Eser 17/12

venerdì 17 dicembre 2021 12:00

EQ DIFF. LINEARI NON OMOGENEE A COEFF. COSTANTI

(1)
$$y^{(n)} + a_1(x) y^{n-1} + \dots + a_{n-1}(x) y' + a_n(x) y = f(x)$$

TEOREMA Sia v_0 un integrale particolare di (1) e siano $y_1, y_2, \dots, y_n$ n integrali lin indip. di

$$y^n + a_1(x) y^{n-1} + \dots + a_{n-1}(x) y' + a_n(x) y = 0$$

allora la sol. generale di (1) è

$$y(x) = C_1 y_1 + \dots + C_n y_n + v_0$$

$a_i(x) \equiv a_i$ $\quad \lambda^n + a_1 \lambda^{n-1} + \dots + a_n = 0$ "$P(\lambda)$"

$f(x) = e^{\bar\lambda x} p_m(x)$ $\quad$ m grado del polin

$P(\bar\lambda) \neq 0$

- $v_0 = e^{\bar\lambda x} q_m(x)$

$P(\bar\lambda) = 0$

- $v_0 = x^h \cdot e^{\bar\lambda x} q_m(x)$

h è la molteplicità di λ

$$\boxed{y'' - 3y' + 2y = 2x^3 - x^2 + 1}$$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda_1 = 2 \quad \lambda_2 = 1$

$f(x) = \underbrace{2x^3 - x^2 + 1}$ $\quad e^{0x}(2x^3 - x^2 + 1)$

- $v_0(x) = b_0 x^3 + b_1 x^2 + b_2 x + b_3$
- $v_0'(x) = 3b_0 x^2 + 2b_1 x + b_2$
- $v_0''(x) = 6b_0 x + 2b_1$

$$6b_0 x + 2b_1 - 3(3b_0 x^2 + 2b_1 x + b_2) + 2(b_0 x^3 + b_1 x^2 + b_2 x + b_3) = 2x^3 - x^2 + 1$$

$2b_0 = 2 \longrightarrow b_0 = 1$ $\qquad 8 - 27 + 2b_3 = 1$

$-9 + 2b_1 = -1 \quad b_1 = 4$ $\qquad b_3 = 10$

$6 - 24 + 2b_2 = 0 \quad b_2 = 9$

$v_0 = x^3 + 4x^2 + 9x + 10$

$y(x) = c_1 e^{x} + c_2 e^{2x} + x^3 + 4x^2 + 9x + 10$

$y'' - 4y' = x^2 + 1$

$\lambda^2 - 4\lambda = 0 \qquad \lambda(\lambda - 4) = 0 \qquad \lambda = 0 \quad \lambda = 4$

$f(x) = (x^2 + 1)e^{0x}$

$v_0 = x(b_0 x^2 + b_1 x + b_2) = b_0 x^3 + b_1 x^2 + b_2 x$

$v_0' = 3b_0 x^2 + 2b_1 x + b_2$

$v_0'' = 6b_0 x + 2b_1$

$6b_0 x + 2b_1 - 4(3b_0 x^2 + 2b_1 x + b_2) = x^2 + 1$

$6b_0 x + 2b_1 - 12b_0 x^2 - 8b_1 x - 4b_2 = x^2 + 1$

$-12b_0 = 1 \qquad b_0 = -1/12$

$6b_0 - 8b_1 = 0 \qquad -\frac{1}{2} - 8b_1 = 0 \qquad 8b_1 = -\frac{1}{2} \qquad b_1 = -\frac{1}{16}$

$2b_1 - 4b_2 = 1 \qquad -\frac{1}{8} - 4b_2 = 1 \qquad -4b_2 = \frac{9}{8} \qquad b_2 = -\frac{9}{32}$

$y(x) = c_1 + c_2 e^{4x} - \frac{1}{12}x^3 - \frac{1}{16}x^2 - \frac{9}{32}x$

$y'' - 2y' - 3y = 8e^{3x}$

$\lambda^2 - 2\lambda - 3 = 0 \qquad 3 \quad -1$

$v_0 = x e^{3x} c \qquad v_0' = ce^{3x} + c\,3x e^{3x}$

$v_0'' = 3ce^{3x} + 3ce^{3x} + 9cxe^{3x}$

$6c\cancel{e^{3x}} + 9cx\cancel{e^{3x}} - 2c\cancel{e^{3x}} - 6cx\cancel{e^{3x}} - 3cx\cancel{e^{3x}} = 8\cancel{e^{3x}}$

$4c = 8 \qquad c = 2$

$v_0 = 2xe^{3x}$

$y(x) = c_1 e^{-x} + c_2 e^{3x} + 2xe^{3x}$

COSA ACCADE SE CERCO $v_0 = ce^{3x}$

$v_0' = 3ce^{3x}$ $\quad v_0'' = 9ce^{3x}$

$$9ce^{3x} - 6ce^{3x} - 3ce^{3x} = 8e^{3x}$$

$$\underline{9c - 6c - 3c} = 8$$

$0 = 8$ IMPOSSIBILE

$y'' - 2y' + y = xe^x$

$\lambda^2 - 2\lambda + 1 = 0 \quad \lambda = 1 \quad y(x) = c_1e^x + c_2xe^x + v_0(x)$

