Eser 27/9

lunedì 27 settembre 2021 12:20

DIS ESPONENZIALI LOGARITMICHE

$a > 0$

$a^0 = 1$ $\qquad a^x \cdot a^y = a^{x+y}$

$\dfrac{a^x}{a^y} = a^{x-y}$ $\qquad a^{-n} = \dfrac{1}{a^n}$ $\qquad m, n \in \mathbb{N}$

$a^{\frac{n}{m}} = \sqrt[m]{a^n}$ $\qquad (a^x)^y = a^{xy}$

$a^x < a^y$

se $0 < a < 1 \Rightarrow x > y$

se $a > 1 \Rightarrow x < y$

$\left(\dfrac{2}{3}\right)^{\frac{x}{2}} - \dfrac{9}{4} > 0$

$4^{2x^2+1} - 16 > 0$

$\sqrt[3]{2^{6x}} < \dfrac{1}{4} 16^{x^2-1}$

21 $\left\{ x < 4 \;/\; \dfrac{2^x - 1}{2^x - 3} > 2^x \right\}$

SUGGERIMENTO

$2^x = t > 0$

$\left(\dfrac{2}{3}\right)^{\frac{x}{2}} - \dfrac{9}{4} > 0$

$\left(\dfrac{2}{3}\right)^{\frac{x}{2}} > \left(\dfrac{3}{2}\right)^2$

$\left(\dfrac{2}{3}\right)^{\frac{x}{2}} \quad \left(\dfrac{2}{3}\right)^{-2}$

$$\left(\frac{\ }{3}\right)^2 \geq \left(\frac{\ }{3}\right) \qquad \frac{\ }{2} \leq -2 \qquad \underline{\underline{x < -4}}$$

equivalente

$$\left(\frac{3}{2}\right)^{-\frac{x}{2}} \geq \left(\frac{3}{2}\right)^2 \qquad -\frac{x}{2} \geq 2 \qquad -x > 4 \qquad \underline{\underline{x < -4}}$$

$$(-\infty, -4)$$

$$4^{2x^2+1} > 4^2$$

$$2x^2+1 > 2 \qquad 2x^2 = 1 \qquad x^2 = \frac{1}{2}$$

$$x = \pm\frac{1}{\sqrt{2}} \qquad x < -\frac{1}{\sqrt{2}} \quad \vee \quad x > \frac{1}{\sqrt{2}}$$

$$\left(-\infty, -\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, +\infty\right)$$

$$|x| > \frac{1}{\sqrt{2}}$$

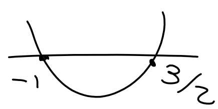
$$\sqrt[3]{2^{6x}} < \frac{1}{4} 16^{x^2-1}$$

$$2^{\frac{6x}{3}} < \frac{\left(2^4\right)^{x^2-1}}{2^2} = 2^{4x^2-4-2}$$

$$2^{2x} < 2^{4x^2-6} \qquad \cancel{2}^{1}x < \cancel{4}^{2}x^2 - \cancel{6}^{3}$$

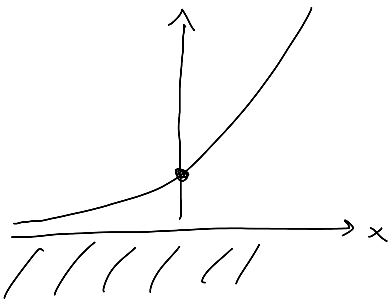
$$\underset{a}{2}x^2 \underset{b}{-} x \underset{c}{-3} > 0 \qquad \frac{1 \pm \sqrt{1+24}}{4} = \frac{1 \pm 5}{4} \begin{cases} \frac{3}{2} = x_2 \\ -1 = x_1 \end{cases}$$

$$x_1 \cdot x_2 = \frac{c}{a} \qquad x_1 + x_2 = -\frac{b}{a}$$



$(-\infty,-1)$ $\quad x<-1 \quad \vee \quad x>\frac{3}{2}$ $\quad \left(\frac{3}{2},+\infty\right)$

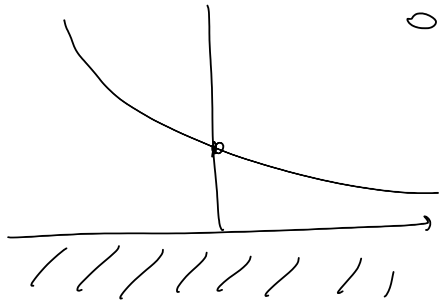
$$\left\{ x<4 : \frac{2^x-1}{2^x-3} > 2^x \right\}$$



a^x

$a>1$

$2^x = t > 0$



$0<a<1$

$$\frac{t-1}{t-3} - t > 0$$

$$\frac{t-1-t^2+3t}{t-3} > 0$$

$$\frac{-t^2+4t-1}{t-3} > 0$$

$a^n \quad n \in \mathbb{N}$

$(-2)^3$

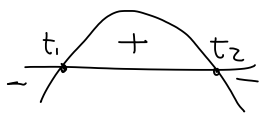
a^{-n}

$$(-2)^{-3} = \frac{1}{(-2)^3}$$

$-t^2+4t-1=0$

$t^2-4t+1=0$

$2 \pm \sqrt{4-1} = 2 \pm \sqrt{3}$



$t-3>0 \quad t>3$

		$2-\sqrt{3}$		3		$2+\sqrt{3}$	
N	$-$	0	$+$		$+$	0	$-$
D	$-$		$-$	0	$+$		$+$
	$+$	0	$-$	$\nexists$	$+$	0	$-$

