AM110 - Analisi matematica 1

Luca Battaglia

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Argomenti: limiti con formula di Taylor

Esercizio 1.
Calcolare, se esiste, il limite

$$\lim_{x\to 0} \frac{\sin(2x) - 2\log(1+x)}{x^2}.$$

Soluzione:

$$\begin{aligned}
\frac{\sin(2x) - 2\log(1+x)}{x^2} &= \frac{\left(2x + o\left((2x)^2\right)\right) - 2\left(x - \frac{x^2}{2} + o\left(x^2\right)\right)}{x^2} \\
&= \frac{2x - \left(2x - x^2\right) + o\left(x^2\right)}{x^2} \\
&= \frac{x^2 + o\left(x^2\right)}{x^2} \\
&= 1 + o(1) \\
&\underset{x\to 0}{\to} 1.
\end{aligned}$$

Esercizio 2.
Calcolare, se esiste, il limite

$$\lim_{x\to 0} \frac{x^4}{e^{-x^2} - \cos\left(\sqrt{2}x\right)}.$$

Soluzione:

$$\begin{aligned}
\frac{x^4}{e^{-x^2} - \cos\left(\sqrt{2}x\right)} &= \frac{x^4}{\left(1 + (-x^2) + \frac{(-x^2)^2}{2} + o\left(\left(-x^2\right)^2\right)\right) - \left(1 - \frac{\left(\sqrt{2}x\right)^2}{2} + \frac{\left(\sqrt{2}x\right)^4}{24} + o\left(\left(\sqrt{2}x\right)^4\right)\right)} \\
&= \frac{x^4}{\left(1 - x^2 + \frac{x^4}{2} + o\left(x^5\right)\right) - \left(1 - x^2 + \frac{x^4}{6} + o\left(x^4\right)\right)} \\
&= \frac{x^4}{\frac{x^4}{3} + o\left(x^4\right)} \\
&= \frac{1}{\frac{1}{3} + o(x)} \\
&\underset{x\to 0}{\to} 3.
\end{aligned}$$

Esercizio 3.
Calcolare, se esiste, il limite

$$\lim_{x\to 0} \frac{\arcsin x - x\cos x}{x - \arctan x}.$$

Soluzione:

$$
\begin{aligned}
\frac{\arcsin x - x\cos x}{x - \arctan x} &= \frac{\left(x + \frac{x^3}{6} + o\left(x^3\right)\right) - x\left(1 - \frac{x^2}{2} + o\left(x^2\right)\right)}{x - \left(x - \frac{x^3}{3} + o\left(x^3\right)\right)} \\
&= \frac{x + \frac{x^3}{6} - \left(x - \frac{x^3}{2}\right) + o\left(x^3\right)}{\frac{x^3}{3} + o\left(x^3\right)} \\
&= \frac{\frac{2}{3}x^3 + o\left(x^3\right)}{\frac{x^3}{3} + o\left(x^3\right)} \\
&= \frac{\frac{2}{3} + o(1)}{\frac{1}{3} + o(1)} \\
&\underset{x\to 0}{\to} 2.
\end{aligned}
$$

Esercizio 4.
Calcolare, se esiste, il limite

$$\lim_{x\to 0} \frac{\frac{2}{2-x} - e^{\frac{x}{2}}}{e^{\frac{x}{e}} - \log(e+x)}.$$

Soluzione:

