Eser 11/10

lunedì 11 ottobre 2021 12:24

LIMITI DI FUNZIONI

$x > 0$

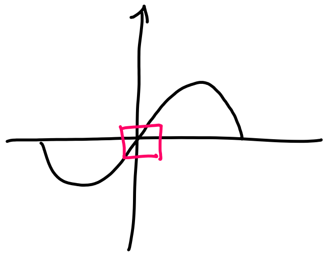
134. $$\lim_{x \to 0^+} \frac{e^{\sqrt{\sin x}} - 1}{\sqrt{x}}$$

94. $$\lim_{x \to 0} \frac{\log(2-\cos x)}{\sin^2 x}$$

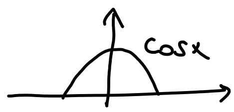
99 $$\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$$

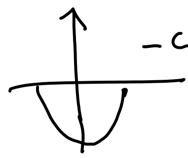
LIMITI NOTEVOLI

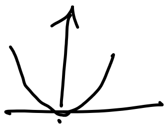
- $$\lim_{x \to 0} \frac{\sin x}{x} = 1$$ $x \to 0$ $\quad \sin x \sim x$



- $$\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$$ $1 - \cos x \sim \frac{x^2}{2}$

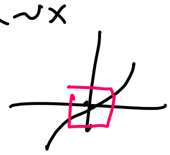




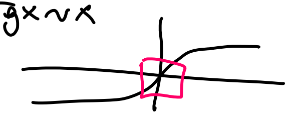


$\arcsin x \sim x$

- $$\lim_{x \to 0} \frac{\arcsin x}{x} = 1$$



- $$\lim_{x \to 0} \frac{\operatorname{arctg} x}{x} = 1$$ $\operatorname{arctg} x \sim x$

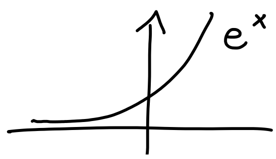


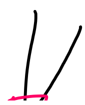
$\operatorname{arctg} x - x \neq 0$

$\pi \sim 3{,}14 \qquad \pi - 3{,}14 \neq 0 \qquad \pi - 3{,}14 \gtrless 0$?

per poter rispondere devo approssimare meglio il π!

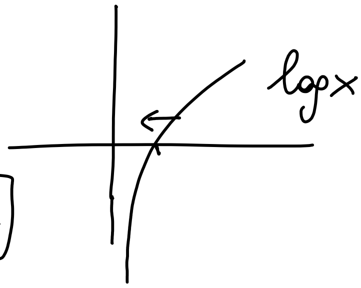
$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1 \qquad \underline{e^x - 1 \sim x}$$

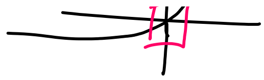


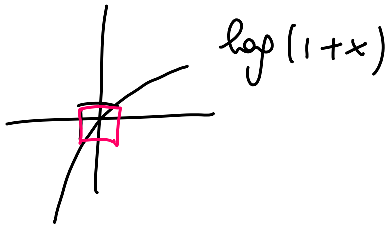


$$\lim_{* \to 0} \frac{\log(1+*)}{*} = 1$$

$$\boxed{\log(1+x) \sim x}$$







$$\lim_{x \to 0} \frac{e^{\sqrt{\sin x}} - 1}{\sqrt{x}} = \lim_{x \to 0} \frac{e^{\sqrt{\sin x}} - 1}{\sqrt{\sin x}} \cdot \frac{\sqrt{\sin x}}{\sqrt{x}} = \lim_{x \to 0} \frac{e^{\sqrt{\sin x}} - 1}{\sqrt{\sin x}} \lim_{x \to 0} \underbrace{\sqrt{\frac{\sin x}{x}}}_{\downarrow \, 1}$$

$$\lim_{y \to 0} \frac{e^y - 1}{y} = 1 \qquad y = \sqrt{\sin x}$$

$$\lim_{x \to 0} \frac{\log(2 - \cos x)}{\sin^2 x} = \lim_{x \to 0} \frac{\log(1 + \overbrace{1 - \cos x}^{y})}{1 - \cos x} \cdot \frac{1 - \cos x}{\sin^2 x} = \qquad \frac{0}{0}$$

