$$\dot{x} = Ax \qquad A = \begin{pmatrix} 1 & -2 & 1 \\ 1 & 2 & 1 \\ 2 & 4 & 2 \end{pmatrix}$$

$$p_c(t) = \det(A - tI_3) = \begin{vmatrix} 1-t & -2 & 1 \\ 1 & 2-t & 1 \\ 2 & 4 & 2-t \end{vmatrix} = (1-t)(2-t)^2 \cancel{-1+1} - 2(\cancel{2-t}) +$$

$$+ 2(\cancel{2-t}) - 4(1-t) =$$

$$= (1-t)(4+t^2-4t-4) =$$

$$= t(1-t)(t-4)$$

$$\lambda_1 = 0 \qquad \lambda_2 = 1 \qquad \lambda_3 = 4$$

$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

<u>$\lambda_1 = 0$</u>

$$\begin{pmatrix} 1 & -2 & 1 \\ 1 & 2 & 1 \\ 2 & 4 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} x - 2y + t = 0 \\ x + 2y + z = 0 \end{cases} \qquad \begin{cases} x + t = 0 \\ y = 0 \end{cases} \qquad \begin{cases} x = -t \\ y = 0 \end{cases} \qquad E_0 = \left\langle \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right\rangle$$

$$\lambda_2 = 1 \qquad \begin{pmatrix} 0 & -2 & 1 \\ 1 & 1 & 1 \\ 2 & 4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} -2y + z = 0 \\ x + y + z = 0 \end{cases} \qquad \begin{cases} z = 2y \\ x = -z - y = -3y \end{cases} \qquad E_1 = \left\langle \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} \right\rangle$$

$\lambda_2 = 4$

$$\begin{pmatrix} -3 & -2 & 1 \\ 1 & -2 & 1 \\ 2 & 4 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix}$$

$$\begin{cases} x - 2y + z = 0 \\ 2x + 4y - 2z = 0 \end{cases} \quad \begin{cases} x - 2y + z = 0 \\ x + 2y - z = 0 \end{cases} \quad \begin{cases} x = 0 \\ z = 2y \end{cases} \qquad E_4 = \left\langle \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \right\rangle$$

$$P = \begin{pmatrix} 1 & -3 & 0 \\ 0 & 1 & 1 \\ -1 & 2 & 2 \end{pmatrix} \qquad \det P = 2 + 3 - 2 = 3.$$

$$P^{-1} = \frac{1}{3} \begin{pmatrix} 0 & 6 & -3 \\ -1 & 2 & -1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\mathrm{cof}(a_{11}) = + \det \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = 0$$

$$\mathrm{cof}(a_{12}) = - \det \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix} = -1$$

$$\mathrm{cof}(a_{13}) = + \det \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix} = 1$$

$$\dot{x} = Ax \rightarrow \dot{x} = PDP^{-1}x \rightarrow P^{-1}\dot{x} = D\,P^{-1}x$$

$$P^{-1}x = y \rightarrow \dot{y} = Dy \rightarrow y(t) = e^{Dt}\,y(0)$$

$$P^{-1}x(t) = e^{Dt}\,P^{-1}x(t)$$

$$x(t) = P\,e^{Dt}\,P^{-1}\,x(0)$$

$$\begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & -3 & 0 \\ 0 & 1 & 1 \\ -1 & 2 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^t & 0 \\ 0 & 0 & e^{4t} \end{pmatrix}\begin{pmatrix} 0 & 6 & -3 \\ -1 & 2 & -2 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{pmatrix} =$$

$$= \frac{1}{3}\begin{pmatrix} 1 & -3e^t & 0 \\ 0 & e^t & e^{4t} \\ -1 & 2e^t & 2e^{4t} \end{pmatrix}\begin{pmatrix} 0 & 6 & -3 \\ -1 & 2 & -2 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{pmatrix} =$$

$$= \frac{1}{3}\begin{pmatrix} 3e^t & 6-6e^t & -3+3e^t \\ -e^t+e^{4t} & 2e^t+e^{4t} & -e^t+e^{4t} \\ -2e^t+2e^{4t} & -6+4e^t+2e^{4t} & 3-2e^t+2e^{4t} \end{pmatrix}\begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{pmatrix} =$$

