Es. 1

$$A = \begin{pmatrix} -1 & 1 & 1 \\ 2 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$$p(\lambda) = \det(A - \lambda \mathbb{1}) = \det\begin{pmatrix} -1-\lambda & 1 & 1 \\ 2 & -\lambda & 0 \\ 1 & 1 & 1-\lambda \end{pmatrix} = (1-\lambda)\begin{vmatrix} -\lambda & 0 \\ 1 & 1-\lambda \end{vmatrix} - \begin{vmatrix} 2 & 0 \\ 1 & 1-\lambda \end{vmatrix}$$

$$+ \begin{vmatrix} 2 & -\lambda \\ 1 & 1 \end{vmatrix} = (-1-\lambda)(-\lambda)(1-\lambda) - 2(1-\lambda) + (2+\lambda)$$

$$= \lambda(1+\lambda)(1-\lambda) - \cancel{2} + 2\lambda + \cancel{2} + \lambda = \lambda(1-\lambda^2) + 3\lambda$$

$$= \lambda(1-\lambda^2+3) = \lambda(4-\lambda^2) = 0 \Rightarrow \lambda = 0, \lambda = 2, \lambda = -2$$

$$\boxed{\lambda = 2} \Rightarrow v_1 = (x, y, z) \text{ t.c. } \begin{pmatrix} -3 & 1 & 1 \\ 2 & -2 & 0 \\ 1 & 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-3x + y + z = 0 \qquad 2x - 2y = 0 \Rightarrow x = y$$

$$-2x + z = 0 \Rightarrow x = 1,\ z = 2,\ y = 1 \Rightarrow \boxed{v_1 = (1, 1, 2)}$$

$\boxed{\lambda_2 = -2} \Rightarrow v_2 = (x,y,z)$ t.q. $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 0 \\ 1 & 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$x + y + z = 0 \qquad 2x + 2y = 0 \Rightarrow x = -y$

$z = 0 \Rightarrow \boxed{v_2 = (-1, 1, 0)}$

$\boxed{\lambda_3 = 0} \Rightarrow v_3 = (x,y,z)$ t.q. $\begin{pmatrix} -1 & 1 & 1 \\ 2 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$-x + y + z = 0 \; , \; x = 0 \Rightarrow y = -z$

$\Rightarrow \boxed{v_3 = (0, -1, 1)}$

$$v_i = \sum_{j=1}^{3} P_{ij} e_j \Rightarrow P = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

$$Q^{-1} = P^T = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & -1 \\ 2 & 0 & 1 \end{pmatrix}$$

$$\det Q^{-1} = 1\cdot\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1 + 3 = 4$$

$$Q = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 \\ -3 & 1 & 1 \\ -2 & -2 & 2 \end{pmatrix}$$

$$QAQ^{-1} = D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$Qe^{At}Q^{-1} = \begin{pmatrix} e^{2t} & 0 & 0 \\ 0 & e^{-2t} & 0 \\ 0 & 0 & 1 \end{pmatrix} = e^{Dt}$$

$$e^{At} = Q^{-1}e^{Dt}Q = \underbrace{\frac{1}{4}\begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & -1 \\ 2 & 0 & 1 \end{pmatrix}\overbrace{\begin{pmatrix} e^{2t} & 0 & 0 \\ 0 & e^{-2t} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ -3 & 1 & 1 \\ -2 & -2 & 2 \end{pmatrix}}^{\begin{pmatrix} e^{2t} & e^{2t} & e^{2t} \\ -3e^{-2t} & e^{-2t} & e^{-2t} \\ -2 & -2 & 2 \end{pmatrix}}}$$

$$e^{At} \Leftarrow \frac{1}{4}\begin{pmatrix} e^{2t}+3e^{-2t} & e^{2t}-e^{-2t} & e^{2t}-e^{-2t} \\ e^{2t}-3e^{-2t}+2 & e^{2t}+e^{-2t}+2 & e^{2t}+e^{-2t}-2 \\ 2e^{2t}-2 & 2e^{2t}-2 & 2e^{2t}+2 \end{pmatrix}$$

soluzione $x(t) = e^{At}\begin{pmatrix}1\\0\\-1\end{pmatrix} =$

$$= \frac{1}{4}\begin{pmatrix}(e^{2t}+3e^{-2t}) - (e^{2t}-e^{-2t}) \\ (e^{2t}-3e^{-2t}+2) - (e^{2t}+e^{-2t}-2) \\ (2e^{2t}-2) - (2e^{2t}+2)\end{pmatrix} = \frac{1}{4}\begin{pmatrix}4e^{-2t} \\ 4-4e^{-2t} \\ -4\end{pmatrix}$$

$$= \begin{pmatrix}e^{-2t} \\ 1-e^{-2t} \\ -1\end{pmatrix}$$

N.B. $t=0 \Rightarrow x(0) = \begin{pmatrix}1\\0\\-1\end{pmatrix}$ <u>come deve essere</u>.

