Nella tavola degli integrali (primitive) di [D] compare

$$\int \frac{dx}{\sin x}$$

Ricorda (formule parametriche) $t = tg\frac{x}{2} \Rightarrow \sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$

$2 \arctg t = x$

Calcoliamo "dx" $\quad \frac{dx}{dt} = x'(t) = \frac{2}{1+t^2}$

$x = 2\arctg t$

$$2\int \frac{dt}{1+t^2} \frac{1}{\frac{2t}{1+t^2}} = \int \frac{1}{t}\,dt = \log|t| = \lg\left|tg\frac{x}{2}\right|$$

$$\boxed{\int \frac{dx}{\sin x} = \log\left|tg\frac{x}{2}\right| + c}$$

2° gruppo ES [D] dal 1051 al 1190

$$\int f\circ u \; u'\,dx = \int f(y)\,dy \Big|_{y=u(x)}$$

$F(y) = \int f(y)\,dy$

$(F\circ u)' = F'\circ u \cdot u' = f\circ u \cdot u'$

$F(y)\big|_{y=u(x)} := (F\circ u)(x)$

Da destra a sinistra:

$$\int f(y)\,dy = \int f\circ u \; u'\,dx$$

$y = u(x)$

$x = u^{-1}(y)$

"$dy = u'\,dx$"

1063 $\displaystyle\int \frac{x}{\sqrt{x^2+1}}\,dx = \int \frac{\left(\frac{x^2}{2}\right)'}{\sqrt{x^2+1}}\,dx$

$$= \int \frac{\left(\frac{x^2}{2}\right)'}{\sqrt{x^2+1}}\,dx$$

$$= \int \frac{du}{\sqrt{2u+1}}$$

$u = \frac{x^2}{2}$

$du = u'dx$

$$= \frac{1}{2}\int \frac{dt}{\sqrt{t}} = \frac{1}{2}\, 2\sqrt{t} = \sqrt{t}$$

$t = 2u+1$

$dt = 2\,du$

$$= \sqrt{2u+1} = \sqrt{2\frac{x^2}{2}+1} = \sqrt{x^2+1}$$

1094 $\displaystyle\int x\, 7^{x^2}\,dx = \frac{1}{2}\int 7^y\,dy = \frac{1}{2}\frac{7^y}{\lg 7} = \frac{1}{2}\frac{7^{x^2}}{\lg 7}$

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$\int a^y = \frac{a^y}{\log a}$

$y = x^2 \quad x = \sqrt{y}$

$(y' = 2x \quad x = \frac{y'}{2})$

$dy = 2x\,dx \quad x\,dx = \frac{dy}{2}$

Controllo

$$\left(\frac{1}{2}\frac{7^{x^2}}{\lg 7}\right)' = \frac{1}{\cancel{2}\,\cancel{\lg 7}}\, 7^{x^2} \cdot \cancel{\lg 7}\; \cancel{2}x$$

$7^y\big|_{y=x^2}$

N.B $\int 7^{x^2}$ non si sa fare nel senso che

$\int 7^{x^2}$ non è esprimibile in termini di funzioni elementari

$\int e^{x^2}$ ← Gaussiana

$\int_0^x e^{t^2}\,dt = F(x)$, $F'(x) = e^{x^2}$

$F = \int e^{x^2}$

Err(x) "funzione di errore"

1075

$$\int \frac{3x+1}{\sqrt{5x^2+1}}\,dx = 3\int \frac{x}{\sqrt{5x^2+1}} + \int \frac{1}{\sqrt{5x^2+1}}$$

$$\int \frac{1}{\sqrt{y^2+1}} = sh^{-1} y$$

$$\int \frac{1}{\sqrt{5x^2+1}} =$$

$$\int \frac{dx}{\sqrt{(\sqrt5 x)^2+1}} = \frac{1}{\sqrt5}\int \frac{1}{\sqrt{t^2+1}}\,dt = \frac{1}{\sqrt5} sh^{-1} t = \frac{1}{\sqrt5} sh^{-1}(\sqrt5 x)$$

$t = \sqrt5 x$

$x = \frac{1}{\sqrt5} t$ $\quad x' = \frac{1}{\sqrt5}$

$$\int \frac{1}{\sqrt{5x^2+1}} = \frac{1}{\sqrt5} sh^{-1}(\sqrt5 x)$$

$$\int \frac{x}{\sqrt{5x^2+1}}\,dx = \frac{1}{10}\int \frac{(5x^2+1)'}{\sqrt{5x^2+1}}\,dx = \frac{1}{10}\int \frac{du}{\sqrt u}$$