$$\begin{cases} x_1(t) = e^t\bar{x}_1 + 2(1-e^t)\bar{x}_2 + (-1+e^t)\bar{x}_3 \\ x_2(t) = \frac{1}{3}(e^{4t}-e^t)\bar{x}_1 + \frac{1}{3}(2e^t+e^{4t})\bar{x}_2 + \frac{1}{3}(e^{4t}-e^t)\bar{x}_3 \\ x_3(t) = \frac{2}{3}(e^{4t}-e^t)\bar{x}_1 + \frac{2}{3}(e^{4t}+2e^t-3)\bar{x}_2 + \frac{1}{3}(2e^{4t}-2e^t+3)\bar{x}_3 \end{cases}$$

$m = 2 \qquad V(x) = 4x^2 - 6x - \log(x^2)$

$$V'(x) = 8x - 6 - \frac{1}{x^2}\cdot 2x = 8x - 6 - \frac{2}{x}$$

$$\begin{cases} \dot{x} = y \\ \dot{y} = -4x + 3 + \frac{1}{x} \end{cases}$$

$$V'(x) = \frac{1}{x}\left(8x^2 - 6x - 2\right) = 0 \iff 4x^2 - 3x - 1 = 0 \iff x = \frac{3 \pm \sqrt{9+16}}{8} \begin{cases} 1 \\ -\frac{1}{4} \end{cases}$$

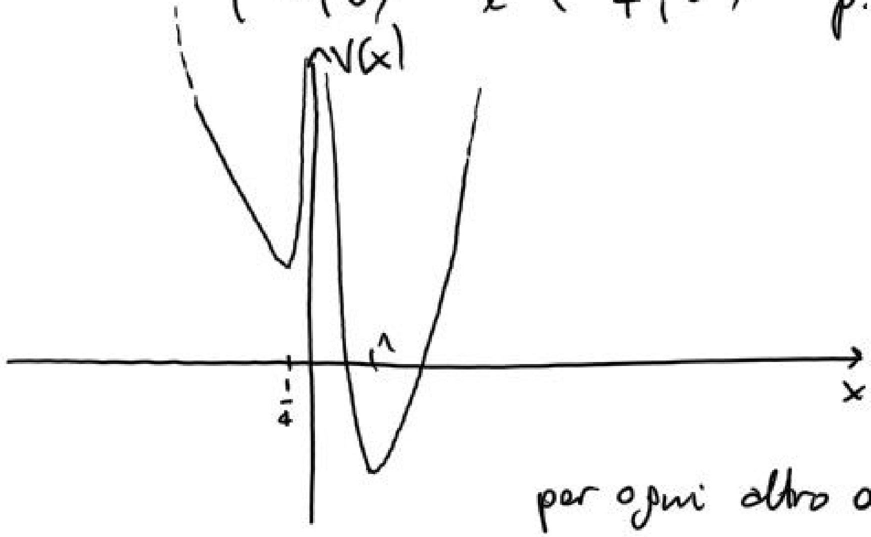
$$V''(x) = 8 + \frac{2}{x^2} \qquad V''(1) = 8 + 2 = 10 > 0 \longrightarrow 1 \text{ minimo}$$

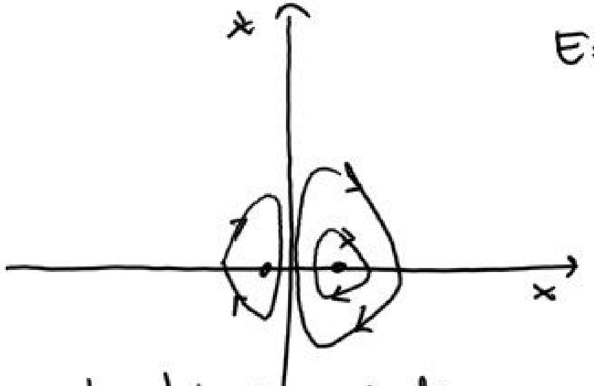
$$V''(-1/4) = 8 + \frac{2}{\frac{1}{16}} > 0 \longrightarrow -\frac{1}{4} \text{ minimo}$$

$(1,0)$ e $(-\frac{1}{4},0)$ p.d.e stabili.

