Eserc 8/10

venerdì 8 ottobre 2021 11:59

- $\lim\limits_{x\to+\infty} \dfrac{x^2+3}{4x^2+x}$ $\qquad$ $\lim\limits_{x\to+\infty} \dfrac{x^2+1}{3^x}$

- $\lim\limits_{x\to+\infty} \dfrac{3-x^3-x}{1-2x^2}$ $\qquad$ $\lim\limits_{x\to+\infty} (x^3+3x)2^{-x}$

$\lim\limits_{x\to+\infty} \dfrac{\operatorname{sen} x}{x}$ $\qquad$ $\lim\limits_{x\to+\infty} (x^2+3^x-\log_3 x)3^{-x}$

$\lim\limits_{x\to+\infty} \dfrac{x^2+\operatorname{sen} x}{2x+x^2+3}$ $\qquad$ $\lim\limits_{x\to+\infty} \left(1+\dfrac{1}{x}\right)^{2x}$

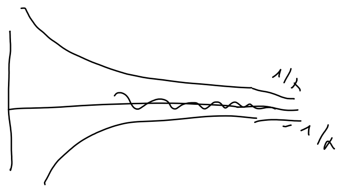
$$\lim_{x\to+\infty} \frac{x^2+3}{4x^2+x} = \lim_{x\to+\infty} \frac{\cancel{x^2}\left(1+\overset{\nearrow 0}{\frac{3}{x^2}}\right)}{4\cancel{x^2}\left(1+\underset{\searrow 0}{\frac{1}{x}}\right)} = \frac{1}{4}$$

$$\lim_{x\to+\infty} \frac{n}{x^b} = 0 \qquad b>0$$

$$\lim_{x\to+\infty} \frac{3-x^3-x}{1-2x^2} = \lim_{x\to+\infty} \frac{+x^{\cancel{3}}\left(-\frac{3}{x^3}+1+\frac{1}{x^2}\right)}{+\cancel{x^2}\left(-\frac{1}{x^2}+2\right)} = +\infty$$

$$\lim_{x\to+\infty} \frac{\operatorname{sen} x}{x} = 0$$

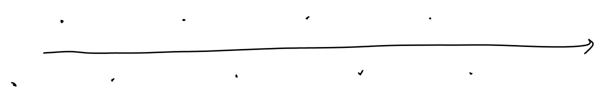
$$\underset{\searrow\, x\to+\infty\, 0}{-\frac{1}{x}} \leq \frac{\operatorname{sen} x}{x} \leq \underset{\swarrow\, x\to+\infty\, 0}{\frac{1}{x}}$$



$f(x) < g(x)$

se $\lim\limits_{x\to+\infty} f(x) = +\infty \Rightarrow \lim\limits_{x\to+\infty} g(x) = +\infty$

$(-1)^n$



$$\lim_{x\to+\infty} \frac{x^2+\operatorname{sen} x}{2x+x^2+3} = \lim_{x\to+\infty} \frac{\cancel{x^2}\left(1+\overset{\nearrow 0}{\frac{\operatorname{sen} x}{x^2}}\right)}{\cancel{x^2}\left(\underset{0\swarrow}{\frac{2}{x}}+1+\underset{\searrow 0}{\frac{3}{x^2}}\right)} = 1$$

$$\lim_{x\to+\infty} \frac{x^2+1}{3^x} = \lim_{x\to+\infty} \frac{x^2}{3^x} + \frac{1}{3^x} = 0$$

perché $\lim\limits_{x\to+\infty} \dfrac{x^b}{a^x} = 0$ $\quad \forall b>0 \quad a>1$

$$\lim_{x\to+\infty} (x^3+3x)2^{-x} = \lim_{x\to+\infty} \frac{x^3+3x}{2^x} = 0$$

