Eserc 15/12

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EQ DIFF. LINEARI OMOGENEE A COEFF. COSTANTI

LIN. II° ORDINE $\quad y'' + a y' + b y = f(x)$

OMOGENEA se $f(x)=0$

$$y'' + a y' + b y = 0$$

EQ. CARATTERISTICA

$$\lambda^2 + a\lambda + b = 0$$

$$\lambda_{1,2} = \frac{-a \pm \sqrt{a^2-4b}}{2} = \frac{-a \pm \sqrt{\Delta}}{2}$$

se $\Delta > 0 \Rightarrow \lambda_1 \neq \lambda_2$

$$y(x) = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$$

se $\Delta = 0 \Rightarrow \lambda_1 = \lambda_2$

$$y(x) = c_1 e^{\lambda_1 x} + c_2 x e^{\lambda_1 x}$$

se $\Delta < 0$, $\alpha = \frac{-a}{2}$ $\beta = \frac{\sqrt{-\Delta}}{2}$

$$y(x) = c_1 e^{\alpha x} \cos\beta x + c_2 e^{\alpha x} \operatorname{sen} \beta x$$

$y'' - 6y' + 5y = 0$ A

$y'' - 2y' + 2y = 0$ B

$y'' - 2y' + y = 0$ C

1. Se trovo due sol. particolari y_1 e y_2 $\Rightarrow$ $\boxed{c_1 y_1 + c_2 y_2 \text{ è sol.}}$

$$y'' + a y' + b y = 0$$

$$(c_1 y_1 + c_2 y_2)'' + a(c_1 y_1 + c_2 y_2)' + b(c_1 y_1 + c_2 y_2)$$

$$= \cancel{c_1 y_1''} + \cancel{c_2 y_2''} + \cancel{a c_1 y_1'} + \cancel{a c_2 y_2'} + \cancel{b c_1 y_1} + \cancel{b c_2 y_2}$$

$= 0$

2. $y_1 = e^{\lambda_1 x}$ $\quad y_1' = \lambda_1 e^{\lambda_1 x}$ $\quad y_1'' = \lambda_1^2 e^{\lambda_1 x}$

$$\lambda_1^2 e^{\lambda_1 x} + a\lambda_1 e^{\lambda_1 x} + b e^{\lambda_1 x} =$$

$$e^{\lambda_1 x}(\lambda_1^2 + a\lambda_1 + b) = 0$$

$y_2 = e^{\lambda_2 x}$ VERIFICO CHE È SOL.

3. Se y_1, y_2 / $\forall x \in \mathbb{R}$

$$c_1 y_1(x) + c_2 y_2(x) = 0 \Rightarrow c_1 = c_2 = 0$$

$\Rightarrow$ TUTTE LE SOL DELL'EQ (*)
SONO $y(x) = c_1 y_1 + c_2 y_2$

EX (A)

$$y'' - 6y' + 5y = 0$$

EQ CARATTER $\lambda^2 - 6\lambda + 5 = 0$

$\lambda_1 = 1 \qquad \lambda_2 = 5$

$$y(x) = c_1 e^{x} + c_2 e^{5x}$$

(B) $y'' - 2y' + 2y = 0$

$\lambda^2 - 2\lambda + 2 = 0 \qquad 1 \pm \sqrt{1-2} = 1 \pm \sqrt{-1} = 1 \pm i$

$\alpha = 1 \qquad \beta = \sqrt{1} = 1$

$$y(x) = c_1 e^{x} \cos x + c_2 e^{x} \operatorname{sen} x$$

(C) $y'' - 2y' + y = 0$

$\lambda^2 - 2\lambda + 1 = 0 \qquad \lambda = 1$

$$y(x) = c_1 e^{x} + c_2 x e^{x}$$

(1) $y'' + y = 0$

(2) $y'' + 3y = 0$

(3) $y'' - 2y' = 0$

(4) PB DI CAUCHY

$$\begin{cases} y'' - 2y' - y = 0 \\ y(0) = 0 \\ y'(0) = 2\sqrt{2} \end{cases}$$

$y'' + y = 0$ (1)

$\lambda^2 + 1 = 0 \qquad \lambda = \pm\sqrt{-1} = \pm i$

$\alpha = 0 \qquad \beta = 1$

$$y(x) = c_1 \cos x + c_2 \operatorname{sen} x$$

$y'' + 3y = 0$ (2)

