$x \mapsto x^{\alpha}$ $\quad \alpha \in \mathbb{R}$, $x > 0$ $\quad$ potenze generalizzate

se $\alpha \in \mathbb{N}$, $x \in \mathbb{R}$

$x \mapsto a^{x}$, $x \in \mathbb{R}$ $\quad a > 0$ $\quad a \neq 1$ $\quad$ funzioni esponenziali

base

$\log_a x$, $x > 0$

funzione inversa di a^x $\quad \log_a a^x = x$, $\quad a^{\log_a x} = x$

Oss il prodotto di due funzioni crescenti e positive è crescente

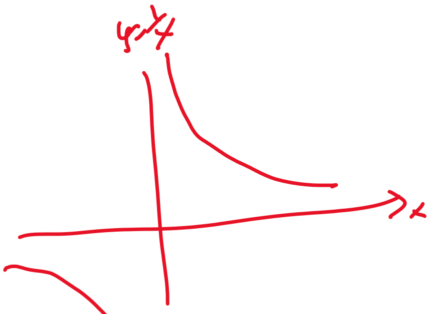
$$f(x)\,g(x) < f(x)\,g(y) < f(y)\,g(y)$$

$x < y$

L'ipotesi di positività è, in generale, necessaria

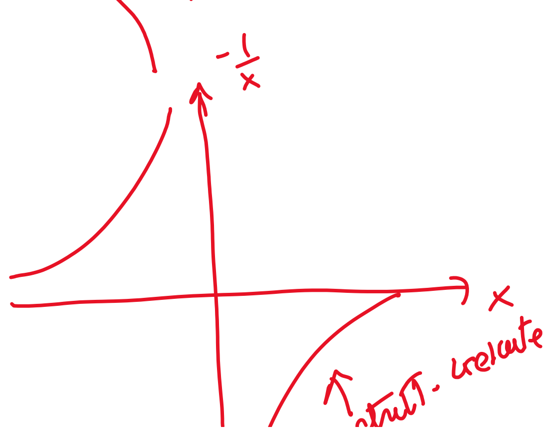
Esempio.

$f(x) = x^2$, $\quad g(x) = -\frac{1}{x}$



$x^2 \cdot \left(-\frac{1}{x}\right) = -x$

strett. decrescente



Es 8 Cap 2 [GE]

Dimostrare che $\forall \; x, y \geq 0$ risulta

$$(*) \quad |\sqrt{x} - \sqrt{y}| \leq \sqrt{|x-y|} \qquad \boxed{|x| = |-x|}$$

$$|\sqrt{y} - \sqrt{x}| \leq \sqrt{|y-x|}$$

Posso assumere che $x \geq y \geq 0 \Rightarrow \sqrt{x} \geq \sqrt{y}$

$$0 \leq \sqrt{x} - \sqrt{y} \overset{?}{\leq} \sqrt{x-y}$$

$$\iff \cancel{x} + y - 2\sqrt{xy} \leq \cancel{x} - y$$

$$\iff y \leq \sqrt{xy}$$

$$\iff y^2 \leq xy$$

se $y=0$ la disug è vera ✓

$$\overset{y>0}{\iff} y \leq x \;✓$$

fatto!

RICEVIMENTO (PROF. L.C.) 17-19 Mercoledì (mio studio)

T.V Es 1, (12)

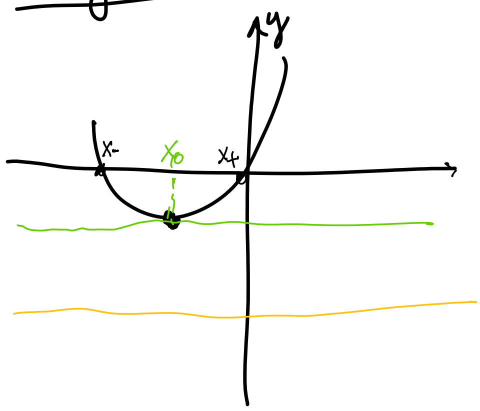
$$A = \{ \underbrace{x^2 - 5x}_{} + 6 \; : \; x^2 - 5x + 4 < 0 \}$$

tale che

verifica $x^2 - 5x + 4 < 0 \}$

$= \{ y = x^2 - 5x + 6 \mid x$ ...