$m = 2 \qquad V(x) = 4x^2 - 6x - \log(x^2)$

$$V'(x) = 8x - 6 - \frac{1}{x^2}\cdot 2x = 8x - 6 - \frac{2}{x}$$

$$\begin{cases} \dot{x} = y \\ \dot{y} = -4x + 3 + \frac{1}{x} \end{cases}$$

$$V'(x) = \frac{1}{x}\left(8x^2 - 6x - 2\right) = 0 \iff 4x^2 - 3x - 1 = 0 \iff x = \frac{3 \pm \sqrt{9+16}}{8} \; \begin{cases} 1 \\ -\frac{1}{4} \end{cases}$$

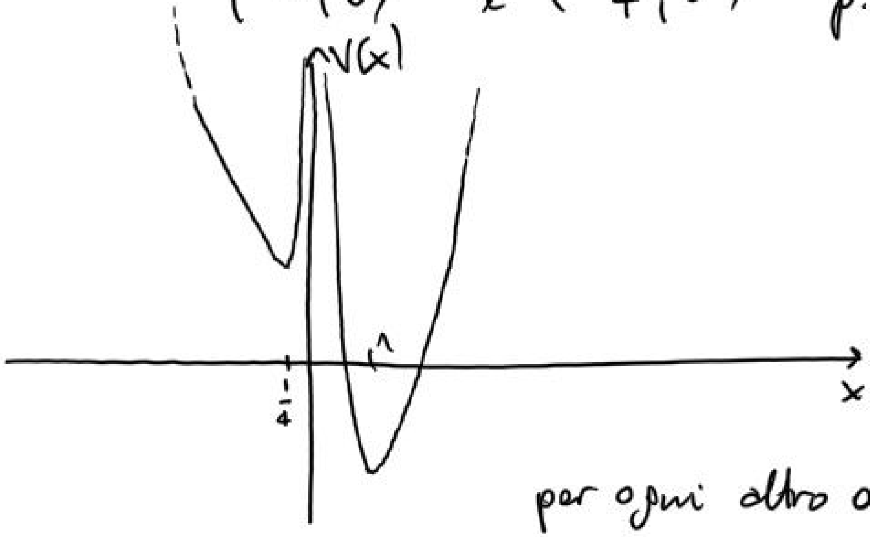
$$V''(x) = 8 + \frac{2}{x^2} \qquad V''(1) = 8 + 2 = 10 > 0 \longrightarrow 1 \text{ minimo}$$

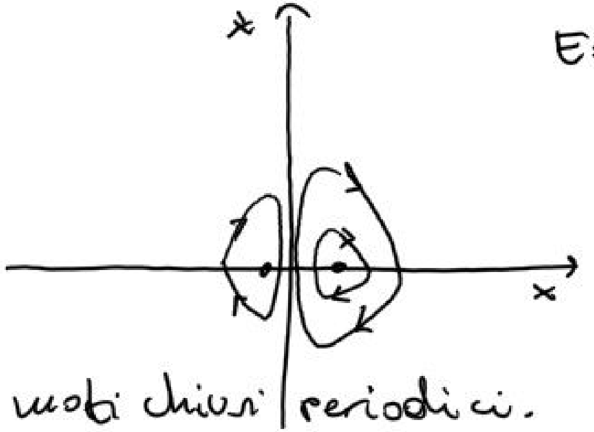
$$V''(-1/4) = 8 + \frac{2}{\frac{1}{16}} > 0 \longrightarrow -\frac{1}{4} \text{ minimo}$$

$(1, 0)$ e $(-\frac{1}{4}, 0)$ p.d.e stabili.

