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On integral domains whose overrings are Kaplansky ideal transforms. (English. English summary)

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Let R be an integral domain with quotient field K and I an ideal of R. The Kaplansky ideal transform of I with respect to R is $\Omega_R(I) = \{z \in K \mid \mathrm{rad}((R{:}_R\, zR)) \supseteq I\}$ or equivalently, $\Omega_R(I)$ is the ring of sections of the affine scheme $\widetilde{R}$ over the open subset $D(I)$ of $\mathrm{Spec}(R)$. The authors define an Ω-domain as an integral domain each of whose overrings is a Kaplansky transform. The paper under review attempts to characterize Ω-domains. The authors are motivated by a paper of J. W. Brewer and R. Gilmer [see Math. Nachr. **51** (1971), 255–267; MR0309925 (46 #9029)] which solved a similar problem for the Nagata transform of finitely generated ideals but left unsolved questions. A particularly good characterization is obtained for Prüfer domains: a Prüfer domain R is an Ω-domain if and only if for each nonzero branched prime ideal P of R the generization of P is open in the Zariski topology. Here are some results of particular interest. If R is a Prüfer Ω-domain and P is a prime ideal of R, then R/P is an Ω-domain. A Prüfer semilocal Ω-domain is a T-domain where T means the Nagata transform. A satisfactory description of general Ω-domains is given in the semilocal case: let R be a semilocal domain with integral closure $\overline{R}$, then R is an Ω-domain if and only if $\overline{R}$ is an Ω-domain and R is a QQR-domain (each overring of R is an intersection of localizations at prime ideals of R). *Gabriel Picavet* (Le Cendre)