$u = 5x^2+1$

$$= \frac{1}{10}\, 2\sqrt u = \frac{1}{5}\sqrt{5x^2+1}$$

Res $\quad \frac{3}{5}\sqrt{5x^2+1} + \frac{1}{\sqrt5} sh^{-1}(\sqrt5 x)$ $\qquad sh^{-1} t = t + lg\sqrt{1+t^2}$

1097

$$\int \frac{e^x}{e^x-1}\,dx = \int \frac{(e^x-1)'}{e^x-1}\,dx = \int \frac{du}{u} = lg|u|$$

$u = e^x - 1$

$$= lg|e^x-1|$$

1070 $\displaystyle\int \frac{x^2-5x+6}{x^2+4}$ 1° passo trasformo in rapp di polinomi con grado numeratore < grado denominatore

$$\frac{x^2-5x+6}{x^2+4} = \frac{x^2+4-5x+2}{x^2+4} = 1 - \frac{5x-2}{x^2+4}$$

o dividere tra polinomi $P(x)$ e $Q(x)$ $\deg P \geq \deg Q$

$P(x) = Q(x)M(x) + R(x)$, $\deg R < \deg Q$.

$$= x - \int \frac{5x-2}{x^2+4}\,dx = x - 5\int \frac{x}{x^2+4} + 2\int \frac{1}{x^2+4}$$

no aspetto $\lg(x^2+4)$ arctg

$$= x - \frac{5}{2}\int \frac{(x^2+4)'}{x^2+4}dx + \frac{1}{2}\int \frac{dx}{\left(\frac{x}{2}\right)^2+1}$$

$$= \int \frac{dt}{t^2+1}$$

$t = \frac{x}{2}$

$x' = 2$

$$= \underbrace{x - \frac{5}{2}\lg(x^2+4) + \operatorname{arctg}\frac{x}{2}}$$

1107 $$\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx = 2\sin\sqrt{x}$$

$$2\int \cos\sqrt{x}\,(\sqrt{x})' \underset{u=\sqrt{x}}{=} 2\int \cos u\,du = 2\sin u = 2\sin\sqrt{x}$$

1119 $$\int \operatorname{tg}x = \int \frac{\sin x}{\cos x}dx = -\int \frac{(\cos x)'}{\cos x}dx \underset{u=\cos x}{=} -\int \frac{du}{u} = -\log|u|$$

$$= -\lg|\cos x| = \lg\frac{1}{|\cos x|}$$

1049 $$\int \operatorname{cth}^2 x = \int \frac{\operatorname{ch}^2 x}{\operatorname{sh}^2 x} = \int \frac{1+\operatorname{sh}^2 x}{\operatorname{sh}^2 x}$$

$\operatorname{ch}^2 x - \operatorname{sh}^2 x = 1$

$$= \int\left(\frac{1}{\operatorname{sh}^2 x}+1\right) = x + \int \frac{1}{\operatorname{sh}^2 x} = x - \operatorname{cth} x$$

$= \int \left(\frac{1}{sh^2 x} \ldots\right) \quad \ldots \, sh\, x$

$$\left(\frac{ch\, x}{sh\, x}\right)' = \frac{sh^2 x - ch^2 x}{sh^2 x} = -\frac{1}{sh^2 x}$$

[D] terzo gruppo da 1191 - 1210

1191 $$\int \frac{dx}{x\sqrt{x^2-2}} = -\int \frac{1}{t^{\cancel{2}}}\,\frac{1}{\cancel{t}\sqrt{\frac{1}{t^2}-2}}\,dt$$

$x = \frac{1}{t}$ $\quad x'(t) \quad dx = x'(t)\,dt$

$|x| > \sqrt{2}$ $\quad |t| < \frac{1}{\sqrt{2}}$ $\quad 0 < t < \frac{1}{\sqrt{2}}$

$x > \sqrt{2}$

$$= -\int \frac{1}{t}\,\frac{1}{\frac{\sqrt{1-2t^2}}{|t|}} \underset{t>0}{=} -\int \frac{1}{\sqrt{1-2t^2}}$$

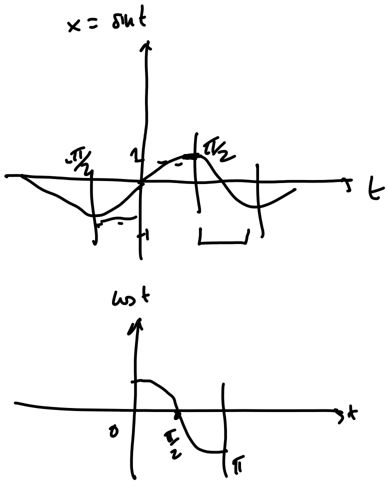
$$= -\int \frac{dt}{\sqrt{1-(\sqrt{2}t)^2}} \underset{u=\sqrt{2}t}{=} -\frac{1}{\sqrt{2}}\int \frac{du}{\sqrt{1-u^2}}$$