$V(1) = -2$

$V(-\frac{1}{4}) = \frac{1}{4} + \frac{3}{2} + \log(16)$





$E = -2$ e $x(0) = 1$
moto fisso
$x(t) \equiv 1$

stessa cosa se
$E = \frac{7}{4} + \log(16)$
e $x(0) = -\frac{1}{4}$
$x(t) \equiv -\frac{1}{4}$

per ogni altro d.i. moti chiusi periodici.

$$\ddot{x} = x - x^3$$

$$V'(x) = x^3 - x = x(x+1)(x-1)$$

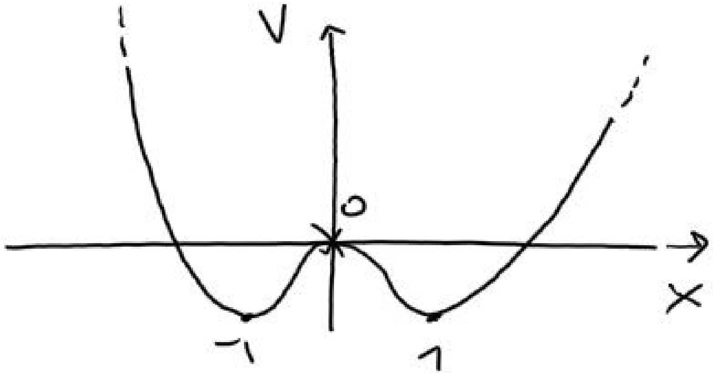
$$V''(x) = 3x^2 - 1$$

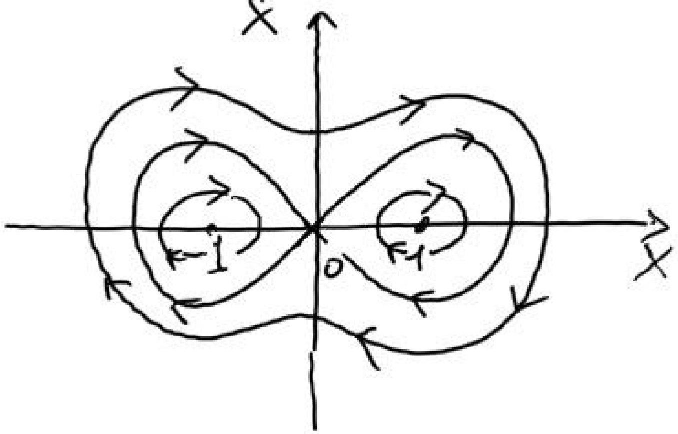
$V''(0) = -1 \to \max \to (0,0)$ pde inst

$V''(\pm 1) = 2 > 0 \to \min \to (\pm 1, 0)$ pde <u>st</u>

$$V(x) = \frac{x^4}{4} - \frac{x^2}{2}$$

$V(\pm 1) = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$ $\qquad V(0) = 0$





$E = -\frac{1}{4}$ e $x(0) = \pm 1 \to$ moto fisso $x(t) \equiv \pm 1$

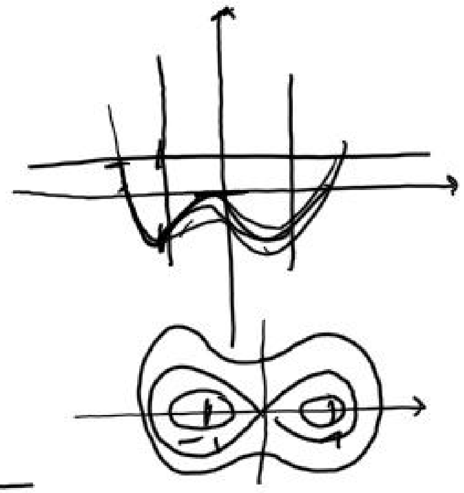
$-\frac{1}{4} < E < 0 \to$ ~~moti chi~~ 2 orbite chiuse periodiche

$E = 0$:
- $x(0) = 0 \to$ moto fisso
- $x(0) \neq 0 \to$ 2 orbite omocline.

$E > 0 \to$ 1 orbita chiusa periodica

•) $\left(x(0), \dot{x}(0)\right) = (-1, v)$ $x = 1$

$$\frac{v^2}{2} + V(-1) = E = \frac{\dot{x}^2}{2} + V(1)$$ (conserv. energia)

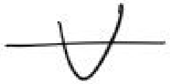


$$V(1) = V(-1)$$

$$\frac{v^2}{\cancel{2}} = \frac{\dot{x}^2}{\cancel{2}} \longrightarrow \dot{x} = |v|$$

$$\frac{\dot{x}^2}{2} + V(1) > 0$$

$$\dot{x}^2 > \frac{1}{8}$$



$$\dot{x} > \frac{1}{2\sqrt{2}} \quad \left(\dot{x} < -\frac{1}{2\sqrt{2}}\right)$$

