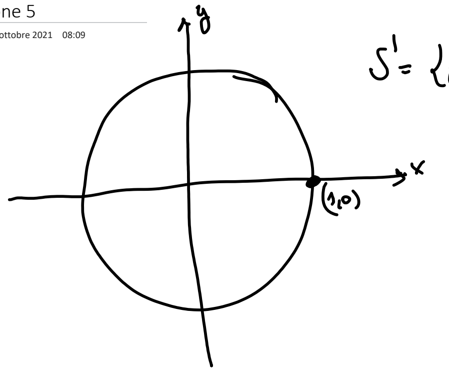
Lezione 5
venerdì 1 ottobre 2021 08:09



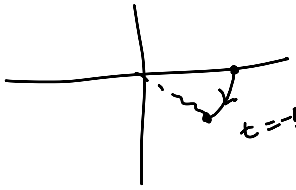
$S^1 = \{(x,y) \mid x^2+y^2=1\}$

lunghezza di $S^1 = 2\pi$

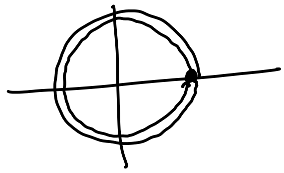
orientamento positivo su S^1 è antiorario
" negativo " " orario

t = lunghezza con segno di un arco

$-\frac{\pi}{4}$

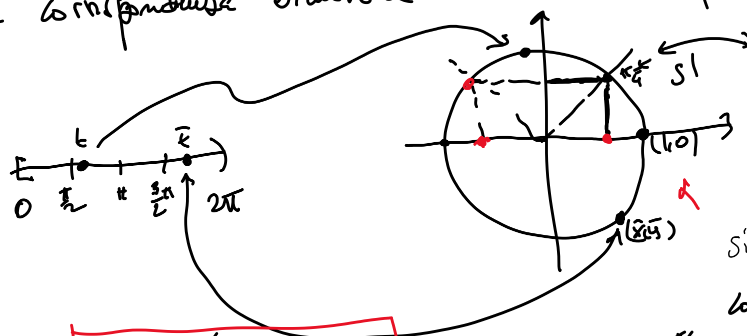


$t=-\frac{\pi}{4}$



$t = 4\pi$ 2 giri in senso antiorario

Mettendo avere preso $t \in [0, 2\pi)$ e in questo caso c'è una corrispondenza biunivoca tra t e i punti di S^1



$x = y$ $\begin{cases} x^2+y^2=1 \\ 2x^2=1 \\ x^2=\frac{1}{2} \\ y = x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \end{cases}$

$\sin\frac{3}{4}\pi = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\cos\frac{3}{4}\pi = -\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$

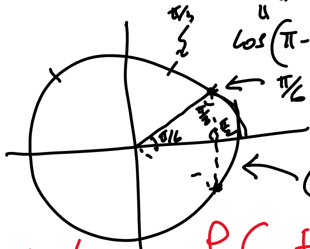
$t=\pi \quad s=-\frac{\pi}{4}$

$\cos\left(\pi-\frac{\pi}{4}\right) = \cos\pi\cos\left(-\frac{\pi}{4}\right) - \cancel{\sin\pi}\sin\left(-\frac{\pi}{4}\right)$ 0

$= -1\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$

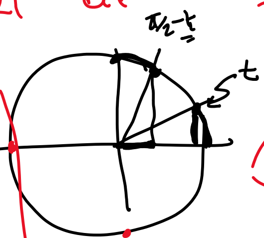
$\boxed{(x,y) = (\cos t, \sin t)}$

$$\boxed{\begin{aligned} \sin(-t) &= -\sin t \\ \cos(-t) &= \cos t \end{aligned}}$$



$\boxed{\sin\frac{\pi}{6} = \frac{1}{2},\ \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}}$ (uso la geometria euclidea)

$\sin t$ è una funzione DISPARI di t $\qquad f(-t) = -f(t)$
$\cos t$ " " " PARI di t $\qquad f(-t) = f(t)$



$\cos\left(\frac{\pi}{2}-t\right) = \cancel{\cos\frac{\pi}{2}}\cos(-t) - \sin\frac{\pi}{2}\sin(-t) = \sin t$

$\boxed{\sin\left(\frac{\pi}{2}-t\right) = \cos t}$

!!!

