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Nagata transforms and localizing systems. (English. English summary)
Let $R$ be an integral domain with quotient field $K$. A set $\mathcal{F}$ of ideals of $R$ is said to be a localizing system if (1) $I \in \mathcal{F}$ and $J \supseteq I$ implies $J \in \mathcal{F}$ and (2) $I \in \mathcal{F}$ and $J$ an ideal of $R$ such that $(J:_R a) \in \mathcal{F}$ for each $a \in I$ implies $J \in \mathcal{F}$. It is easy to show that a localizing system is a special type of generalized multiplicative system and that therefore one can form the ring $R_\mathcal{F} = \{ x \in K | xI \subseteq R \text{ for some } I \in \mathcal{F} \}$. Now recall that the Nagata transform of an ideal $I$ of $R$ is the ring $T_R(I) = \bigcup_{n=1}^{\infty} (R : I^n)$. It is well known that if $I$ is a finitely generated ideal, then $T_R(I) = \bigcap R_P$, where the intersection is taken over those prime ideals of $R$ which do not contain $I$ [J. Brewer, Math. Z. 107 (1968), 301–306; MR0236158 (38 #4456)]; it is also easy to see that this intersection of localizations equals $R_\mathcal{F}$ for an appropriate localizing system $\mathcal{F}$. The purpose of the paper under review is to characterize when the Nagata transform of an ideal is of the form $R_\mathcal{F}$. Denoting by $N(I)$ the smallest localizing system of $R$ which contains $I$, the authors’ main result is the equivalence of the following statements: (1) $T(I) = R_\mathcal{F}$ for some localizing system $\mathcal{F}$; (2) $T(I) = R_{N(I)}$; and (3) $T_R(I) = T_{T_R(I)}(IT_R(I))$. The proofs require transfinite induction. The paper concludes with an example of an $R$ and $I$ for which $T_R(I)$ is not of the form $R_\mathcal{F}$.

[References]


17. Kaplansky, I. Topics in Commutative Rings, Mimeographed Notes: Univ. Chicago, 1974. MR0345945 (49 #10674)


23. Ratliff, L.J., Jr. A Brief History and Survey on the Catenary Chain