$V(1) = -2$

$V(-\frac{1}{4}) = \frac{1}{4} + \frac{3}{2} + \log(16)$





$E = -2$ e $x(0) = 1$
moto fisso
$x(t) \equiv 1$
stesse cose se
$E = \frac{7}{4} + \log(16)$
e $x(0) = -\frac{1}{4}$
$x(t) \equiv -\frac{1}{4}$

per ogni altro d.i. moti chiusi periodici.

$$\ddot{x} = x - x^3$$

$$V'(x) = x^3 - x = x(x+1)(x-1)$$

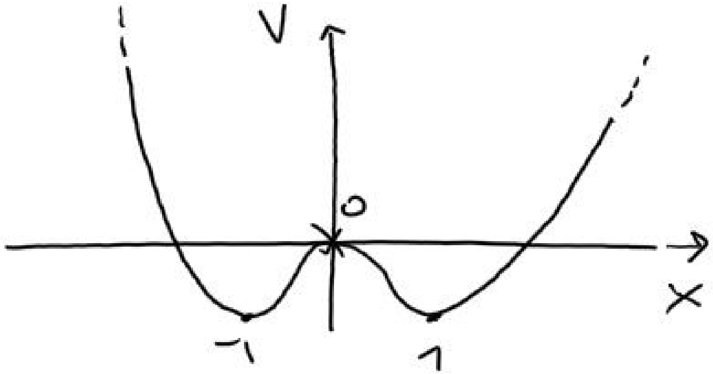
$$V''(x) = 3x^2 - 1$$

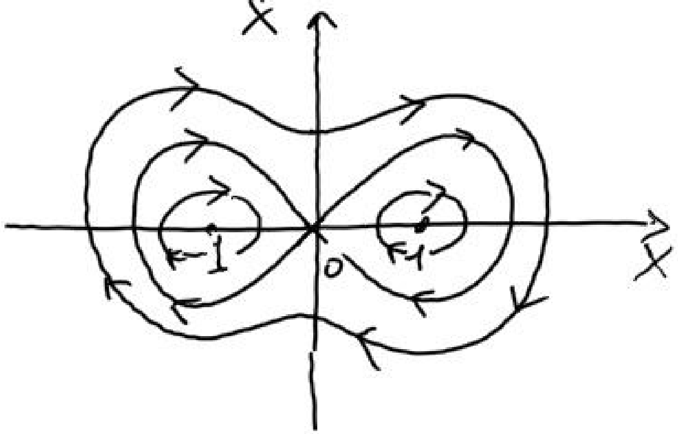
$V''(0) = -1 \to \max \to (0,0)$ p.de inst

$V''(\pm 1) = 2 > 0 \to \min \to (\pm 1, 0)$ p.de $\underline{\underline{st}}$

$$V(x) = \frac{x^4}{4} - \frac{x^2}{2}$$

$$V(\pm 1) = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4} \qquad V(0) = 0$$





$E = -\frac{1}{4}$ e $x(0) = \pm 1 \to$ moto fisso $x(t) \equiv \pm 1$

$-\frac{1}{4} < E < 0 \to$ ~~moti chi~~ 2 orbite chiuse periodiche

$E = 0$ — $x(0) = 0 \to$ moto fisso

$E = 0$ — $x(0) \neq 0 \to$ 2 orbite omocline.

$E > 0 \to$ 1 orbite chiusa periodica

•) $\left(x(0), \dot{x}(0)\right) = (-1, v)$ $\qquad x = 1$

$$\frac{v^2}{2} + V(-1) = E = \frac{\dot{x}^2}{2} + V(1)$$

(conserv. energia)

