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Nagata rings, Kronecker function rings, and related semistar operations. (English. English summary)

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For a semistar operation $\star$ on an integral domain D, the authors in the present paper investigate the following domain, called the Kronecker function ring of D with respect to $\star$: $\mathrm{Kr}(D,\star)=\{f/g \mid f,g\in D[X]-\{0\}$ and there exists $h\in D[X]-\{0\}$ such that $(c(f)c(h))^\star\subseteq(c(g)c(h))^\star\}\cup\{0\}$, where $c(j)$ denotes the content of a polynomial j. They associate an e.a.b. semistar operation $\star_a$ to $\star$, and show that $\mathrm{Kr}(D,\star)=\mathrm{Kr}(D,\star_a)$.

For a star operation on D, B. G. Kang [J. Algebra **123** (1989), no. 1, 151–170; MR1000481 (90e:13017)] considered a generalization of the Nagata ring construction. In the present paper the authors generalize his construction so that, given a semistar operation $\star$ on D, they define the semistar Nagata ring as follows: $\mathrm{Na}(D,\star)=\{f/g \mid f,g\in D[X]$ and $c(g)^\star=D^\star\}$. They then study the ideal structure of $\mathrm{Na}(D,\star)$ and compare it to that of $\mathrm{Kr}(D,\star)$. They also show how $\mathrm{Na}(D,\star)$ gives rise to a natural semistar operation $\tilde{\star}$, which plays a role analogous to that of $\star_a$.

For instance, they show that there is a natural 1-1 correspondence between the maximal ideals of $\mathrm{Na}(D,\star)$ and the maximal elements in the set of all proper quasi-$\star$-ideals of D. They prove also that $\mathrm{Na}(D,\star)=\mathrm{Na}(D,\tilde{\star})$. Furthermore, they show that there is a strict link between the semistar operation $\tilde{\star}$, the maximal elements P in the set of all proper quasi-$\star$-ideals of D and the valuation overrings of D_P. They also show that there is a natural 1-1 correspondence between the maximal ideals of $\mathrm{Kr}(D,\star)$ and the minimal $\star$-valuation overrings of D. *Ryūki Matsuda* (Ibaraki)

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