$v_0 = x^2e^x(bx + c) = e^x(bx^3 + cx^2)$

$v_0' = e^x(bx^3 + cx^2) + e^x(3bx^2 + 2cx)$

$v_0'' = e^x(bx^3 + cx^2) + 2e^x(3bx^2 + 2cx) + e^x(6bx + 2c)$

$\cancel{bx^3} + \cancel{cx^2} + \cancel{6bx^2} + \cancel{4cx} + 6bx + 2c - \cancel{2bx^3} - \cancel{2cx^2} - \cancel{6bx^2} - \cancel{4cx}$

$+\cancel{bx^3} + \cancel{cx^2} = x \qquad 6b = 1 \quad b = 1/6$

$c = 0$

$$y(x) = c_1e^x + c_2xe^x + \frac{1}{6}x^3e^x$$

$f(x) = e^{\lambda x}[\underline{p_m(x)}\cos\mu x + \underline{r_k(x)}\operatorname{sen}\mu x]$

$P(\lambda \pm i\mu) \neq 0$

$v_0 = e^{\lambda x}[q_{\overline{m}}\cos\mu x + q_{\overline{m}}\operatorname{sen}\mu x]$

$\overline{m} = \max(m, k)$

$P(\lambda \pm i\mu) = 0$

$$V_0 = x^h e^{\lambda x}\left[q_{\bar m}\cos\mu x + q_{\bar m}\operatorname{sen}\mu x\right]$$

h molteplicità di $\lambda \pm i\mu$ e $\bar m = \max(m,k)$

$$y'' - 2y' - 3y = \cos 2x$$

$\lambda^2 - 2\lambda - 3 = 0$ radici -1 e 3

$$y(x) = c_1 e^{-x} + c_2 e^{3x} + v_0(x)$$

$f(x) = (\cos 2x)e^{0x}$ $\quad \lambda \pm \mu i$: $0 \pm 2i$ non sono soluzioni

$$v_0(x) = b\cos 2x + c\operatorname{sen} 2x$$

$$v_0' = -2b\operatorname{sen} 2x + 2c\cos 2x$$

$$v_0'' = -4b\cos 2x - 4c\operatorname{sen} 2x$$

$$-4b\cos 2x - 4c\operatorname{sen} 2x + 4b\operatorname{sen} 2x - 4c\cos 2x - 3b\cos 2x - 3c\operatorname{sen} 2x = \cos 2x$$

$$\begin{cases} -7b - 4c = 1 \\ -7c + 4b = 0 \end{cases} \qquad b = \frac{7}{4}c$$

$$-\frac{49}{4}c - 4c = 1$$

$$-65c = 4$$

$$c = -4/65$$

$$b = \frac{7}{4}\left(-\frac{4}{65}\right) = -7/65$$

$$y(x) = -\frac{7}{65}\cos 2x - \frac{4}{65}\operatorname{sen} 2x + c_1 e^{-x} + c_2 e^{3x}$$

$$y'' + y = x + 1 \qquad (y = c_1 \cos x + c_2 \operatorname{sen} x + x + 1)$$

$$y'' - y' - 2y = 2\operatorname{sen} x \qquad (y = c_1 e^{-x} + c_2 e^{2x} + (\cos x - 3\operatorname{sen} x)/5$$

$$\lambda^2 + 1 = 0 \qquad \lambda = \pm\sqrt{-1} = \pm i$$

$$y(x) = c_1 \cos x + c_2 \operatorname{sen} x + v_0$$

$$v_0 = bx + c \qquad v_0' = b \qquad v_0'' = 0$$

$$bx + c = x + 1 \qquad b = 1 \quad c = 1$$

$$y(x) = c_1 \cos x + \underline{c_2 \operatorname{sen} x + x + 1}$$

$$y'' - y' - 2y = 0 \qquad f(x) = 2\operatorname{sen} x \qquad \lambda \pm i\mu$$

$$0 \pm i$$

$$\lambda^2 - \lambda - 2 = 0$$

$$\frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} \begin{cases} 4/2 = 2 \\ -2/2 = -1 \end{cases}$$

$$y(x) = c_1 e^{-x} + c_2 e^{2x} + v_0$$

$y_0 = c\cos x + b\,\mathrm{sen}\,x$

$y_0' = -c\,\mathrm{sen}\,x + b\cos x \qquad y_0'' = -c\cos x - b\,\mathrm{sen}\,x$

$-c\cos x - b\,\mathrm{sen}\,x + c\,\mathrm{sen}\,x - b\cos x - 2c\cos x - 2b\,\mathrm{sen}\,x = 2\,\mathrm{sen}\,x$

$\begin{cases} -3c - b = 0 \\ -3b + c = 2 \end{cases} \qquad \begin{cases} -6 - 9b - b = 0 \\ c = 2 + 3b \end{cases} \qquad \begin{cases} -6 = 10b \\ c = 2 + 3b \end{cases} \qquad \begin{cases} b = -3/5 \\ c = 2 - 9/5 = \dfrac{1}{5} \end{cases}$

$$y(x) = c_1 e^{-x} + c_2 e^{2x} + \frac{1}{5}\cos x - \frac{3}{5}\,\mathrm{sen}\,x$$