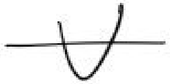
$$V(1) = V(-1)$$

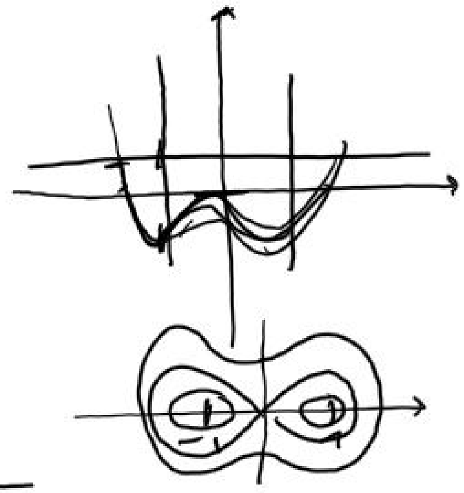
$$\frac{v^2}{\cancel{2}} = \frac{\dot{x}^2}{\cancel{2}} \longrightarrow \dot{x} = |v|$$

$$\frac{\dot{x}^2}{2} + V(1) > 0$$

$$\dot{x}^2 > \frac{1}{8}$$

$$\dot{x} > \frac{1}{2\sqrt{2}} \qquad \left(\dot{x} < -\frac{1}{2\sqrt{2}}\right)$$





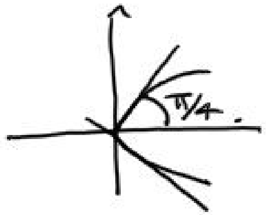
•) $\dot{x} = \pm\sqrt{2\left(\frac{x^2}{2} - \frac{x^4}{4}\right)}$

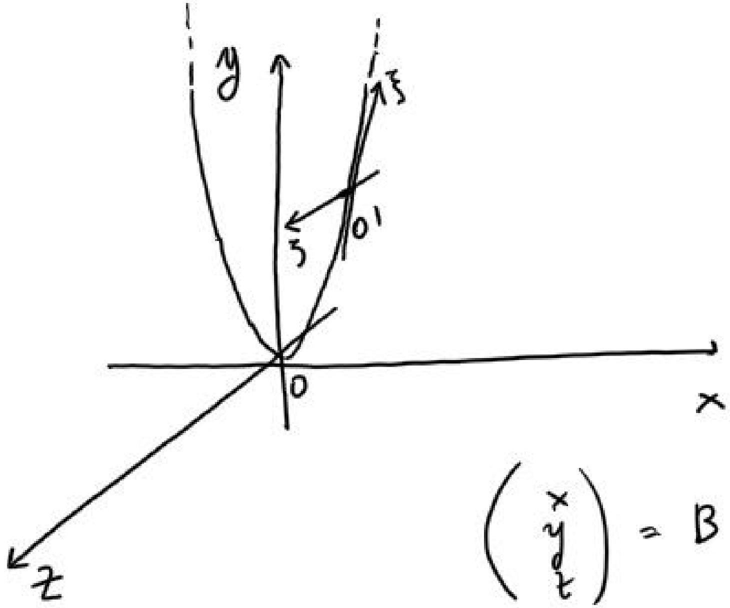
$$\tan\alpha = \lim_{x\to 0^+} \frac{d}{dx}\sqrt{x^2 - \frac{x^4}{2}} =$$

$$= \lim_{x\to 0^+} \frac{1}{2\sqrt{x^2 - \frac{x^4}{2}}}\left(2x - 2x^3\right) =$$

$$= \lim_{x\to 0^+} \frac{\cancel{2x}\left(1 - x^3\right)}{\cancel{2x}\sqrt{1 - \frac{x^2}{2}}} = 1$$

$$\alpha = \frac{\pi}{4}$$





$$y(x) = x^2$$

$$x_{O'} = \text{sen}\, t$$

$$\underline{z} = (\text{sen}\, t,\ \text{sen}^2 t,\ 0)$$

$$\xi(t) = t \qquad P = (t, 0, 0)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = B \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} + \underline{z} \qquad B = \begin{pmatrix} \cos\theta(t) & -\text{sen}\,\theta(t) & 0 \\ \text{sen}\,\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\theta(t) = \arctan\left(\frac{d}{dx} y_{O'}\right) = \arctan(2\,\text{sen}\, t)$$

$$\underline{Q} = \begin{pmatrix} t \\ 0 \\ 0 \end{pmatrix}$$

$$q = \begin{pmatrix} \cos\theta & -\text{sen}\,\theta & 0 \\ \text{sen}\,\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \text{sen}\, t \\ \text{sen}^2 t \\ 0 \end{pmatrix} =$$