$t<2-\sqrt{3} \qquad 3<t<2+\sqrt{3}$

$2^x < 2-\sqrt{3}$

$$\begin{cases} 3 < 2^x \\ 2^x < 2+\sqrt{3} \end{cases}$$

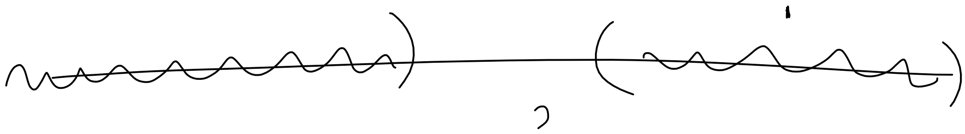
$\log_2 2^x < \log_2(2-\sqrt{3})$

$x < \log_2(2-\sqrt{3})$

$x < \log_2(2-\sqrt{3})$

$$\begin{cases} \log_2 3 < x \\ x < \log_2(2+\sqrt{3}) \end{cases}$$

$(-\infty, \log_2(2-\sqrt{3}))$ $\qquad$ $(\log_2 3, \log_2(2+\sqrt{3}))$



?

$$2^{\log_2 3} \overset{?}{<} 2^4$$

$3 < 16$ SÌ È VERO

?

$$4 < \log_2(2+\sqrt{3}) \quad ?$$

$16 = 2^4 < 2+\sqrt{3}$ $\quad$ FALSO

24) $\left\{ x \in \mathbb{R} \,/\, 0 < \log_{10}\left(\frac{x+2}{x+1}\right) < 1 \right\}$

- $\log_5(x^2+x+1) > 1$

- $\ln(4x+5) + \ln(x-2) = \ln 3 + \ln(5-x)$

$a^x = y$ $\qquad$ x è l'esponente da dare alla base a per ottenere y. $x = \log_a y$

$a > 0$

$y > 0$

$$0 < \log_{10} \frac{x+2}{x+1} < 1$$

$$\begin{cases} \frac{x+2}{x+1} > 1 \\ \dots \end{cases}$$

$$10^0 < \frac{x+2}{\dots} < 10$$

$1 = 10 < \frac{\quad}{x+1} < 10$ $\quad \Big| \; \frac{x+2}{x+1} < 10$

$$\begin{cases} \frac{\cancel{x}+2-\cancel{x}-1}{x+1} > 0 \\ \frac{x+2-10x-10}{x+1} < 0 \end{cases} \quad \begin{cases} \frac{1}{x+1} > 0 \\ \frac{-9x-8}{x+1} < 0 \end{cases} \quad \begin{cases} x+1 > 0 \\ -9x-8 < 0 \end{cases}$$

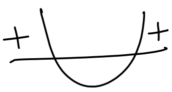
$$\begin{cases} x > -1 \\ -\frac{8}{9} < \frac{\cancel{9}x}{\cancel{9}} \end{cases} \quad x > -\frac{8}{9}$$

$$\log_5 (x^2+x+1) > 1$$

$$\begin{cases} \cancel{x^2+x+1>0} \\ x^2+x+1 > 5 \end{cases}$$

$$x^2+x-4 > 0 \qquad x^2+x-4=0$$

$$\frac{-1 \pm \sqrt{1+16}}{2} = \frac{-1 \pm \sqrt{17}}{2}$$



$$x < \frac{-1-\sqrt{17}}{2} \quad \vee \quad x > \frac{-1+\sqrt{17}}{2}$$

$$\left(-\infty, \frac{-1-\sqrt{17}}{2}\right) \cup \left(\frac{-1+\sqrt{17}}{2}, +\infty\right)$$

$$\ln(4x+5) + \ln(x-2) = \ln 3 + \ln(5-x)$$

$$\ln(4x+5)(x-2) = \ln 3(5-x)$$

$$(4x+5)(x-2) = 3(5-x)$$

$$\begin{cases} 4x+5>0 \\ x-2>0 \\ 5-x>0 \end{cases} \qquad \begin{cases} x>-5/4 \\ x>2 \\ x<5 \end{cases} \qquad \underline{\underline{2<x<5}}$$

$$4x^2 - \cancel{8x} + \cancel{5x} - 10 = 15 - \cancel{3x}$$

$$4x^2 = 25 \qquad x^2 = \frac{25}{4} \qquad x = \pm\frac{5}{2}$$

$x = 5/2$ È L'UNICA SOLUZIONE

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29) $\left\{ x \in \mathbb{R} \,/\, \log_{(x+1)}(x^2-4x+5) > 1 \right\}$

$$\begin{cases} \underline{x^2-4x+5>0} \\ \underline{\underline{0 < x+1 < 1}} \\ x^2-4x+5 < x+1 \end{cases} \qquad \begin{cases} x^2-4x+5>0 \\ x+1>1 \\ x^2-4x+5 > x+1 \end{cases}$$

$-1 \quad -1 \quad -1$

$x^2-4x+5=0$

$2 \pm \sqrt{4-5}$ NO SOL EQ. SEMPRE POSITIVA

$$\begin{cases} -1<x<0 \\ x^2-5x+4<0 \end{cases} \qquad \begin{cases} x>0 \\ x^2-5x+4>0 \end{cases}$$

$4 \quad 1$

$$\begin{cases} -1<x<0 \\ 1<x<4 \end{cases} \qquad \begin{cases} x>0 \\ x<1 \quad \vee \quad x>4 \end{cases}$$

$\varnothing$

$0<x<1 \quad \vee \quad x>4$

$(0, 1) \cup (4, +\infty)$