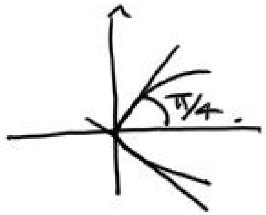
•) $$\dot{x} = \pm\sqrt{2\left(\frac{x^2}{2} - \frac{x^4}{4}\right)}$$

$$\tan\alpha = \lim_{x \to 0^+} \frac{d}{dx}\sqrt{x^2 - \frac{x^4}{2}} =$$

$$= \lim_{x \to 0^+} \frac{1}{2\sqrt{x^2 - \frac{x^4}{2}}}\left(2x - 2x^3\right) =$$

$$= \lim_{x \to 0^+} \frac{\cancel{2x}\left(1 - x^3\right)}{\cancel{2x}\sqrt{1 - \frac{x^2}{2}}} = 1$$

$$\alpha = \frac{\pi}{4}$$



Es.4

$$V_{eff}(\rho) = \rho^4\left(\rho^2 + \frac{\alpha}{\rho^2}\right) + \frac{L^2}{2\mu\rho^2} \qquad \boxed{\underline{\underline{\rho > 0}}}$$

$$= \rho^6 + \alpha\rho^2 + \frac{L^2}{2\mu\rho^2}$$

$$\lim_{\rho\to 0} V_{eff}(\rho) = \lim_{\rho\to+\infty} V_{eff}(\rho) = +\infty$$

$$V'_{eff}(\rho) = 6\rho^5 + 2\alpha\rho - \frac{L^2}{\mu\rho^3}$$

$$= \frac{1}{\rho^3}\left[6\rho^8 + 2\alpha\rho^4 - \frac{L^2}{\mu}\right] = 0$$

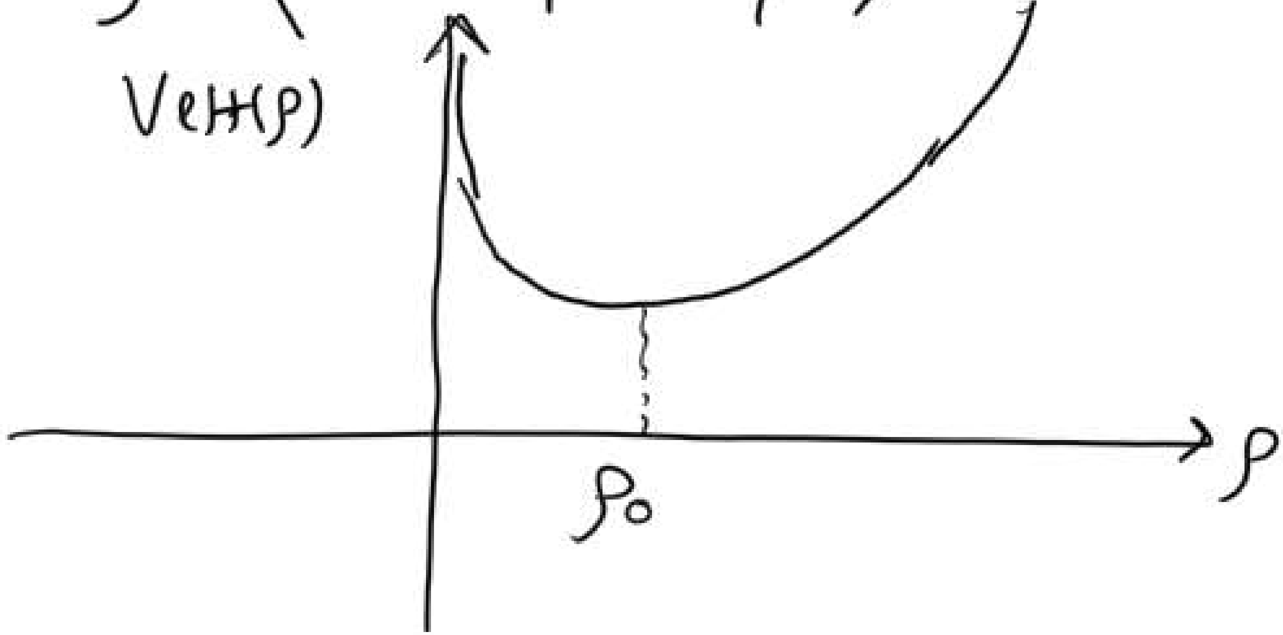
$$\rho^4 = x \Rightarrow 6x^2 + 2\alpha\rho - \frac{L^2}{\mu} = 0 \Leftrightarrow x = -\alpha \pm \sqrt{\underbrace{\alpha^2 + \frac{6L^2}{\mu}}_{>\alpha^2}}$$

