## 7.3 Exercises

1. (A) Show that $t, -2t^2, t^3$ are linearly independent.
   (B) Show that $e^{-t}, e^t$, and $e^{2t}$ are linearly independent.
2. (A) Show that if $u_1, u_2, \ldots, u_k$, $2 \le k < n$, are linearly dependent, then any larger set $u_1, u_2, \ldots, u_k, u_{k+1}, \ldots, u_n$ are all linearly dependent.
   (B) Evaluate the wronskian $W(2, \sin^2 t, t^7 e^t, e^{t^2}, \cos^2 t, t^3, t^5 - 1)$.
3. Prove that pairwise linear independence does not imply linear independence.
4. Solve $x''' - 4x' = 0$.
5. Solve $x''' - 3x'' - x' + 3x = 0$.
6. Solve $x''' - 2x'' + x' - 2x = 0$.
7. Solve $x''' - 5x'' + 3x' + 9x = 0$.
8. Solve $x'''' - x' = 0$.
9. Solve $x'''' - 3x'' + 2x = 0$.
10. Solve $x'''' - 4x''' = 0$.
11. Solve $x''' - 25x' = e^t + 3\sin t$.
12. Solve $x''' - x = e^t$.
13. Solve $x'''' - 2x''' = -12$.
14. Solve the ivp $x''' + x'' = 0$, $x(0) = 1$, $x'(0) = 1$, $x''(0) = 0$.
15. Solve the ivp $x'''' - x = 0$, $x(0) = x'(0) = x''(1) = 0$, $x'''(0) = -1$.
16. Find the solution of the beam equation $\frac{d^4 u(x)}{dx^4} = h(x)$ when the distributed load is $h(x) = ax^2$, $a \in \mathbb{R}$.
17. Prove that there are solutions of a third order equation which do not vanish.
18. Find $p, q$ such that the solutions of the ivp $x''' - x' = 0$, $x(0) = p$, $x'(0) = q$, $x''(0) = p$, tends to 0 as $t \to +\infty$.
19. Find $b$ such that none of the nontrivial solutions of $x''' - (3+b)x'' + (2+3b)x' - 2bx = 0$ tend to zero as $t \to +\infty$. [Hint: notice that $\lambda = b$ is a root of the characteristic equation.]
20. Show that for every $b \neq 0$ there is a one-parameter family of nontrivial solutions of $x'''' - b^2 x'' = 0$ which tend to zero at $t \to +\infty$.
21. Solve $t^3 x''' - 3t^2 x'' + 6tx' - 6x = 0$.
22. Show that $t^3 x''' + 3t^2 x'' + tx + x = 0$ has a one-parameter family of solutions that tends to 0 as $t \to \infty$ and two-parameter families of solutions with infinitely many zeros.
23. Solve $t^3 x''' + 2t^2 x'' + tx' - x = 0$.
24. Solve $t^3 x''' + 2t^2 x'' - 4tx' + 4x = 0$.
25. Solve $t^3 x''' + 3t^2 x'' + tx' + x = 0$.