$$= \begin{pmatrix} t\cos\theta(t) + \text{sen}\, t \\ t\,\text{sen}\,\theta(t) + \text{sen}^2(t) \\ 0 \end{pmatrix}$$

$$v = \dot q = \begin{pmatrix} \cos\theta(t) - t\,\dot\theta(t)\,\mathrm{sen}\,\theta(t) + \cos t \\ \mathrm{sen}\,\theta(t) + t\,\dot\theta(t)\cos\theta(t) + \mathrm{sen}(2t) \\ 0 \end{pmatrix}, \quad \dot\theta = \frac{2\cos t}{1 + 4\,\mathrm{sen}^2 t}$$

$$v' = B\dot Q = \begin{pmatrix} \cos\theta & -\mathrm{sen}\,\theta & 0 \\ \mathrm{sen}\,\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta(t) \\ \mathrm{sen}\,\theta(t) \\ 0 \end{pmatrix}$$

$$V_0 = \dot r = \begin{pmatrix} \cos t \\ \overset{\mathrm{sen}}{\cancel{\cos}}(2t) \\ 0 \end{pmatrix}$$

$$v_T = \omega \wedge (q - r) = \begin{vmatrix} i & j & k \\ 0 & 0 & \dot\theta \\ t\cos\theta & t\,\mathrm{sen}\,\theta & 0 \end{vmatrix} = \begin{pmatrix} -t\dot\theta\,\overset{\mathrm{sen}}{\cancel{\cos}}\,\theta \\ t\dot\theta\cos(\theta) \\ 0 \end{pmatrix}$$

$$|\omega| = \dot\theta$$

$$z \parallel \zeta \to \omega = (0, 0, \dot\theta)$$

$$F_{cf} = -\Omega \wedge (\Omega \wedge Q) \qquad \Omega = B\omega = \omega.$$

$$\Omega \wedge Q = \begin{vmatrix} i & j & k \\ 0 & 0 & \dot\theta \\ t & 0 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ t\dot\theta \\ 0 \end{pmatrix}$$

$$-\Omega \wedge (\Omega \wedge Q) = - \begin{vmatrix} i & j & k \\ 0 & \cancel{t\dot\theta}\,0 & \dot\theta \\ 0 & t\dot\theta & 0 \end{vmatrix} = +\begin{pmatrix} t\dot\theta^2 \\ 0 \\ 0 \end{pmatrix}$$

$$F_{cor} = -2(\Omega \wedge \dot{Q}) = -2 \begin{vmatrix} i & j & k \\ 0 & 0 & \dot{\theta} \\ 1 & 0 & 0 \end{vmatrix} = -2 \begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -2\dot{\theta} \\ 0 \end{pmatrix}$$

$$t \longmapsto q(t) = \big(x(t), y(t), 0 \big)$$

$$y(t) = 0 \iff t \operatorname{sen}\theta(t) + \operatorname{sen}^2 t = 0$$

$$\operatorname{sen} x = \frac{\tan x}{\sqrt{1 + \tan^2 x}} \qquad |x| < \frac{\pi}{2}$$

$$\operatorname{sen} t \left(\frac{2t}{\sqrt{1 + 4\operatorname{sen}^2 t}} + \operatorname{sen} t \right) = 0$$

$$\operatorname{sen}\theta(t) = \frac{2 \operatorname{sen} t}{\sqrt{1 + 4\operatorname{sen}^2 t}}$$

$$y(t_k) = 0 \qquad t_k = k\pi \quad k \in \mathbb{Z}.$$

$$V(\rho) = 2\log\rho - \frac{1}{2}\log(1+2\rho^2+2\rho^4) \qquad L \neq 0 \qquad \mu$$

$$V_{eff} = 2\log\rho - \frac{1}{2}\log(1+2\rho^2+2\rho^4) + \frac{L^2}{2\mu\rho^2}, \quad L \neq 0$$

$$V'_{eff}(\rho) = \frac{2}{\rho} - \frac{\cancel{4}^{2}\rho + \cancel{8}^{4}\rho^3}{\cancel{2}(1+2\rho^2+2\rho^4)} - \frac{L^2}{\mu\rho^3} \qquad \frac{L^2}{\mu} = \beta > 0$$