$$\boxed{\begin{aligned} \cos(t+s) &= \cos t\cos s - \sin t\sin s \\ \sin(t+s) &= \sin t\cos s + \sin s\cos t \end{aligned}}$$

FORMULE DI ADDIZIONE

Bisogna sapere $\sin\left(\frac{\pi}{2}+k\frac{\pi}{2}\right)$ $\forall k$
$\cos\left(\frac{\pi}{2}+k\frac{\pi}{2}\right)$ $\forall k$

$\sin\frac{\pi}{6} = \frac{1}{2}$

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ $\qquad$ e "multipli"

$\sin\frac{\pi}{3} = \sqrt{3}/2$

$$\cos\frac{\pi}{3} = \frac{1}{2}$$
$$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$
$$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

(1) $\sin^2 t + \cos^2 t = 1 \quad \left(\Longleftrightarrow (\cos t, \sin t) \in S^1 \right)$

Es. Ricavare la formula (1) dalle formule di addizione

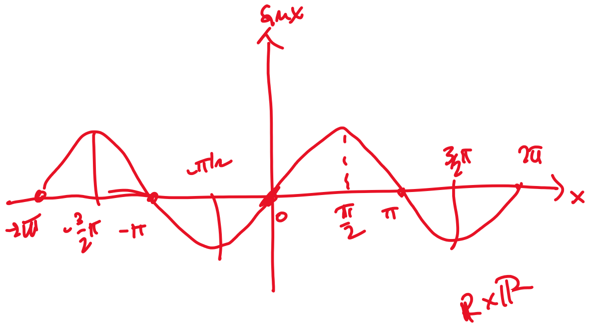
Sol. $s = -t$

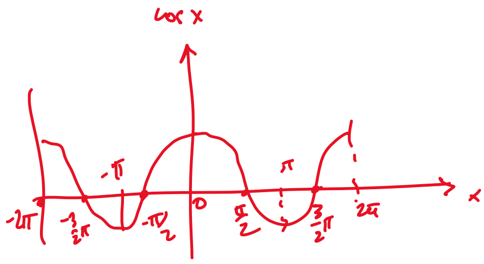
$$1 = \underset{\substack{\uparrow \\ t+s = t+(-t) = 0}}{\cos 0} = \cos t \cos(-t) - \sin t \sin(-t) = \cos^2 t + \sin^2 t.$$

Es. Dedurre le formule di duplicazione dalle formule di addizione

! $\left[\begin{array}{l} \cos 2t = \cos^2 t - \sin^2 t \\ \sin 2t = 2 \sin t \cos t \end{array} \right] \leftarrow$

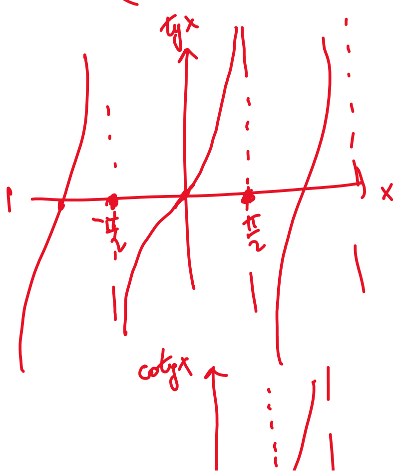
Def. $\operatorname{tg} x = \frac{\sin x}{\cos x} \qquad \operatorname{cotg} x = \frac{\cos x}{\sin x}$





$\mathbb{R} \times \mathbb{R}$

$\cos(t + 2\pi) = \cos t$ e $\sin(2\pi + t) = \sin t$ (segue dalle formule di addizione)



$\operatorname{tg} x = \frac{\sin x}{\cos x}$

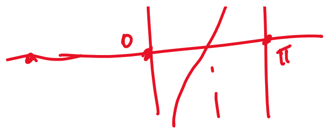
NB $\operatorname{tg}(x + \pi) = \operatorname{tg} x$

Es. ↑ dimostrare questa formula usando le formule di addizione

Dominio della tg è $\mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi \mid k \in \mathbb{Z} \right\}$

$\operatorname{cotg} x = \frac{\cos x}{\sin x}$

Dominio della cotg è $\mathbb{R} \setminus \left\{ k\pi \mid k \in \mathbb{Z} \right\}$



Es. Sia $t := \operatorname{tg}\frac{x}{2}$ Calcolare $\frac{1-t^2}{1+t^2}$ e $\frac{2t}{1+t^2}$