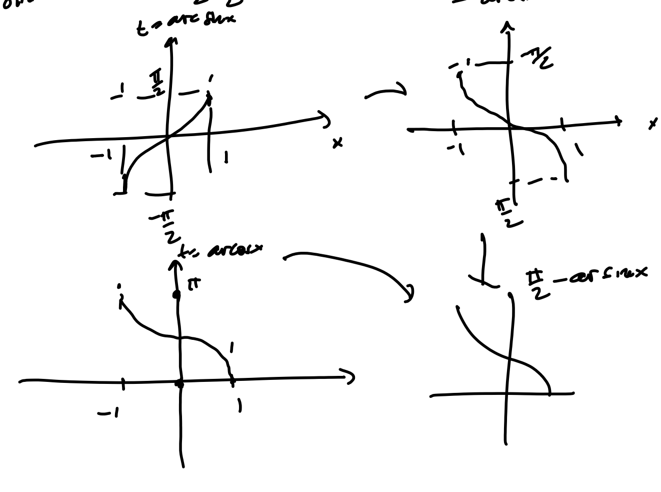
$$= -\frac{1}{\sqrt{2}}\arcsin u = -\frac{1}{\sqrt{2}}\arcsin \sqrt{2}t$$

$$= -\frac{1}{\sqrt{2}}\arcsin \frac{\sqrt{2}}{x}$$

Ris $\frac{1}{\sqrt{2}}\arccos\frac{\sqrt{2}}{x}$ $\qquad x > \sqrt{2}$

$\sin t$ in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ è invertibile $\Rightarrow$ derivabile



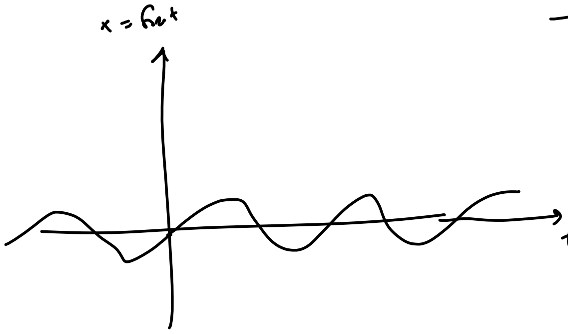


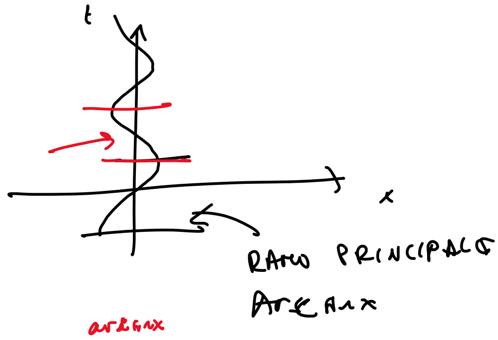
$\arccos x = \frac{\pi}{2} - \arcsin x$

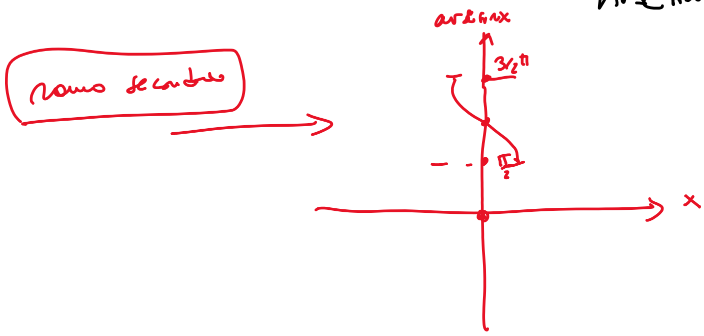
$\arccos 0 = \pi$ $\quad\quad$ $\arcsin 0 = 0$

$\mathrm{Arccos}\, x$ $\quad\quad$ $\mathrm{Arcsin}\, x$

RAMI PRINCIPALI







$\arcsin 0 = \pi$

è l'inversa di $\sin x$ con dominio $\left[\frac{\pi}{2}, \frac{3}{2}\pi\right]$

$$\boxed{\arcsin x = \pi + \mathrm{Arcsin}\, x}$$