$$\mu\ddot{\rho} = -V'_{eff}$$

$$\begin{cases} \dot{\rho} = y \\ \dot{y} = -\dfrac{V'_{eff}}{\mu}. \end{cases}$$

$$V'_{eff} = \frac{2 + 4\rho^2 + \cancel{4\rho^4} - 2\rho^2 - \cancel{4\rho^4}}{\rho(1+2\rho^2+2\rho^4)} - \frac{L^2}{\mu\rho^3} =$$

$$= \frac{2(-2\rho^4-3\rho^2+1)}{\rho(1+2\rho^2+2\rho^4)} - \frac{\beta}{\rho^3} =$$

$$= \frac{1}{\rho^3}\left(\frac{-4\rho^6-6\rho^4+2\rho^2-\beta}{1+2\rho^2+2\rho^4}\right) \geq 0$$

$$-4\rho^3 - 6\rho^4 + 2\rho^2 - \beta \geq 0.$$

$$V'_{eff} = \frac{2+2\rho^2}{\rho(1+2\rho^2+2\rho^4)} - \frac{\beta}{\rho^3}$$

$$= \frac{2\rho^2+2\rho^4-\beta-2\beta\rho^2-2\beta\rho^4}{\rho^3(1+2\rho^2+2\rho^4)} \geq 0$$

$$\rho > 0 \iff 2(1-\beta)\rho^4 + 2(1-\beta)\rho^2 - \beta = 0$$

$$\rho^4 + \rho^2 - \frac{\beta}{2(1-\beta)} = 0$$

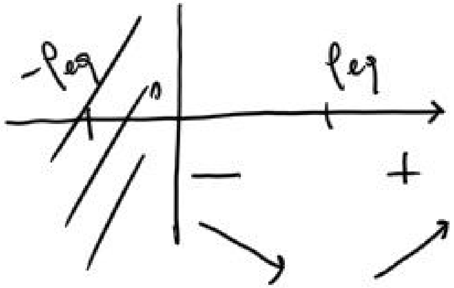
$$\gamma = \frac{\beta}{2(1-\beta)}$$

$$\rho^4 + \rho^2 - \gamma \geq 0$$

$$\rho^2 = \frac{-1 \pm \sqrt{1+4\gamma}}{2}$$

$$\rho^2 = \frac{-1 + \sqrt{1+4\gamma}}{2}$$

$$\boxed{\rho_{eq} = \sqrt{\frac{-1+\sqrt{1+4\gamma}}{2}}}$$

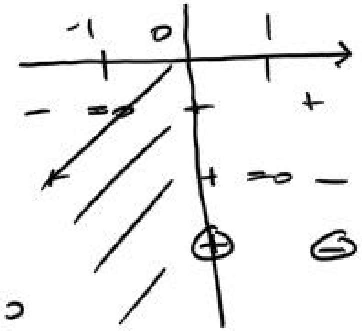


$(\rho_{eq}, 0)$ stabile

*) $1 + 4\dfrac{\beta}{2(1-\beta)} \geq 0$

$$\frac{2-2\beta+4\beta}{1-\beta} \geq 0$$

$$\frac{2\beta+2}{1-\beta} \geq 0 \qquad \frac{\beta+1}{1-\beta} \geq 0$$

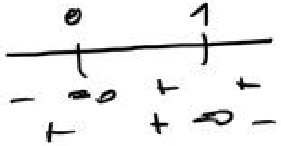


$0 < \beta \leq 1 \to$ la radice $\exists$

$\beta > 1 \to$ la radice $\nexists \to$ no pti di eq.