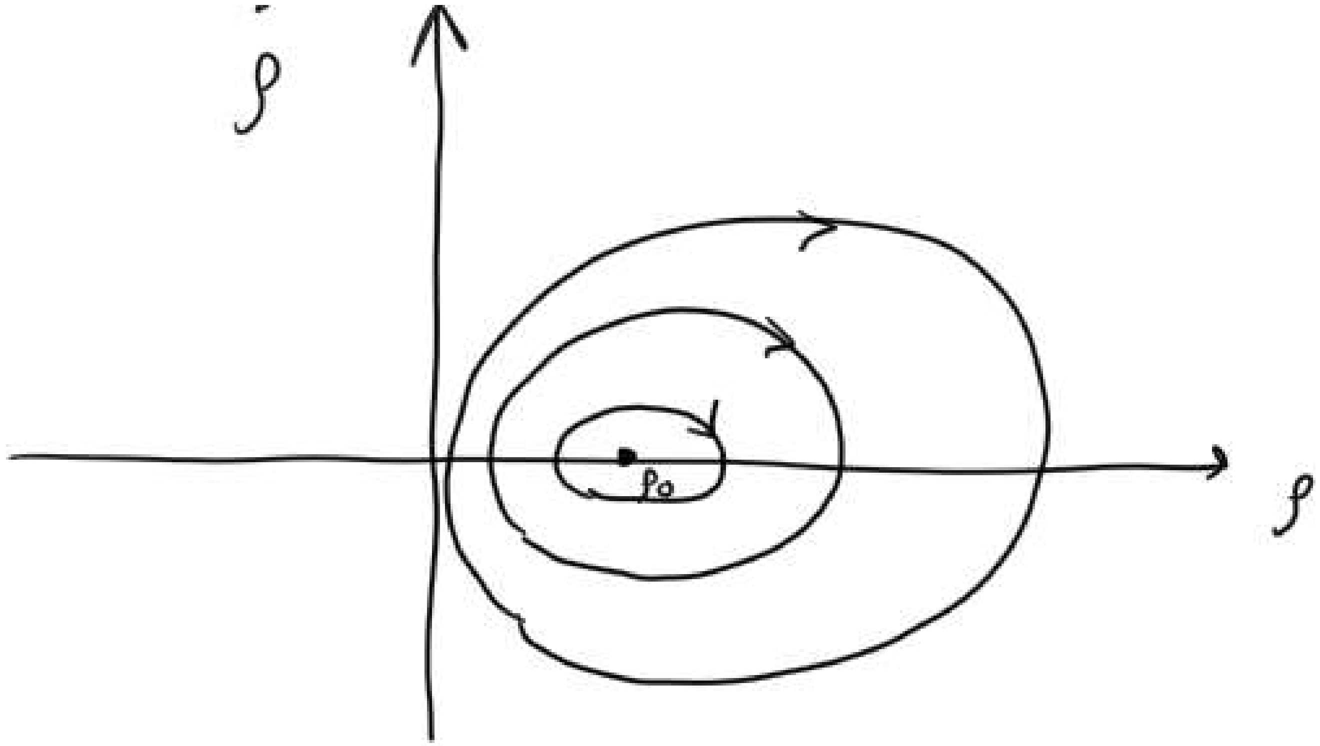
$\alpha > 0$ $\quad x_\pm = -\alpha \pm \sqrt{\alpha^2 + \frac{\sigma L^2}{\mu}} \Rightarrow \cancel{x_- < 0}, \boxed{x_+ > 0}$

$\alpha < 0$ $\quad x_\pm = \dots\dots \Rightarrow x_- < 0, \boxed{x_+ > 0}$

$\alpha = 0$ $\quad x_- < 0, \; x_+ = L\sqrt{\frac{\sigma}{\mu}} > 0$

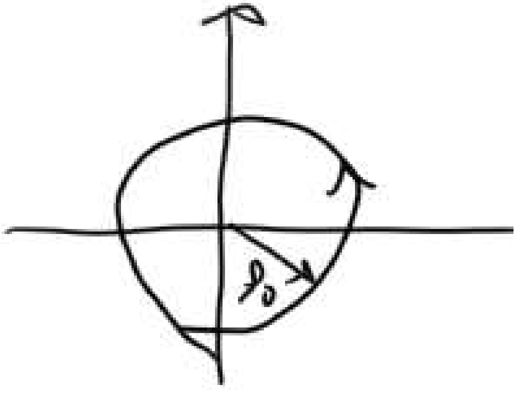
$$\Rightarrow \forall \alpha \quad \rho = \left(-\alpha + \sqrt{\alpha^2 + \frac{\sigma L^2}{\mu}}\right)^{\frac{L}{q}} = \rho_0$$