$\nearrow 0$ $\qquad$ $\searrow 0$

$$\lim_{x\to+\infty}\left(x^2+3^x-\log_3 x\right)3^{-x}=\lim_{x\to+\infty}\frac{x^2}{3^x}+1-\frac{\log_3 x}{3^x}=1$$

perché $\frac{\log_a x}{a^x}\xrightarrow[x\to+\infty]{}0$ $\quad a>1$

$b>0$

$$\lim_{x\to+\infty}\frac{\log x^b}{\log x}=\lim_{x\to+\infty}\frac{b\cdot\log x}{\log x}=b$$

$$\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^{2x}=\lim_{x\to+\infty}\left[\left(\boxed{1+\frac{1}{x}}\right)^x\right]^2=e^2$$

$\frac{\infty}{\infty}$ $\quad\infty-\infty\quad$ $0\cdot\infty$ $\begin{cases}\frac{\infty}{\infty}\\ \frac{0}{0}\end{cases}$ $\quad 1^\infty$

$f(x)\xrightarrow[x\to+\infty]{}1\qquad g(x)\xrightarrow[x\to+\infty]{}+\infty\quad\Rightarrow\quad\lim_{x\to+\infty}f(x)^{g(x)}$

$$\lim_{x\to+\infty}1^x=1\qquad f(x)^{g(x)}=e^{\log f(x)^{g(x)}}=e^{g(x)\cdot\log f(x)}$$

$$\lim_{x\to+\infty}\frac{x^2}{x+1}-x$$

$$\lim_{x\to+\infty}\frac{\sqrt{1+x^2}+\sqrt{x}}{\sqrt{x}-x}$$

$$\lim_{x\to+\infty}x\left(\sqrt{1+x^4}-x^2\right)$$

$$\lim_{x\to+\infty}\frac{x^2}{x+1}-x=\lim_{x\to+\infty}\frac{\cancel{x^2}-\cancel{x^2}-x}{x+1}=-1$$

$$\lim_{x\to+\infty}\frac{\sqrt{1+x^2}+\sqrt{x}}{\sqrt{x}-x}=\lim_{x\to+\infty}\frac{\sqrt{x^2\left(\frac{1}{x^2}+1\right)}+\sqrt{x}}{\sqrt{x}-x}=\lim_{x\to+\infty}\frac{x\sqrt{\frac{1}{x^2}+1}+\sqrt{x}}{\sqrt{x}-x}$$

$$\sqrt{x^2}=|x|=\begin{cases}x & \text{se } x\geq 0\\ -x & \text{se } x<0\end{cases}$$

$$\lim_{x\to+\infty}\frac{\sqrt{\frac{1}{x^2}+1}+\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}-1}=-1$$

$x\left(1+\cancel{x^4}-\cancel{x^4}\right)$

$$\lim_{x \to +\infty} x\left(\sqrt{1+x^4} - x^2\right) = \lim_{x \to +\infty} \frac{\cdots}{\sqrt{1+x^4} + x^2}$$

$$\lim_{x \to +\infty} \frac{\cancel{x}}{x^{\cancel{2}}\left(\sqrt{\frac{1}{x^4}+1} + 1\right)} = 0$$

$$\lim_{n \to +\infty} \sqrt[n]{2^n + 3^n}$$

$$3 = \sqrt[n]{3^n} < \sqrt[n]{2^n + 3^n} < \sqrt[n]{3^n + 3^n} = \sqrt[n]{2 \cdot 3^n} = 3\sqrt[n]{2}$$

$$3 \leq \lim_{n \to \infty} \sqrt[n]{2^n + 3^n} \leq 3 \lim_{n \to +\infty} \sqrt[n]{2} = 3$$

$$\lim_{n \to +\infty} \sqrt[n]{a^n + 1} \qquad a \geq 0$$

$$= \max(a; 1)$$

Sia $a > 1$

$$a = \sqrt[n]{a^n} < \sqrt[n]{a^n + 1} < \sqrt[n]{a^n + a^n} = \sqrt[n]{2a^n} = a\sqrt[n]{2}$$

$\downarrow$

a

se $0 \leq a < 1$

$$1 = \sqrt[n]{1} < \sqrt[n]{a^n + 1} < \sqrt[n]{1+1} = \sqrt[n]{2}$$

$\downarrow$

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