Lezione 3

lunedì 27 settembre 2021 08:30

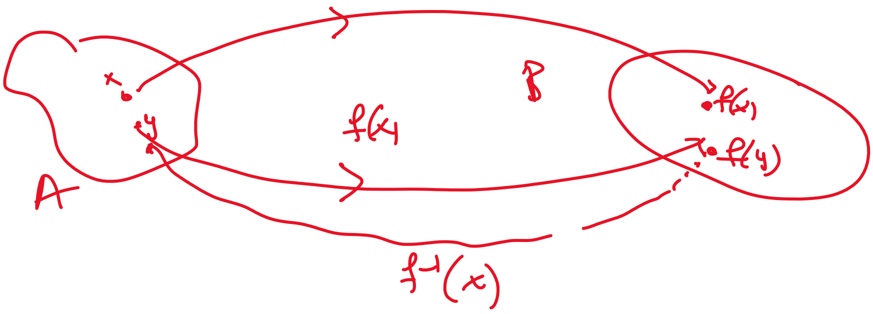
Base $a>0$, $a\neq 1$ $\quad$ $\left(\log x := \log_e x\right)$ ↑ base e numero di Eulero

$$\boxed{a^{\log_a x} = x}$$

Teor. Dato $x>0$ $\exists!$ $y\in\mathbb{R}$ | $a^y = x$, $y := \log_a x$

Ossia $x \mapsto \log_a x$ è la funzione INVERSA di $y \to a^y$

$f: A \to B$ iniettiva (1-1) $\quad (x\neq y \Rightarrow f(x)\neq f(y))$



$$\underbrace{f^{-1}(f(x))} = x$$

$$\boxed{f^{-1}\circ f = id}$$

$id(x) = x$ funzione identità

Def f è strettamente crescente se

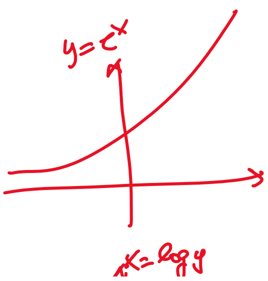
$\forall$ $x>y \Rightarrow f(x)>f(y)$

nel dominio di f

(in altri termini f strettam. crescente conserva l'ordine)

Esempi

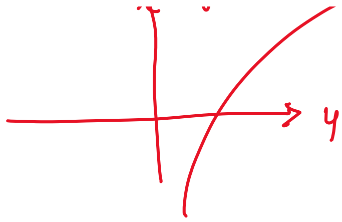
1. $x\to e^x$, $x\in\mathbb{R}$



$(a^x,\ a>1)$

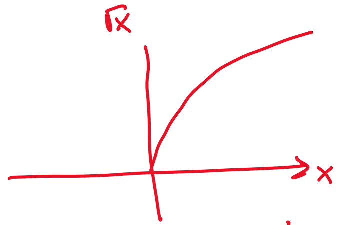
2. $y \to \log y$ $\left(\log_a y,\ a>1\right)$

$\underline{y>0}$ $\log_e y)$



3. $\sqrt{x} = x^{\frac{1}{2}}$

$x \geq 0$

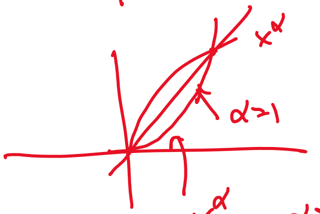


4. x^α

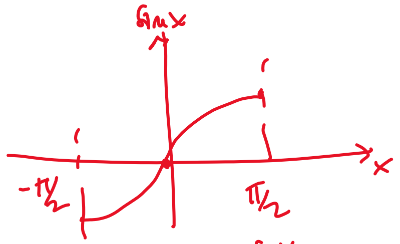
$x \geq 0$

$\underline{\alpha \geq 0}$

potenze (generalizzate)



5.



$\sin: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to \mathbb{R}$

è strett. crescente

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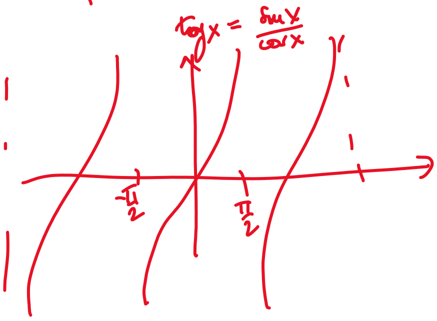


dominio è $\mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\right\}$

Torniamo ai logaritmi:

(1) $\log_a(xy) = \log_a x + \log_a y$

(2) $\log_a x^y = y \log_a x$ $x>0,\ y \in \mathbb{R}$

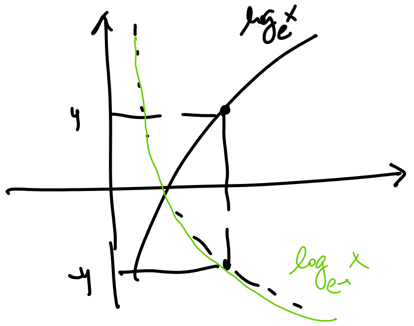
Cambio di base $\boxed{\log_a b \, \log_b x = \log_a x}$, $\left(a,b>0,\ a,b \neq 1,\ x \in \mathbb{R}\right)$

(Verifica $\underset{\text{"def"}}{x =} a^{\log_a x} \overset{?}{=} a^{\log_a b \, \log_b x} \underset{\text{proprietà degli esponenziali}}{=} \left(a^{\log_a b}\right)^{\log_b x} \overset{\text{def}}{=} b^{\log_b x} \overset{\text{def.}}{=} x$ OK!)