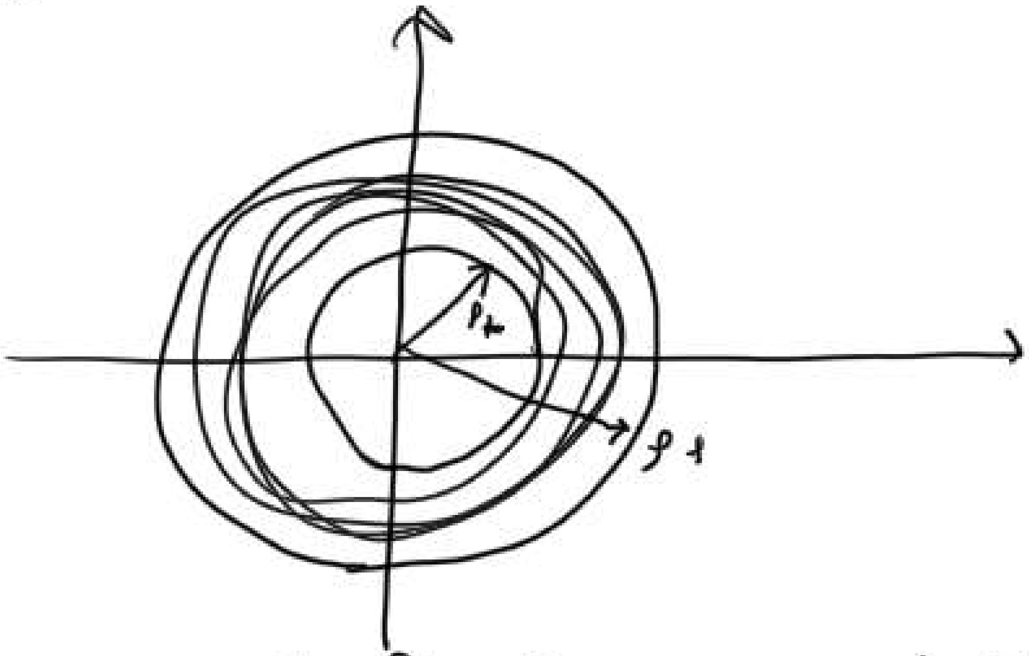


$\Rightarrow$ tutti i moti sono limitati

1) $\underbrace{\rho(t) = \rho_0}\ \forall t \Rightarrow \dot\theta = \frac{L}{\mu\rho_0^2} = \omega \Rightarrow \theta(t) = \theta(0) + \underbrace{\omega t}$



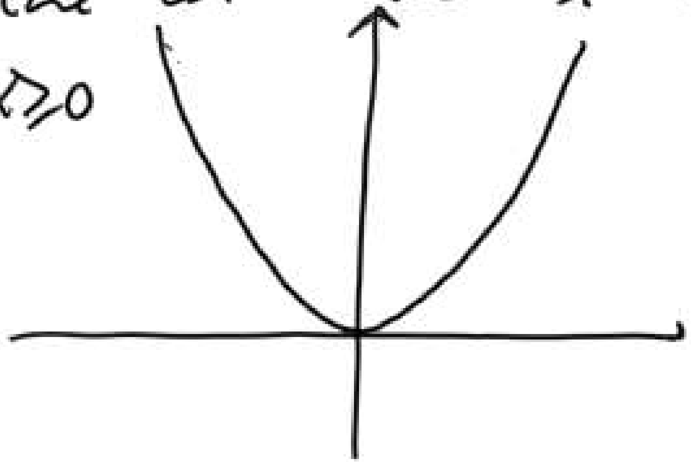
2) $\rho_- \le \rho(t) \le \rho_+$, dove $E - V_{eff}(\rho_\pm) = 0$, se $E > V_{eff}(\rho_0)$

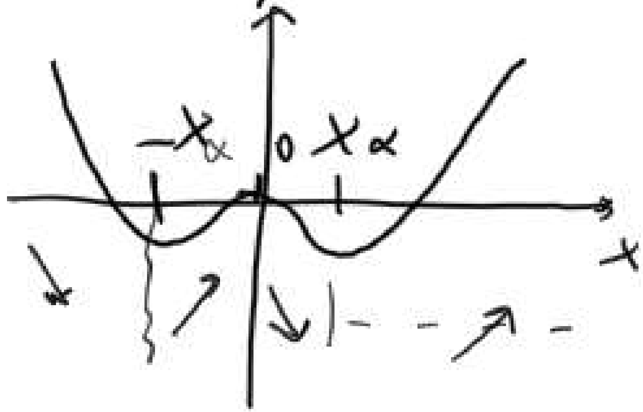


Se $\Delta\vartheta = \{$incremento di ϑ in un periodo T del moto di $\rho(t)\}$ è t.c. $\frac{\Delta\vartheta}{2\pi} \in \mathbb{Q} \Rightarrow$ la traiettoria nel piano $\perp$ a L è periodica, altrimenti riempie densamente la corona circolare $\rho_- \le \rho \le \rho_+$.