$\lambda^2 + 3 = 0 \qquad \lambda^2 = -3 \qquad \lambda = \pm\sqrt{-3} = \sqrt{-1}\sqrt{3} = \pm\sqrt{3}\,i$

$\alpha = 0 \qquad \beta = \sqrt{3}$

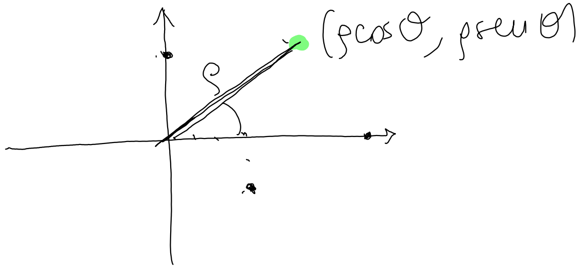
$y(x) = c_1 \cos\sqrt{3}\,x + c_2 \operatorname{sen}\sqrt{3}\,x$

$y'' - 2y' = 0$ (3)

$\lambda^2 - 2\lambda = 0 \qquad \lambda(\lambda - 2) = 0 \qquad \lambda = 0, \quad \lambda = 2$

$y(x) = c_1 + c_2 e^{2x}$

COMPLESSI



$\sqrt{-1} = i$

$i^2 = -1$

reale

$x + i\,y$ complessa immag. (x, y)

$\boxed{3-2i} \qquad 5+7i \qquad \mathbb{R} \subset \mathbb{C}$

$\sqrt{3}\,i \qquad 7$

$(0, \sqrt{3})$

$y'' - 2y' + 5y = 0$

$\lambda^2 - 2\lambda + 5 = 0$

$\lambda = 1 \pm \sqrt{1-5} = 1 \pm \sqrt{-4}$

$= 1 \pm 2i$

$y(x) = c_1 e^{x} \cos 2x + c_2 e^{x} \operatorname{sen} 2x$

(4)
$$\begin{cases} y'' - 2y' - y = 0 \\ y(0) = 0 \\ y'(0) = 2\sqrt{2} \end{cases}$$

$$\lambda^2 - 2\lambda - 1 = 0 \qquad \lambda = 1 \pm \sqrt{1+1} = 1 \pm \sqrt{2}$$

$$y(x) = c_1 e^{(1-\sqrt{2})x} + c_2 e^{(1+\sqrt{2})x}$$

$$y(0) = \boxed{c_1 + c_2 = 0}$$

$$y'(x) = c_1 (1-\sqrt{2}) e^{(1-\sqrt{2})x} + c_2 (1+\sqrt{2}) e^{(1+\sqrt{2})x}$$

$$\begin{cases} c_1 + c_2 = 0 \\ c_1(1-\sqrt{2}) + c_2(1+\sqrt{2}) = 2\sqrt{2} \end{cases} \qquad \begin{cases} c_1 = -c_2 \\ \cancel{c_1} - c_1\sqrt{2} - \cancel{c_1} - c_1\sqrt{2} = 2\sqrt{2} \end{cases}$$

$$\begin{cases} c_1 = -c_2 \\ c_1 = -1 \end{cases} \qquad c_2 = 1$$

$$y(x) = -e^{(1-\sqrt{2})x} + e^{(1+\sqrt{2})x}$$

$$\begin{cases} y'' - y' - 2y = 0 \\ y(0) = 0 \end{cases} \qquad \begin{cases} xy' = 1 + y^2 \\ y(-1) = 1 \end{cases} \qquad \begin{cases} xy' = 1 + y^2 \\ y(1) = 1 \end{cases}$$

① $\left\{ y'(0)=3 \right.$ ② ③

① $\lambda^2-\lambda-2=0$

$$\lambda=\frac{1\pm\sqrt{1+8}}{2}=\frac{1\pm 3}{2}\begin{cases}2\\-1\end{cases}$$

$$y(x)=c_1e^{-x}+c_2e^{2x} \quad \Big| \quad y'(x)=-c_1e^{-x}+2c_2e^{2x}$$

$$\begin{aligned} y(0)&=c_1+c_2=0\\ y'(0)&=-c_1+2c_2=3\end{aligned} \quad \begin{cases}c_1=-c_2\\ c_2+2c_2=3\end{cases} \quad \begin{cases}c_1=-1\\ c_2=1\end{cases}$$