$$
\begin{aligned}
\frac{\frac{2}{2-x} - e^{\frac{x}{2}}}{e^{\frac{x}{e}} - \log(e+x)} &= \frac{\frac{1}{1-\frac{x}{2}} - e^{\frac{x}{2}}}{e^{\frac{x}{e}} - 1 - \log\left(1 + \frac{x}{e}\right)} \\
&= \frac{\left(1 + \frac{x}{2} + \left(\frac{x}{2}\right)^2 + o\left(\left(\frac{x}{2}\right)^2\right)\right) - \left(1 + \frac{x}{2} + \frac{\left(\frac{x}{2}\right)^2}{2} + o\left(\left(\frac{x}{2}\right)^2\right)\right)}{\left(1 + \frac{x}{e} + \frac{\left(\frac{x}{e}\right)^2}{2} + o\left(\left(\frac{x}{e}\right)^2\right)\right) - 1 - \left(\frac{x}{e} - \frac{\left(\frac{x}{e}\right)^2}{2} + o\left(\left(\frac{x}{e}\right)^2\right)\right)} \\
&= \frac{\left(1 + \frac{x}{2} + \frac{x^2}{4} + o\left(x^2\right)\right) - \left(1 + \frac{x}{2} + \frac{x^2}{8} + o\left(x^2\right)\right)}{\left(1 + \frac{x}{e} + \frac{x^2}{2e^2} + o\left(\left(x^2\right)\right)\right) - 1 - \left(\frac{x}{e} - \frac{x^2}{2e^2} + o\left(\left(x^2\right)\right)\right)} \\
&= \frac{\frac{x^2}{8} + o\left(x^2\right)}{\frac{x^2}{e^2} + o\left(x^2\right)} \\
&\underset{x\to 0}{\to} \frac{e^2}{8}.
\end{aligned}
$$

Esercizio 5 (Assegnato per casa)**.**
Calcolare, se esiste, il limite

$$\lim_{x\to 0} \frac{6x - 2\sin(3x)}{\arctan(2x)\arcsin(2x)\tan(2x)}.$$

Soluzione:

$$
\begin{aligned}
\frac{6x - 2\sin(3x)}{\arctan(2x)\arcsin(2x)\tan(2x)} &= \frac{6x - 2\left(3x - \frac{(3x)^3}{6} + o\left((3x)^3\right)\right)}{(2x + o(x))(2x + o(x))(2x + o(x))} \\
&= \frac{6x - \left(6x - 9x^3 + o\left(x^3\right)\right)}{8x^3 + o\left(x^3\right)} \\
&= \frac{9x^3 + o\left(x^3\right)}{8x^3 + o\left(x^3\right)} \\
&\underset{x\to 0}{\to} \frac{9}{8}.
\end{aligned}
$$

Esercizio 6 (Assegnato per casa)**.**
Calcolare, se esiste, il limite

$$\lim_{x\to 0} \frac{\sqrt{1+2x^2}-\sqrt[3]{1+3x^2}}{\log\left(1+x+\frac{x^2}{2}\right)-\sin x}.$$

Soluzione:

$$\begin{aligned}
&\frac{\sqrt{1+2x^2}-\sqrt[3]{1+3x^2}}{\log\left(1+x+\frac{x^2}{2}\right)-\sin x} \\
= \quad & \frac{\left(1+\frac{1}{2}\left(2x^2\right)-\frac{\left(2x^2\right)^2}{8}+o\left(\left(2x^2\right)^2\right)\right)-\left(1+\frac{1}{3}\left(3x^2\right)-\frac{\left(3x^2\right)^2}{9}+o\left(\left(3x^2\right)^2\right)\right)}{\left(x+\frac{x^2}{2}-\frac{\left(x+\frac{x^2}{2}\right)^2}{2}+\frac{\left(x+\frac{x^2}{2}\right)^3}{3}-\frac{\left(x+\frac{x^2}{2}\right)^4}{4}+o\left(\left(x+\frac{x^2}{2}\right)^4\right)\right)-\left(x-\frac{x^3}{6}+o\left(x^4\right)\right)} \\
= \quad & \frac{\left(1+x^2-\frac{x^4}{2}+o\left(x^4\right)\right)-\left(1+x^2-x^4+o\left(x^4\right)\right)}{\left(x+\frac{x^2}{2}-\frac{x^2+x^3+\frac{x^4}{4}}{2}+\frac{x^3+\frac{3}{2}x^4+o(x^4)}{3}-\frac{x^4+o(x^4)}{4}+o\left(x^4\right)\right)-\left(x-\frac{x^3}{6}+o\left(x^4\right)\right)} \\
= \quad & \frac{\frac{x^4}{2}+o\left(x^4\right)}{\left(x-\frac{x^3}{6}+\frac{x^4}{8}+o\left(x^4\right)\right)-\left(x-\frac{x^3}{6}+o\left(x^4\right)\right)} \\
= \quad & \frac{\frac{x^4}{2}+o\left(x^4\right)}{\frac{x^4}{8}+o\left(x^4\right)} \\
\underset{x\to 0}{\to} \quad & 4.
\end{aligned}$$