$\boxed{L=0}$ $\Rightarrow$ il moto avviene lungo una retta che si può identificare con l'asse x con energia potenziale

$V(x) = x^6 + \alpha x^2$ $\qquad \alpha \geq 0$





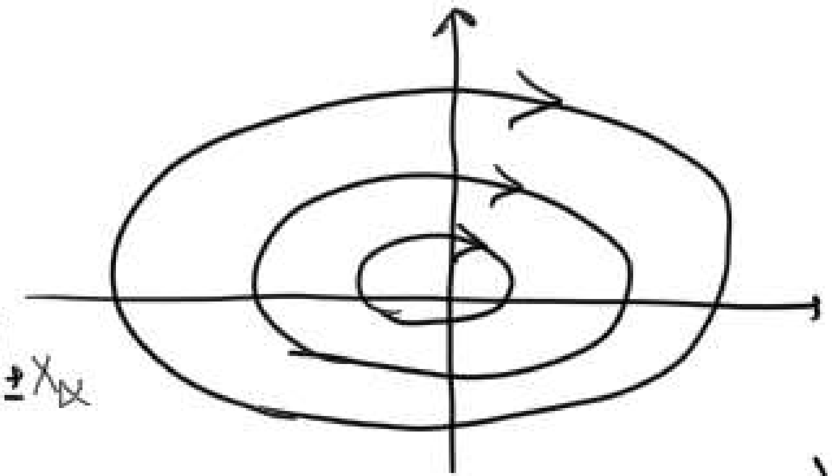
$$V'(x) = 6x^5 + 2\alpha x = 2x(3x^4 + \alpha)$$

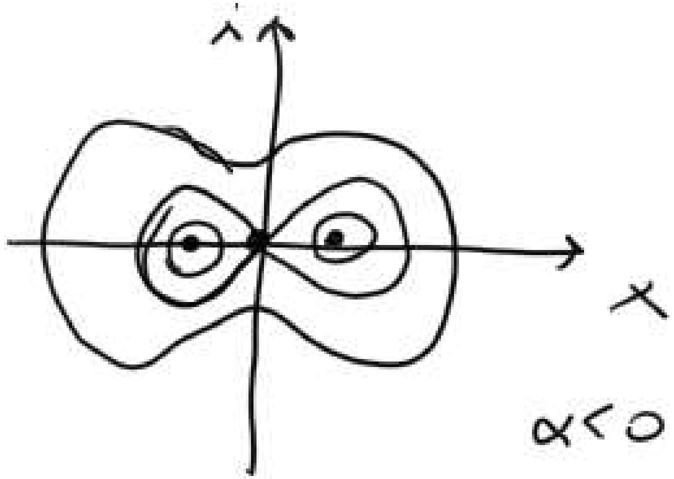
$$\updownarrow$$

$3x^4 + \alpha = 0$

se $\alpha < 0$

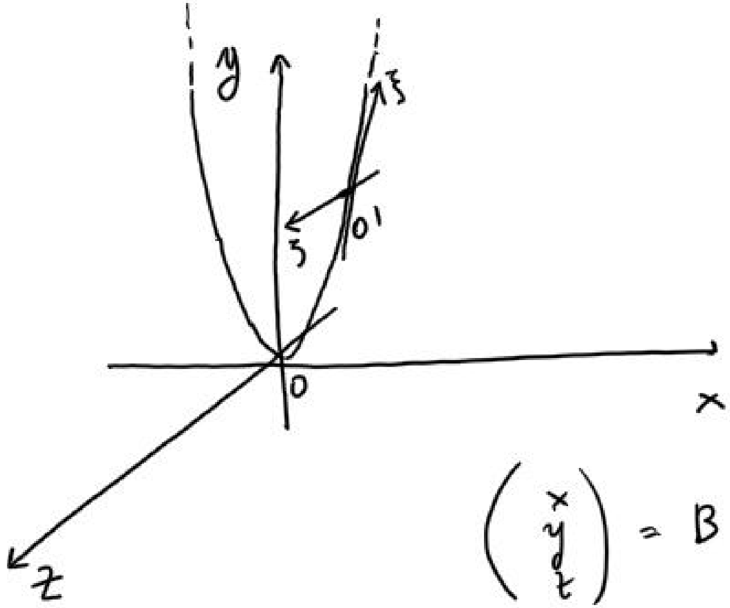
e $x = \pm\left(-\frac{\alpha}{3}\right)^{\frac{1}{4}} = \pm x_\alpha$





