Esrc 6/10

mercoledì 6 ottobre 2021 12:19

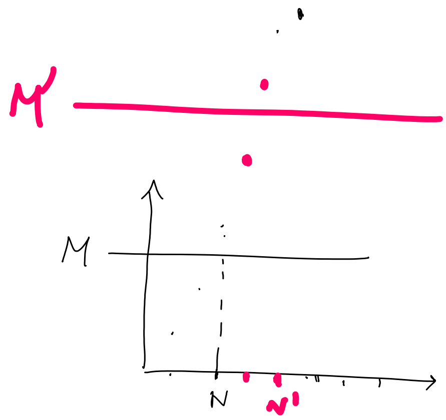
LIMITI

$a_n \xrightarrow[n\to+\infty]{} +\infty \qquad b_n \xrightarrow[n\to\infty]{} +\infty$

$\frac{a_n}{b_n} \xrightarrow[n\to\infty]{} ? \qquad a_n - b_n \xrightarrow[n\to\infty]{} ?$

$\frac{\infty}{\infty} \qquad \infty - \infty \qquad \lim_{n\to\infty} \frac{n^b}{n!} = 0 \qquad \lim_{n\to\infty} \frac{n^b}{a^n} = 0 \quad \begin{matrix} b>0 \\ a>1 \end{matrix}$

$\log_a n \xrightarrow[n\to+\infty]{} +\infty \qquad a>1$



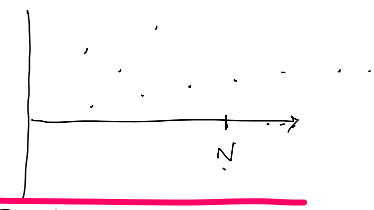
$n^b \xrightarrow[n\to+\infty]{} +\infty \qquad b>0$

$a^n \xrightarrow[n\to+\infty]{} +\infty \qquad a>1$

$n > N$

$a^n > M$

$n! \xrightarrow[n\to+\infty]{} +\infty$



$n^n \xrightarrow[n\to+\infty]{} +\infty$

$$\boxed{\lim_{n\to\infty} \frac{\log_a n}{n^b} = 0}$$

$$\boxed{\forall \varepsilon>0 \ \exists N\in\mathbb{N} : \left|\frac{\log_a n}{n^b}\right| < \varepsilon \quad \forall n>N}$$

$a_n \longrightarrow +\infty$

$$\lim_{n\to+\infty} \frac{1}{n} = 0 \qquad \lim_{n\to\infty} \frac{1}{a_n} = 0$$

$\forall \varepsilon>0$

$\exists M : \quad \frac{1}{n} < \varepsilon \qquad M = \frac{1}{\varepsilon} < n$

$\varepsilon = 0{,}01$

$\frac{1}{n} < 0{,}01 \qquad 100 = \frac{1}{\frac{1}{100}} < n$

$$\lim_{n\to+\infty} \frac{3n^4 - \sqrt[5]{n} + 7n^3}{2n - 3n^2 + \sqrt[3]{n^2}} = \lim_{n\to+\infty} \frac{n^{\cancel{4}\,2}\left(3 - \frac{\sqrt[5]{n}}{n^4} + \frac{7}{n}\right)}{\cancel{n^2}\left(\frac{2}{n} - 3 + \frac{\sqrt[3]{n^2}}{n^2}\right)}$$

$$= -\infty$$

$$\lim_{n \to \infty} \frac{n^4+3}{n} - \frac{n^4+3n}{n-2}$$

$$\lim_{n \to +\infty} \frac{n^{20}+4n^4+1}{n^2-3n^{20}} = \lim_{n \to +\infty} \frac{\cancel{n^{20}}\left(1+\frac{4}{n^{16}}+\frac{1}{n^{20}}\right)}{\cancel{n^{20}}\left(-3+\frac{1}{n^{18}}\right)} = -\frac{1}{3}$$

$$\lim_{n \to +\infty} \frac{n^6-\sqrt[4]{n}}{\sqrt[5]{n^4}+n^7} = \lim_{n \to \infty} \frac{\cancel{n^6}\left(1-\frac{1}{n^{6-1/4}}\right)}{n^{\cancel{7}}\left(\frac{1}{n^{7-4/5}}+1\right)} = 0$$

$$\lim_{n \to +\infty} \frac{n^4+3}{n} - \frac{n^4+3n}{n-2} = \lim_{n \to +\infty} \frac{\cancel{n^5}-2n^4+3n-6-\cancel{n^5}-3n^2}{n(n-2)}$$

$$= \lim_{n \to +\infty} \frac{-2n^4-3n^2+3n-6}{n^2-2n} = \lim_{n \to +\infty} \frac{n^2\,\cancel{n^4}\left(-2-\frac{3}{n^2}+\frac{3}{n^3}-\frac{6}{n^4}\right)}{\cancel{n^2}\left(1-\frac{2}{n}\right)}$$

$$= -\infty$$

$$\lim_{n \to +\infty} \frac{n}{2^n-3^n} = \lim_{n \to \infty} \frac{n}{3^n\left(\left(\frac{2}{3}\right)^n-1\right)} = 0$$

$$\lim_{n \to +\infty} \frac{n^5-3n}{n!} = \lim_{n \to \infty} \frac{n^5}{n!} - \frac{3n}{n!} = \lim_{n \to \infty} \frac{n^5}{n!} - \frac{3}{(n-1)!} = 0$$

$$\lim_{n \to +\infty} \frac{n^2}{n!} = 0 \qquad \lim_{n \to \infty} \frac{\cancel{n}\cdot n}{\cancel{n}(n-1)(n-2)\dots 1} = \lim_{n \to \infty} \frac{n}{n-1} \cdot \frac{1}{(n-2)!} = 0$$

$$\lim_{n \to +\infty} \frac{n^3}{n^2+1} - \frac{n^3-1}{n^2} = \lim_{n \to \infty} \frac{\cancel{n^5}-\cancel{n^5}-n^3+n^2+1}{n^2(n^2+1)} =$$

$$\lim_{n \to +\infty} \frac{\cancel{n^3}\left(-1+\frac{1}{n}+\frac{1}{n^3}\right)}{n^4\left(1+\frac{1}{\ }\right)} = 0$$

$$\lim_{n\to\infty} \frac{\sqrt{n^3+9n^2}-\sqrt{n^4+1}}{n^2+2} = -1$$

Sugg. $\dfrac{\sqrt{n^3+9n^2}+\sqrt{n^4+1}}{\sqrt{n^3+9n^2}+\sqrt{n^4+1}}$

$(a-b)(a+b)=a^2-b^2$

$$\lim_{n\to\infty} \frac{n^3+9n^2-n^4-1}{(n^2+2)\left(\sqrt{n^3\left(1+\frac{9}{n}\right)}+\sqrt{n^4\left(1+\frac{1}{n^4}\right)}\right)} =$$

$$\lim_{n\to\infty} \frac{-n^4+n^3+9n^2-1}{(n^2+2)\left(n^{3/2}\sqrt{1+\frac{9}{n}}+n^2\sqrt{1+\frac{1}{n^4}}\right)} = -1$$