Svolgimento. Sia $E = \{ x \in \mathbb{R} \mid \underline{x^2 - 5x + 4 < 0} \}$



$$y < 0 \iff \begin{cases} x_- < x < x_+ \\ \emptyset \\ \emptyset \end{cases}$$

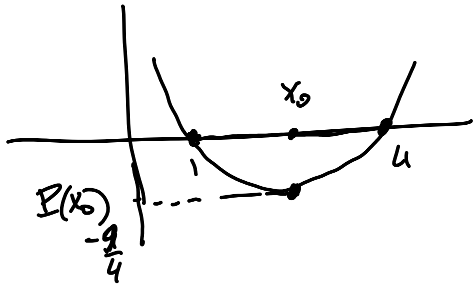
$$= (x-1)(x-4)$$

$$x_- = 1 \qquad x_+ = 4$$

$$1 < x < 4$$

$$E = (1, 4)$$

$$y = P(x) + 2 \qquad P(x) = x^2 - 5x + 4$$



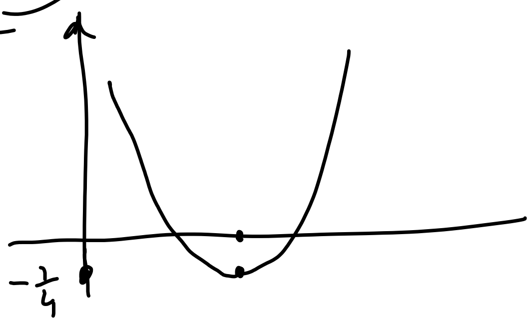
$$x_0 = \frac{4+1}{2} = \frac{5}{2}$$

punto di mezzo tra x_- e x_+

$$P(x_0) = \frac{25}{4} - \frac{25}{2} + 4$$

$$= -\frac{25}{4} + 4 = -\frac{9}{4}$$

$P+2$



$$2 - \frac{9}{4} = -\frac{1}{4}$$

$$A \quad \left[-\frac{1}{4}, 2\right)$$

$$P(x) = x^2 - 5x + 4 = x^2 - 2\left(\frac{5}{2}\right)x + 4 =$$

COMPLETAMENTO DEL QUADRATO:

$$= \left(x - \frac{5}{2}\right)^2 + 4 - \frac{25}{4}$$

Quindi

$$P(x) = \left(x - \frac{5}{2}\right)^2 - \frac{9}{4}$$

$\Longleftrightarrow$ il valore minimo di $P(x)$

è $P\left(\frac{5}{2}\right) = -\frac{9}{4}$

Radici $\quad P(x) = 0 \Longleftrightarrow \left(x - \frac{5}{2}\right)^2 = \frac{9}{4}$

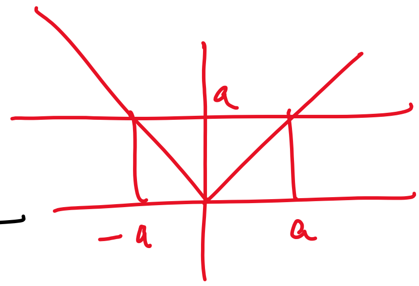
$$\Longleftrightarrow \left|x - \frac{5}{2}\right| = \frac{3}{2}$$

$$\Longleftrightarrow \begin{cases} x - \frac{5}{2} = \frac{3}{2} \\ \frac{5}{2} - x = \frac{3}{2} \end{cases}$$

$$\Longleftrightarrow \begin{cases} x = 4 \\ x = 1 \end{cases}$$

$|x| = a \quad a > 0$

$\Longleftrightarrow \begin{cases} x = a \\ \text{oppure} \\ x = -a \end{cases}$



CASO GENERALE

$P(x) = ax^2 + bx + c, \quad a \neq 0$

$$= a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$$

$$\left(x^2 \quad \frac{b}{a}x + \frac{c}{a}\right)$$

$$= a\Big(\underbrace{x + \cdots \tfrac{\ }{2a}}\ \cdots\ \tfrac{\ }{a}\Big)$$

$$= a\Big(\underbrace{\Big(x+\frac{b}{2a}\Big)^2} + \frac{c}{a} - \underbrace{\frac{b^2}{4a^2}}\Big)$$

ho aggiunto e tolto $\frac{b^2}{4a^2}$

$$= a\left(\Big(x+\frac{b}{2a}\Big)^2 - \frac{b^2-4ac}{4a^2}\right)$$

$$:= a\underbrace{\left(\Big(x+\frac{b}{2a}\Big)^2 - \frac{\Delta}{4a^2}\right)}$$

