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Factoring ideals in Prüfer domains. (English summary)

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Let R be a Prüfer domain, and let I^v denote the divisorial closure of an ideal I of R. The authors say that R has the strong factorization property if, for each nonzero ideal I of R, we have (1) $I = I^v M_1 \cdots M_n$, where $M_1, \dots, M_n$ are the (distinct) nondivisorial maximal ideals of R containing I for which IR_M is also nondivisorial, and (2) this factorization is unique in the sense that no M_i can be omitted. The Prüfer domain R is said to have the weak factorization property if each nonzero ideal I of R can be written as $I = I^v M_1 \cdots M_n$, where $M_1, \dots, M_n$ are (not necessarily distinct) maximal ideals. Among the many results on these factorization properties, it is shown that a Prüfer domain R has the strong factorization property if and only if R is h-local if and only if for each nonzero ideal I of R, I is divisorial if and only if IR_M is divisorial for each maximal ideal M of R. It is shown that a Prüfer domain of finite character with the weak factorization property is h-local. Several further properties are given of Prüfer domains which satisfy either of these factorization properties, including behavior of factorizations of II^{-1}, IJ, $I \cap J$, $I + J$, $\mathrm{rad}(I)$ in terms of factorizations of I and J. Several interesting examples are also given.

Reviewed by *David E. Rush*

References

1. V. Barucci, E. Houston, T. Lucas, I. Papick, m-Canonical ideals in integral domains II, in: D.D. Anderson, I. Papick (Eds.), Ideal Theoretic Methods in Commutative Algebra, in: Lecture Notes in Pure and Appl. Math., vol. 220, Dekker, New York, 2001, pp. 89–108. MR1836594 (2002e:13007)
2. S. Bazzoni, L. Salce, Warfield domains, J. Algebra 185 (1996) 836–868. MR1419726 (97j:13014)
3. E. Bastida, R. Gilmer, Overrings and divisorial ideals of rings of the form $D + M$, Michigan Math. J. 20 (1973) 79–95. MR0323782 (48 #2138)
4. M. Fontana, J. Huckaba, Localizing systems and semistar operations, in: S.T. Chapman, S. Glaz (Eds.), Non-Noetherian Commutative Ring Theory, Kluwer Academic Publishers, 2000, pp. 169–198. MR1858162 (2002k:13001)
5. M. Fontana, E. Houston, T. Lucas, Toward a classification of prime ideals in Prüfer domains (manuscript).
6. M. Fontana, J. Huckaba, I. Papick, Prüfer Domains, Marcel Dekker Inc., New York, 1997. MR1413297 (98d:13021)
7. R. Gilmer, Overrings of Prüfer domains, J. Algebra 4 (1966) 331–340. MR0202749 (34 #2609)
8. R. Gilmer, Multiplicative Ideal Theory, M. Dekker, New York, 1972. MR0427289 (55 #323)

9. R. Gilmer, W. Heinzer, Overrings of Prüfer domains. II, J. Algebra 7 (1967) 281–302. MR0217055 (36 #150)
10. W. Heinzer, Integral domains in which each non-zero ideal is divisorial, Mathematika 15 (1968) 164–170. MR0236161 (38 #4459)
11. W. Heinzer, J. Huckaba, I. Papick, m-canonical ideals in integral domains, Comm. Algebra 26 (1998) 3021–3043. MR1635902 (99h:13024)
12. I. Kaplansky, Commutative Rings, Allyn and Bacon, Inc., Boston, 1970. MR0254021 (40 #7234)
13. K.A. Loper, T. Lucas, Factoring ideals in almost Dedekind domains, J. Reine Angew. Math. 565 (2003) 61–78. MR2024646 (2005b:13036)
14. T. Lucas, The radical trace property and primary ideals, J. Algebra 184 (1996) 1093–1112. MR1407887 (97f:13002)
15. A. Okabe, R. Matsuda, Semistar-operations on integral domains, Math. J. Toyama Univ. 17 (1994) 1–21. MR1311837 (95k:13027)
16. B. Olberding, Globalizing local properties of Prüfer domains, J. Algebra 205 (1998) 480–504. MR1632741 (2000f:13040)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.