Svolgimento

$$1+t^2 = 1+\frac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}} = \frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}} = \frac{1}{\cos^2\frac{x}{2}}$$

$$1-t^2 = 1-\frac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}} = \frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}} = \frac{\cos x}{\cos^2\frac{x}{2}}$$

! $\boxed{\frac{1-t^2}{1+t^2} = \cos x}$ $\boxed{\frac{2t}{1+t^2} = \sin x}$

$$2t = 2\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} \qquad \frac{2t}{1+t^2} = 2\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\cos^2\frac{x}{2} = 2\sin\frac{x}{2}\cos\frac{x}{2} = \sin x$$

T.V, Es 1, (33)

$$A = \{x \in [-\pi, 4\pi] \mid \sin^2 x + 3\cos^2 x + \sin x - 2 = 0\}$$

$$\sin^2 x + 3\cos^2 x + \sin x - 2 = 0$$

1ª idea $\rightarrow$ $\sin^2 x + 3(1-\sin^2 x) + \sin x - 2 = 0$

$$-2\sin^2 x + \sin x + 1 = 0$$

$$2\sin^2 x - \sin x - 1 = 0$$

2ª idea $\rightarrow$ $2y^2 - y - 1 = 0$, $y := \sin x$

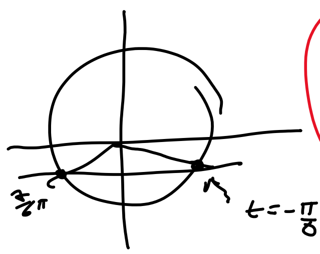
$$y_\pm = \frac{1 \pm \sqrt{1+8}}{4} = \begin{cases} 1 \\ -\frac{1}{2} \end{cases}$$

- $\sin x = 1$ oppure $\sin x = -\frac{1}{2}$

$\Updownarrow$ $\Updownarrow$

- $x = \frac{\pi}{2} + 2k\pi$ $\qquad x = -\frac{\pi}{6} + 2k\pi$

$= \frac{7}{6}\pi + 2k\pi$



DEF
$x \in [a, b)$
$\Updownarrow$
$a \leq x < b$

$[-\pi, 4\pi]$

$x \in [a, b]$ DEF
$\Leftrightarrow a \leq x \leq b$

$[-\pi, 4\pi] \to \frac{\pi}{2} + 2k\pi = \frac{\pi + 4k\pi}{2}$, $\frac{\pi}{2} = k=0$

$-\pi \le \frac{\pi + 4\pi k}{2} \le 4\pi$

$-2\pi \le \pi + 4k\pi \le 8\pi$

$[-2\pi, 8\pi] \ni \pi + 4k\pi$

ok. $k = 0, 1$ $\hookrightarrow$ $\frac{\pi}{2}, \frac{5}{2}\pi$

$-\pi \le -\frac{\pi}{6} + 2k\pi \le 4\pi \iff -6\pi \le -\pi + 12k\pi \le 24\pi$

$\iff -6 \le 12k - 1 \le 24$, $k \in \mathbb{Z}$

ok. $0, 1, 2$

$-\frac{\pi}{6}, \frac{11}{6}\pi, \frac{23}{6}\pi$

$-\frac{1}{6} + 2 = \frac{11}{6}$

$-\frac{1}{6} + 4 = \frac{23}{6}$

$-\pi \le \frac{7}{6}\pi + 2k\pi \le 4\pi$

$-6 \le 7 + 12k \le 24$

$k = 0$ ✓.

$k = -1$

$k = 1$

$\frac{7 + 12k}{6}\pi$, $\frac{7}{6}\pi, \frac{19}{6}\pi, -\frac{5}{6}\pi$

tutte le soluzioni sono. $A = \left\{ \frac{\pi}{2}, \frac{5}{2}\pi, -\frac{\pi}{6}, \frac{11}{6}\pi, \frac{23}{6}\pi, \frac{7}{6}\pi, \frac{19}{6}\pi, -\frac{5}{6}\pi \right\}$

$= \left\{ -\frac{5}{6}\pi, -\frac{\pi}{6}, \frac{\pi}{2}, \frac{7}{6}\pi, \frac{11}{6}\pi, \frac{5}{2}\pi, \frac{19}{6}\pi, \frac{23}{6}\pi \right\}$