8.5 Exercises

1. Solve

$$\begin{cases} x' - y = 1, \\ y' - 2x' = -x. \end{cases}$$

2. Solve

$$\begin{cases} x'' - x' - y = 0, \\ y' - x' = -x. \end{cases}$$

3. Use the eigenvalue method to find the general solution of

$$\begin{cases} x' = 4x + 6y, \\ y' = x + 3y. \end{cases}$$

4. Find the general solution of problem 3 by converting the equation to a second order linear equation suggested in Theorem 8.5.
5. Solve

$$\overline{x}' = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} \overline{x}.$$

6. Solve

$$\overline{x}' = \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix} \overline{x}, \quad \overline{x}(0) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

7. Solve problem 6 by converting the system to a linear scalar equation suggested by Theorem 8.5.
8. Solve

$$\begin{cases} x_1' = x_1 + x_2 + 3e^t, \\ x_2' = 2x_1 + 2x_2 + e^t. \end{cases}$$

9. Solve

$$\begin{cases} x_1' = x_1 + \sin t, \\ x_2' = -x_2 + \cos t. \end{cases}$$

10. Show that any system

$$\begin{cases} x_1' = ax_1 + bx_2, \\ x_2' = cx_1 + dx_2, \end{cases}$$

has a nontrivial constant solution if and only if $\left|\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right| = 0$.

11. Solve

$$\overline{x}' = \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix} \overline{x}.$$

12. Solve problem 11 by converting the system to a second order scalar equation.
13. Use both methods (eigenvalue and converting it to a scalar equation) to solve

$$\overline{x}' = \begin{pmatrix} 4 & 6 \\ 1 & 3 \end{pmatrix} \overline{x}.$$

14. Use the eigenvalue method to find the solution to the initial value problem

$$\overline{x}' = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \overline{x}, \quad \overline{x}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

15. Find a fundamental set of solutions of the system below, first by the method of eigenvalues and then by converting the equation to a scalar equation suggested by Theorem 8.5.

$$\overline{x}' = \begin{pmatrix} 5 & -8 \\ 1 & 1 \end{pmatrix} \overline{x}.$$

16. Show that

$$\overline{x}' = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \overline{x}$$

cannot have periodic solutions if b and c are either both positive or both negative.

17. Prove that the following vectors are linearly dependent:

$$\overline{x}_1 = \begin{pmatrix} 2t^2 \\ t \\ \sin^2 t \end{pmatrix}, \quad \overline{x}_2 = \begin{pmatrix} -3t^2 \\ 4t \\ 2\cos^2(t) \end{pmatrix}, \quad \overline{x}_3 = \begin{pmatrix} t^2 \\ 6t \\ 2 \end{pmatrix}.$$

18. Solve

$$\overline{x}' = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & -1 \end{pmatrix} \overline{x}.$$

19. Solve the Cauchy problem

$$\overline{x}' = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix} \overline{x}, \quad \overline{x}(0) = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}.$$

20. Find a fundamental set of solutions for

$$\overline{x}' = \begin{pmatrix} 3 & 4 & 1 \\ 1 & 3 & -2 \\ 0 & 0 & 3 \end{pmatrix} \overline{x}.$$