Eser 22/12

mercoledì 22 dicembre 2021 11:59

CALCOLA

1) $\int \cos(\log x)\,dx$

VERIFICA

2) $\int_0^{+\infty} x\, e^{-\sqrt{1+x^2}} = \frac{2}{e}$ 3) $\int_1^{+\infty} e^{-x^2}\,dx$ È CONVERGENTE

4) RISOLVI $y'' - 2y' - 3y = (2x+1)e^x$

5) $\begin{cases} y' = e^{x-y}\cos x \\ y(0) = \log \frac{3}{2} \end{cases}$

$\int \cos(\log x)\,dx$

$\log x = t$

$x = e^t$

$dx = e^t\,dt$

$\int \cos t \cdot e^t\,dt$

$\int f \cdot g'\,dt = f\cdot g - \int f' g\,dt$

$f = \cos t \quad f' = -\sin t$

$g' = e^t \quad g = e^t$

$\int \cos t \cdot e^t\,dt = e^t \cos t + \int e^t \sin t\,dt$

$f = \sin t \quad f' = \cos t$

$g' = e^t \quad g = e^t$

$\underbrace{\int \cos t \cdot e^t\,dt}_{I(t)} = e^t \cos t + e^t \sin t - \underbrace{\int e^t \cos t\,dt}_{-I(t)}$

SE DEVO RISOLVERE $z = 5 - z \Rightarrow 2z = 5 \Rightarrow z = 5/2$

$2\int \cos t \cdot e^t\,dt = e^t(\sin t + \cos t) + c$

$e^t(\sin t + \cos t)$

$$\int \cos t \cdot e^t \, dt = \frac{\cdot}{2} + c$$

$$\int \cos(\log x)\,dx = \frac{x\,(\operatorname{sen}(\log x) + \cos(\log x))}{2} + c$$

$$\int_0^{+\infty} x e^{-\sqrt{1+x^2}}\,dx$$

$$\lim_{x \to +\infty} x e^{-\sqrt{1+x^2}} = 0 \qquad \lim_{M \to +\infty} \int_0^M x e^{-\sqrt{1+x^2}}\,dx$$

$$-\sqrt{1+x^2} = t \qquad 1+x^2 = t^2 \qquad 2x\,dx = 2t\,dt$$

$$\int_{-1}^{-\sqrt{1+M^2}} t e^t\,dt = \left[t e^t\right]_{-1}^{-\sqrt{1+M^2}} - \int_{-1}^{-\sqrt{1+M^2}} e^t\,dt$$

$f = t \quad f' = 1$

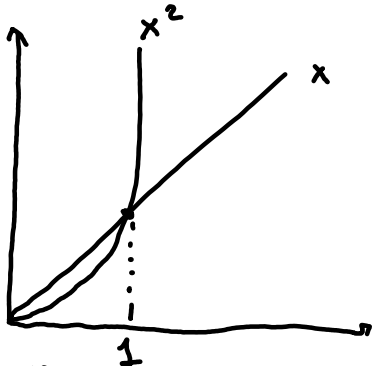
$g' = e^t \quad g = e^t$

$$= -\sqrt{1+M^2}\left(e^{-\sqrt{1+M^2}}\right) + e^{-1} - \left[e^t\right]_{-1}^{-\sqrt{1+M^2}}$$

$$= -\sqrt{1+M^2}\, e^{-\sqrt{1+M^2}} + \frac{1}{e} - e^{-\sqrt{1+M^2}} + \frac{1}{e}$$

$$\xrightarrow[M \to +\infty]{} \frac{1}{e} + \frac{1}{e} = \frac{2}{e}$$

$\int_1^{+\infty} e^{-x^2}\,dx$ è convergente



$x \geq 1$

$x^2 \geq x$

$\Rightarrow$

$-x^2 \leq -x$

$e^{-x^2} \leq e^{-x}$

$$\int_1^{+\infty} e^{-x^2}\,dx \leq \int_1^{+\infty} e^{-x}\,dx = \lim_{M \to +\infty} \int_1^M e^{-x}\,dx = \lim_{M \to +\infty} \left[-e^{-x}\right]_1^M =$$