$\left((a^x)^y = a^{xy} \right)$

(*) $\log_a x = -\log_{a^{-1}} x$

segue da (1) con $b = a^{-1}$ $\qquad \log_a b = \log_a a^{-1} = -1 \log_a a = -1$



Es. IV $\quad$ Es 1 - (25)

$$A = \{ y = 2^x \mid \underbrace{\log_{11}(x+5) + \log_{11}(x-2) < \log_{11}(3x-1)}\}$$

$$B = \{ x \in \mathbb{R} \mid \text{vale} \}$$

1. Dominio di def. delle funzioni coinvolte

Devo avere

$$\begin{cases} x+5>0 \\ x-2>0 \\ 3x-1>0 \end{cases} \iff \begin{cases} x>-5 \\ x>2 \\ x>\frac{1}{3} \end{cases}$$

$x>y \Rightarrow 0>y-x$

$-x<-y \quad \Downarrow$

$-y>-x$

($ax>ay$ se $x>y$, $a>0$)

Quindi $\boxed{x>2}$

$$\underbrace{\log_{11}(x+5)(x-2)} < \underbrace{\log_{11}(3x-1)}$$

$$\iff (x+5)(x-2) < 3x-1$$

y $\log_{11}$ è strett. crescente ($f(x)>f(y) \iff x>y$)

$$\iff x^2+\cancel{3x}-10 < \cancel{3x}-1$$

$|x|<3 \iff -3<x<3$

$\iff x^2 < 9$

$\iff |x| < 3$

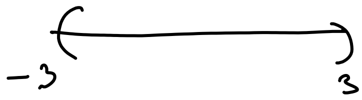
$|x| = \sqrt{x^2} = \begin{cases} x & \text{se } x>0 \\ -x & \text{se } x\le 0 \end{cases}$

$\forall x,y \in \mathbb{R}$

$|x+y| \le |x| + |y|$

$|xy| = |x|\,|y|$

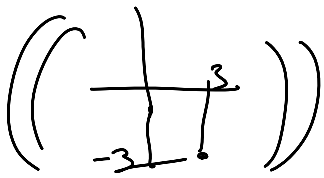
$|x-y| \ge \big||x|-|y|\big|$



$(-3,3) := \{|x|<3\}$

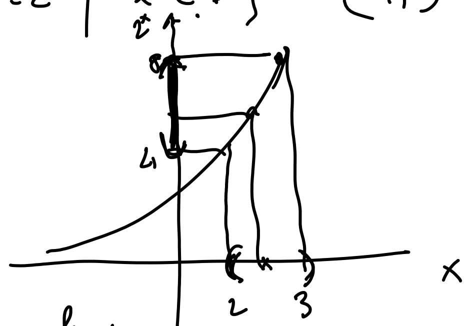
Def

$\{x \mid -3 < x < 3\}$



$B = (-3,3) \cap (2,+\infty) = (2,3) = \{x \mid 2<x<3\}$

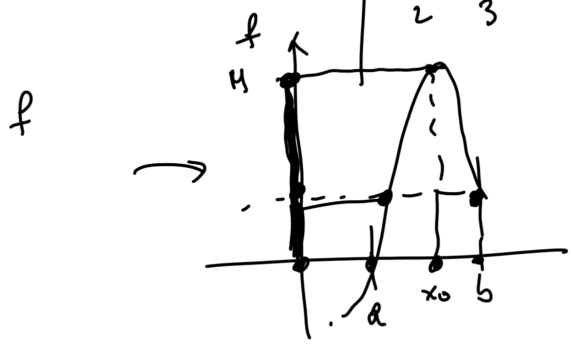
$A = \{y = 2^x \mid x \in B\} = (4,8)$



Ho usato 2 cose

$x \to 2^x$ è strett. crescente

e continua



$f((a,b)) = \{f(x) \mid x \in (a,b)\}$ $\quad a < x < b$

$= (0, M]$

MEDITARE SU QUESTO ESEMPIO

[GE] (Giusti Esercizi e complementi)

Cap 2 Es. 24.

$2^{(1+x^2)} \log_{10}(1+x^2) < 2^{10}$ cambio di variabile !!

$y := 1+x^2$

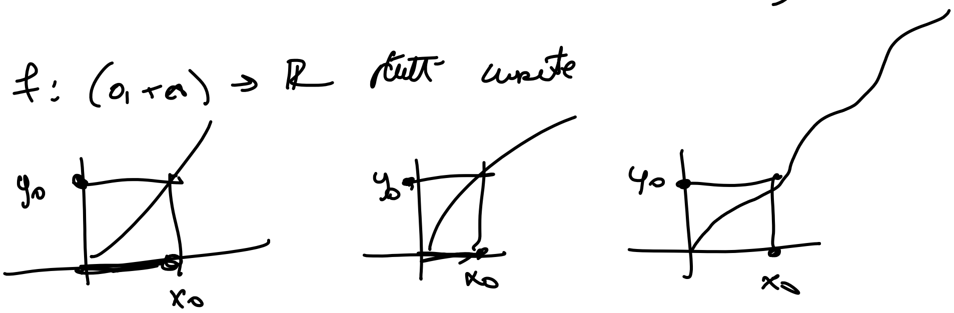
$\rightarrow \boxed{2^y \log_{10} y < 2^{10}}$ $\qquad 2^{y_0} \log_{10} y_0 = 2^{10} \iff y_0 = 10$!!

$y \rightarrow 2^y \log_{10} y$ è strett crescente

$y > 0$

(essendo il prodotto di due funzioni strett. crescenti)

Data $f : (0, +\infty) \rightarrow \mathbb{R}$ strett crescente



$f(x) < y_0 \iff 0 < x < x_0$

$\boxed{y \in (0, 10)}$

$y = 1+x^2$ $\qquad 0 < 1+x^2 < 10$

$x^2 < 9$

! $|x| < 3$

Risposta $\boxed{x = (-3, 3)}$