$$y(x)=-e^{-x}+e^{2x}$$

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② / ③

$$xy'=1+y^2$$

$$\int\frac{dy}{1+y^2}=\int\frac{dx}{x} \qquad x\neq 0$$

$$\operatorname{arctg} y = \log|x| + c \qquad x \in (-\infty, 0) \cup (0, +\infty)$$

$$\begin{cases} x y' = 1 + y^2 \\ y(-1) = 1 \end{cases}$$

$$\operatorname{arctg} y = \log(-x) + c$$

$$y = \operatorname{tg}(\log(-x) + c)$$

$$1 = \operatorname{tg}(\log 1 + c)$$

$$1 = \operatorname{tg} c$$

$$c = \pi/4$$

$$y(x) = \operatorname{tg}\left(\log(-x) + \frac{\pi}{4}\right)$$

$$\begin{cases} \\ y(1) = 1 \end{cases}$$

$$\operatorname{arctg} y = \log x + c$$

$$y = \operatorname{tg}(\log x + c)$$

$$1 = y(1) = \operatorname{tg} c$$

$$c = \pi/4$$

$$y(x) = \operatorname{tg}\left(\log x + \frac{\pi}{4}\right)$$

① $\begin{cases} y' = \dfrac{y^2-1}{x^2-1} \\ y(0)=0 \end{cases}$

② $\begin{cases} y'' - 10y' + 25y = 0 \\ y(0)=0 \\ y'(0) = 1 \end{cases}$

③ $\begin{cases} y' + \dfrac{1}{x}y = x^3 \\ y(1) = \dfrac{1}{5} \end{cases}$

① $\begin{cases} y' = \dfrac{y^2-1}{x^2-1} \\ y(0)=0 \end{cases}$

$$\int \frac{dy}{y^2-1} = \int \frac{dx}{x^2-1}$$

$$\frac{1}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$$

$\begin{cases} A+B=0 \\ A-B=1 \end{cases}$

$A = -B \qquad A = 1/2$

$$\begin{cases} A = \ldots \\ B = -1/2 \end{cases}$$

$$\int \frac{dx}{x^2-1} = \frac{1}{2}\log|x-1| - \frac{1}{2}\log|x+1| + c$$

$$= \log\sqrt{\left|\frac{x+1}{x-1}\right|} + c$$

$$\log\sqrt{\left|\frac{y+1}{y-1}\right|} = \log\sqrt{\left|\frac{x+1}{x-1}\right|} + c$$

$x = 0 \qquad y = 0$

$$0 = 0 + c \rightarrow c = 0$$

$$\log\sqrt{\left|\frac{y+1}{y-1}\right|} = \log\sqrt{\left|\frac{x+1}{x-1}\right|}$$

$$y = x$$

② $$\begin{cases} y'' - 10y' + 25y = 0 \\ y(0) = 0 \\ y'(0) = 1 \end{cases}$$

$$\lambda^2 - 10\lambda + 25 = 0 \quad (\lambda - 5)^2 = 0$$

$$\lambda = 5$$

$$y(x) = c_1 e^{5x} + c_2 x e^{5x}$$

$$y'(x) = 5c_1 e^{5x} + c_2 e^{5x} + 5c_2 x e^{5x}$$

$$\begin{cases} y(0) = c_1 = 0 \\ y'(0) = c_2 = 1 \end{cases}$$

$$y(x) = x e^{5x}$$

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$$\begin{cases} y' = -\frac{1}{x} y + x^3 \\ y(1) = 1/5 \end{cases}$$

$$y(x) = e^{A(x)}\left(\int e^{-A(x)} b(x)\,dx + c\right)$$

$$A(x) = -\int \frac{1}{x}\,dx = \ln|x|^{-1} = \ln x^{-1}$$

$$A(x) = \int -\frac{1}{x}\,dx = -\log|x| = \log x^{-1}$$

$$y(x) = \frac{1}{x}\left(\int x^4\,dx + c\right)$$

$$y(x) = \frac{1}{x}\left[\frac{x^5}{5} + c\right]$$

$$y(1) = \frac{1}{5} + c = \frac{1}{5} \Rightarrow c = 0$$

$$y(x) = \frac{x^4}{5}$$