con $\Delta := b^2-4ac$

= discriminante della parabola

RADICI $\quad P(x)=0 \iff \Big(x+\frac{b}{2a}\Big)^2 = \frac{\Delta}{4a^2}$

- Nessuna soluzione $\qquad \Delta < 0$
- $x = -\frac{b}{2a} \qquad \Delta = 0$
- $x + \frac{b}{2a} = \pm\frac{\sqrt{\Delta}}{2a} \qquad \Delta > 0 \qquad (\sqrt{\Delta} > 0)$

$$= \frac{-b\pm\sqrt{\Delta}}{2a}$$

Se $a>0$, P ha un minimo assoluto in $x_0 = -\frac{b}{2a}$

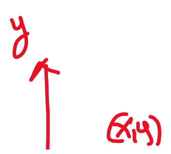
e $P\Big(-\frac{b}{2a}\Big) = -\frac{\Delta}{4a}$

Se $a<0$, P ha un massimo assoluto in ....)

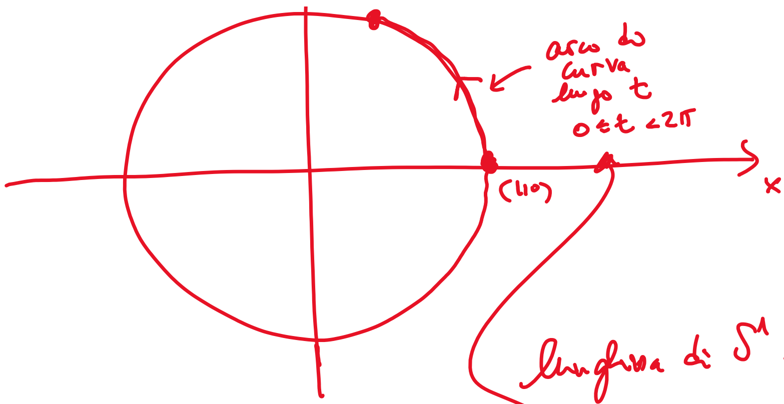
Vertice della parabola $(x_0,y_0) = \Big(-\frac{b}{2a}, -\frac{\Delta}{4a}\Big)$

FUNZIONI TRIGONOMETRICHE

"Def"



$S^1 := \{(x,y)\in\mathbb{R}^2 \mid x^2+y^2=1\}$



$\mathbb{R}\times\mathbb{R}$ = coppie ordinate di numeri reali

$(x,y)\neq(y,x)$ se $x\neq y$

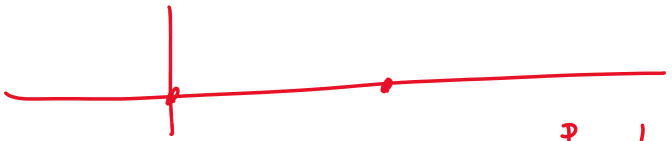
lunghezza di S^1 =: 2π

$(x(t), y(t))$:= coordinate del punto (x,y)

Def $\begin{cases} \cos t := x(t) \\ \sin t := y(t). \end{cases}$ $\quad 0\le t<2\pi$

estendo per periodicità $\forall\ t$

$\cos(t+2\pi)=\cos t$, $\quad \sin(t+2\pi)=\sin t \quad \forall\ t\in\mathbb{R}.$



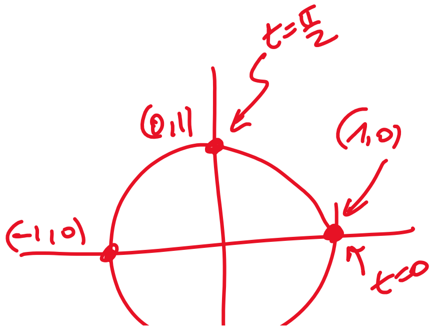
(1) Per def $\sin t$ e $\cos t$ sono funzioni definite $\forall\ t\in\mathbb{R}$ e periodiche di periodo 2π.

