - 3\cos t\right]_{\text{arc sen}\,\frac{2}{3}}^{a} =$$

$$= \lim_{a \to \frac{\pi}{2}} \left[\frac{9}{2}a - \frac{9}{2}\,\text{sen}\,a \cos a - 3\cos a - \frac{9}{2}\,\text{arc sen}\,\frac{2}{3} + \frac{\cancel{9}}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{3}} \frac{\sqrt{5}}{\cancel{3}} + \cancel{3}\frac{\sqrt{5}}{\cancel{3}}\right]$$

$$= \frac{9}{4}\pi - \frac{9}{2}\,\text{arc sen}\,\frac{2}{3} + 2\sqrt{5}$$