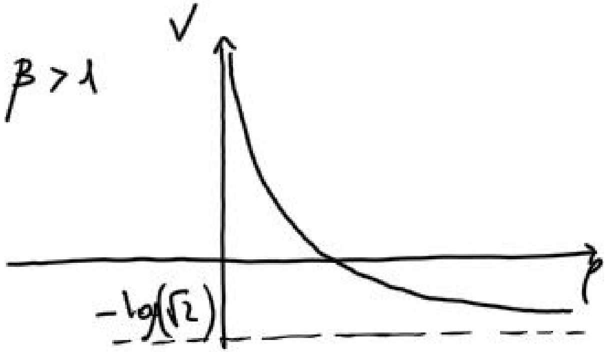
**) $\sqrt{1+4\beta\gamma} > 1$

$$\cancel{1} + \frac{\cancel{4}^{2}\beta}{\cancel{2}(1-\beta)} > \cancel{1} \qquad \frac{\beta}{1-\beta} > 0$$

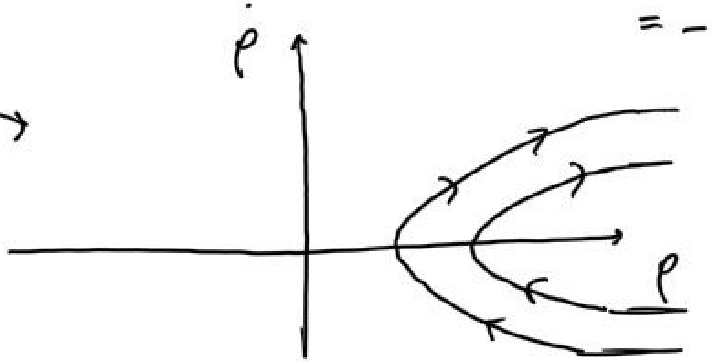


$$\boxed{0 < \beta < 1}$$

$\beta > 1$

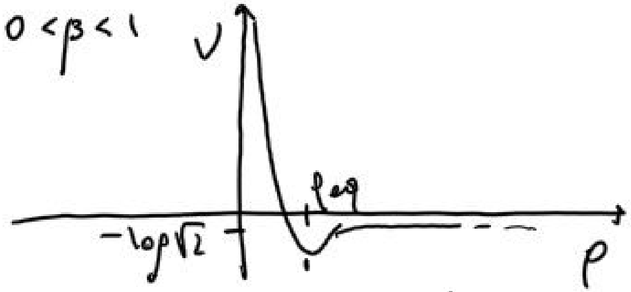


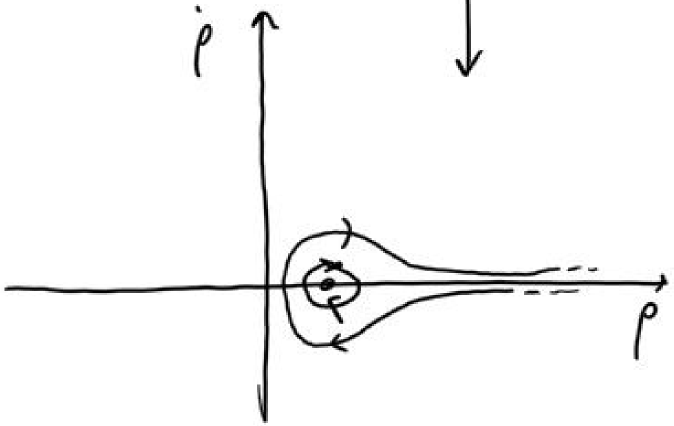
$$\lim_{\rho\to+\infty} V(\rho) = \lim_{\rho\to+\infty} \log\left(\frac{\rho^2}{\sqrt{1+2\rho^2+2\rho^4}}\right)+\frac{\beta}{2\rho^2} = -\log(\sqrt{2})$$



tutti moti aperti.

$0 < \beta < 1$





$$\begin{cases} V_{eff}(\rho_{eq}) < E < -\log(\sqrt{2}) \rightarrow \text{moti chiusi periodici} \\ \text{se } E = V_{eff}(\rho_{eq}) \text{ e } \rho(0)=\rho_{eq} \rightarrow \text{moto fisso.} \\ E > -\log(\sqrt{2}) \rightarrow \text{moti aperti.} \end{cases}$$

$$V(\rho) = 2\log\rho - \frac{1}{2}\log(1 + 2\rho^2 + 2\rho^4)$$

$$V'(\rho) = \frac{2(\rho^2+1)}{\rho(2\rho^4+2\rho^2+1)} \geq 0 \quad \forall \rho > 0 .$$