$$\lim_{M\to\infty} -e^{-M} + e^{-1} = e^{-1} \Rightarrow \int_1^{+\infty} e^{-x^2}\,dx \text{ converge}$$

$$\int_1^{+\infty} \frac{1}{x^p}\,dx < \infty \quad \text{si } p>1 \qquad = +\infty \quad \text{si } p\le 1$$

$$\int_0^{1} \frac{1}{x^p}\,dx \quad \begin{cases} < \infty & p<1 \\ = +\infty & p\ge 1 \end{cases}$$

$y'' - 2y' - 3y = (2x+1)e^x \qquad \lambda = 1$

$\lambda^2 - 2\lambda - 3 = 0 \qquad \lambda = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2\pm 4}{2} \begin{cases} 3 \\ -1 \end{cases}$ (−1), (3)

$y(x) = c_1 e^{-x} + c_2 e^{3x} + v_0(x)$

$v_0(x) = e^x(ax+b)$

$v_0'(x) = e^x(ax+b) + e^x(a) \qquad v_0'' = e^x(ax+b) + ae^x + ae^x$

$e^x(ax+b+2a) - 2e^x(ax+b+a) - 3e^x(ax+b) = (2x+1)e^x$

$ax + b + 2a - 2ax - 2b - 2a - 3ax - 3b = 2x+1$

$-4a = 2 \qquad \begin{cases} a = -1/2 \\ -4b = 1 \end{cases} \qquad \begin{cases} a = -1/2 \\ b = -1/4 \end{cases}$

$b + 2a - 2b - 2a - 3b = 1$

$y(x) = c_1 e^{-x} + c_2 e^{3x} + e^x\left(-\frac{1}{2}x - \frac{1}{4}\right)$

$f(x) = p_m(x)e^{\lambda x} \begin{cases} P(\lambda)=0 & v_0(x) = x^q q_m(x) e^{\lambda x} \\ P(\lambda)\neq 0 & v_0(x) = q_m(x) e^{\lambda x} \end{cases}$

$$\begin{cases} y' = e^{x-y}\cos x \\ y(0) = \log\frac{3}{2} \end{cases}$$

$$\int e^y\,dy = \int e^x \cdot \cos x\,dx$$

$$e^y = \frac{e^x(\sin x + \cos x)}{2} + c$$

$$e^{y-x} = \frac{\sin x + \cos x}{2} + c$$

$$y = \log\left(\frac{\sin x + \cos x}{2} + c\right) + x$$

$$\log\frac{3}{2} = \log\left(\frac{1}{2} + c\right) \Rightarrow c = 1$$

$g(x) = \log\left(\frac{\sin x - \cos x}{2} + 1\right) + x$

STABILIRE SE CONVERGONO

$$\int_1^{+\infty} \left(1 - \cos\frac{1}{x}\right) dx \qquad \int_1^{+\infty} \log\left(1 + \frac{1}{x^2}\right) dx \qquad \int_0^1 \frac{1}{\sqrt[4]{(1-x^3)^3}}\, dx$$

$\lim_{x \to +\infty} 1 - \cos\frac{1}{x} = 0$

$1 - \cos\frac{1}{x} \sim \frac{1}{x^2}$ per confr. converge

$\lim_{x \to +\infty} \log\left(1 + \frac{1}{x^2}\right) = 0$ $\log\left(1 + \frac{1}{x^2}\right) \sim \frac{1}{x^2}$ per conf. converge

$$\frac{1}{\sqrt[4]{(1-x^3)^3}} = \frac{1}{(1-x)^{3/4}(1+x+x^2)^{3/4}} \sim \frac{1}{(1-x)^{3/4}}$$

per confronto $\frac{3}{4} < 1$ l'int. converge