$$\lim_{y \to 0} \underbrace{\frac{\log(1+y)}{y}}_{\downarrow \, 1} \lim_{x \to 0} \underbrace{\left(\frac{1 - \cos x}{x^2}\right)}_{\downarrow \, \frac{1}{2}} \cdot \underbrace{\left(\frac{x}{\sin x}\right)^2}_{\downarrow \, 1} = \frac{1}{2}$$

$$\left(\frac{\sin x}{x}\right) \to 1$$

$$\left(\frac{x}{\sin x}\right) = \left(\frac{\sin x}{x}\right)^{-1} \to 1$$

$$\lim_{x \to 0} \frac{\operatorname{tg} x}{x} = 1$$

$$\lim_{x \to 0} \frac{(\sqrt{1+x} - 1)}{x} \cdot \frac{(\sqrt{1+x} + 1)}{(\sqrt{1+x} + 1)} = \lim_{x \to 0} \frac{\cancel{1} + \cancel{x} - \cancel{1} \quad 1}{\cancel{x}(\sqrt{1+x} + 1)}$$

$$= \frac{1}{2}$$

$\downarrow$

Alg dei limiti

$a > 0$

$a^x - 1 \qquad 0 \qquad a \neq 1$

$$\lim_{x\to 0} \frac{a^x - 1}{x} = \log a \qquad a \neq 1$$

$$\lim_{x\to 0} \frac{a^x - 1}{x} = \lim_{x\to 0} \frac{e^{x\log a} - 1}{x} = \lim_{x\to 0} \frac{e^{x \log a} - 1}{\underbrace{x \cdot \log a}_{=y}} \cdot \log a$$

$$= \lim_{y\to 0} \frac{e^y - 1}{y} \cdot \log a = \log a$$

$$\lim_{x\to 0} \frac{\sqrt{1+x} - e^x}{x^2+1} = 0 \qquad \lim_{x\to 0} \frac{\sqrt{1+x} - e^x}{x^2} = \lim_{x\to 0} \frac{\sqrt{1+x} - 1 + 1 - e^x}{x^2}$$

$$\lim_{x\to 0} \frac{\log(1+x)}{e^x - \sqrt{1+x}}$$

$$\lim_{x\to -1} \frac{x^2 - 1}{x^2+3x+2} = \lim_{x\to -1} \frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}(x+2)} = \frac{-2}{1} = -2$$

$(x+1)$

$$\lim_{x\to 0} \frac{\sqrt{1+x} - e^x}{x^2} = \lim_{x\to 0} \frac{\sqrt{1+x} - 1 + 1 - e^x}{x^2} = \lim_{x\to 0} \frac{1}{x} \cdot \left(\overbrace{\frac{\sqrt{1+x} - 1}{x}}^{\frac{1}{2}} + \overbrace{\frac{1 - e^x}{x}}^{-1} \right)$$

BISOGNA DISTINGUERE $\lim_{x\to 0^+} = -\infty \qquad \lim_{x\to 0^-} = +\infty$

$$\lim_{x\to 0} \frac{\log(1+x)}{e^x - \sqrt{1+x}} = \lim_{x\to 0} \frac{\log(1+x)}{x} \cdot \lim_{x\to 0} \frac{x}{e^x - \sqrt{1+x}}$$

$$= \lim_{x\to 0} \frac{1}{\frac{e^x - 1 + 1 - \sqrt{1+x}}{x}} = \lim_{x\to 0} \frac{1}{\underbrace{\frac{e^x - 1}{x}}_{1} + \underbrace{\frac{1 - \sqrt{1+x}}{x}}_{-\frac{1}{2}}} = 2$$

$$\lim_{x\to 0} \frac{1 - \sqrt{1+x}}{x} \cdot \frac{1 + \sqrt{1+x}}{1 + \sqrt{1+x}} = \lim_{x\to 0} \frac{\cancel{1} - \cancel{1} - \cancel{x}^{\,-1}}{\cancel{x}(1 + \sqrt{1+x})} = -\frac{1}{2}$$

$$\lim_{x \to 0^+} \frac{1}{x} = +\infty$$

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$