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A generalization of Kronecker function rings and Nagata rings. (English summary)

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Summary: “Let D be an integral domain with quotient field K . The Nagata ring $D(X)$ and the Kronecker function ring $\text{Kr}(D)$ are both subrings of the field of rational functions $K(X)$ containing as a subring the ring $D[X]$ of polynomials in the variable X . Both of these function rings have been extensively studied and generalized. The principal interest in these two extensions of D lies in the reflection of various algebraic and spectral properties of D and $\text{Spec}(D)$ in algebraic and spectral properties of the function rings. Despite the obvious similarities in definitions and properties, these two kinds of domains of rational functions have been classically treated independently, when D is not a Prüfer domain. The purpose of this note is to study two different unified approaches to the Nagata rings and the Kronecker function rings, which yield these rings and their classical generalizations as special cases.”

Reviewed by *R. K. Markanda*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.