Proprietà

(2) Per def. $\sin^2 t+\cos^2 t=1$

$\left((\sin t)^2+(\cos t)^2=1 \right)$

(3) Valori speciali



$t=0$, $\cos 0 = 1$, $\sin 0 = 0$

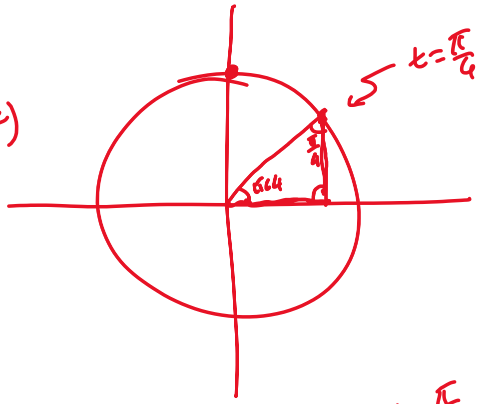
$t=\frac{\pi}{2}$, $\cos\frac{\pi}{2}=0$, $\sin\frac{\pi}{2}=1$

$t=\pi$, $\cos\pi=-1$, $\sin\pi=0$

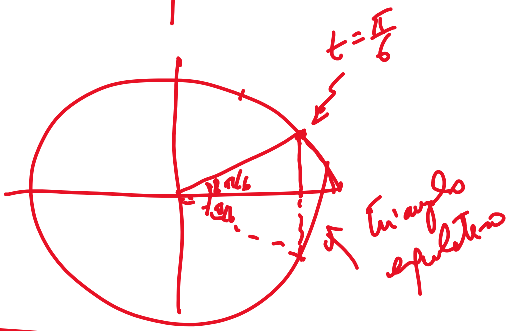
$t=\frac{3}{2}\pi$, $\cos\frac{3}{2}\pi=0$, $\sin\frac{3}{2}\pi=-1$

$t=\frac{\pi}{4}$ $\sin\frac{\pi}{4}=\cos\frac{\pi}{4}$ (grado euclideo)

$= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

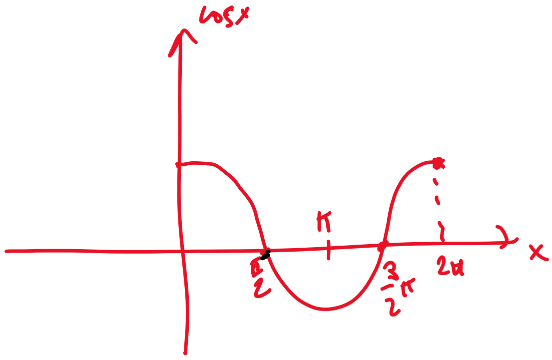


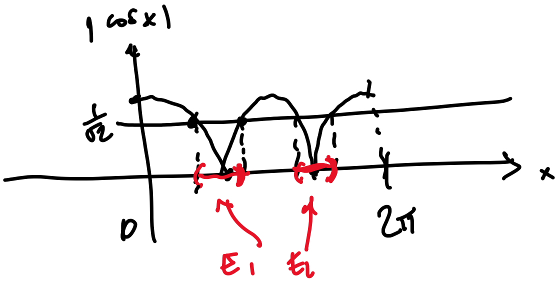
$\sin\frac{\pi}{6}=\frac{1}{2}$, $\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$



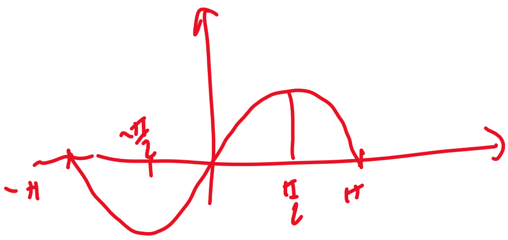
[GE] Es. 21 cap 2.

$A=\{x \mid |\cos x| < \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}\}$

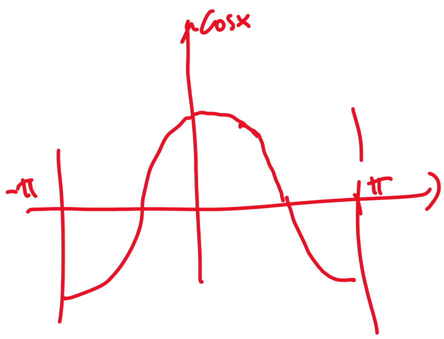




$E = E_1 \cup E_2$



$A = \bigcup_{k\in\mathbb{Z}} (E + 2k\pi)$



$$\begin{cases} \cos(t+s) = \cos t \cos s - \sin t \sin s \\ \sin(t+s) = \sin t \cos s + \cos t \sin s \end{cases}$$

FORMULE DI ADDIZIONE

$$\left[e^{it} = \cos t + i \sin t \right]$$

$$e^{i\pi} = -1$$