$\alpha < 0$

$$V''(x)\Big|_{x=0} = 30x^4 + 2\alpha\Big|_{x=0} = 2\alpha$$



$$y(x) = x^2$$

$$x_{O'} = \operatorname{sen} t$$

$$\underline{z} = (\operatorname{sen} t\,,\ \operatorname{sen}^2 t\,,\ 0)$$

$$\xi(t) = t \qquad P = (t, 0, 0)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = B \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} + \underline{z} \qquad B = \begin{pmatrix} \cos\theta(t) & -\operatorname{sen}\theta(t) & 0 \\ \operatorname{sen}\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\theta(t) = \arctan\left(\frac{d}{dx} y_{O'}\right) = \arctan(2\operatorname{sen} t)$$

$$\underline{Q} = \begin{pmatrix} t \\ 0 \\ 0 \end{pmatrix}$$

$$q = \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta & 0 \\ \operatorname{sen}\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \operatorname{sen} t \\ \operatorname{sen}^2 t \\ 0 \end{pmatrix} =$$

$$= \begin{pmatrix} t\cos\theta(t) + \operatorname{sen} t \\ t\operatorname{sen}\theta(t) + \operatorname{sen}^2(t) \\ 0 \end{pmatrix}$$

$$v = \dot q = \begin{pmatrix} \cos\theta(t) - t\,\dot\theta(t)\,\text{sen}\,\theta(t) + \cos t \\ \text{sen}\,\theta(t) + t\,\dot\theta(t)\cos\theta(t) + \text{sen}(2t) \\ 0 \end{pmatrix}, \quad \dot\theta = \frac{2\cos t}{1 + 4\,\text{sen}^2 t}$$

$$v' = B\dot Q = \begin{pmatrix} \cos\theta & -\text{sen}\,\theta & 0 \\ \text{sen}\,\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta(t) \\ \text{sen}\,\theta(t) \\ 0 \end{pmatrix}$$

$$V_0 = \dot r = \begin{pmatrix} \cos t \\ \overset{\text{sen}}{\cancel{\cos}}(2t) \\ 0 \end{pmatrix}$$

$$v_T = \omega \wedge (q - r) = \begin{vmatrix} i & j & k \\ 0 & 0 & \dot\theta \\ t\cos\theta & t\,\text{sen}\,\theta & 0 \end{vmatrix} = \begin{pmatrix} -t\dot\theta\,\overset{\text{sen}}{\cancel{\cos}}\,\theta \\ t\dot\theta\cos(\theta) \\ 0 \end{pmatrix}$$

$$|\omega| = \dot\theta$$

$$z \parallel \zeta \to \omega = (0, 0, \dot\theta)$$

$$F_{cf} = -\Omega \wedge (\Omega \wedge Q) \qquad \Omega = B\omega = \omega.$$

$$\Omega \wedge Q = \begin{vmatrix} i & j & k \\ 0 & 0 & \dot\theta \\ t & 0 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ t\dot\theta \\ 0 \end{pmatrix}$$

$$-\Omega \wedge (\Omega \wedge Q) = -\begin{vmatrix} i & j & k \\ 0 & 0 & \dot\theta \\ 0 & t\dot\theta & 0 \end{vmatrix} = +\begin{pmatrix} t\dot\theta^2 \\ 0 \\ 0 \end{pmatrix}$$

$$F_{cor} = -2(\Omega \wedge \dot{Q}) = -2 \begin{vmatrix} i & j & k \\ 0 & 0 & \dot{\theta} \\ 1 & 0 & 0 \end{vmatrix} = -2 \begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -2\dot{\theta} \\ 0 \end{pmatrix}$$

$$t \longmapsto q(t) = (x(t), y(t), 0)$$

$$y(t) = 0 \iff t \operatorname{sen}\theta(t) + \operatorname{sen}^2 t = 0$$

$$\operatorname{sen} x = \frac{\tan x}{\sqrt{1+\tan^2 x}} \qquad |x| < \frac{\pi}{2}$$

$$\operatorname{sen} t \left(\frac{2t}{\sqrt{1+4\operatorname{sen}^2 t}} + \operatorname{sen} t \right) = 0$$

$$\operatorname{sen}\theta(t) = \frac{2\operatorname{sen} t}{\sqrt{1+4\operatorname{sen}^2 t}}$$

$$y(t_k) = 0 \qquad t_k = k\pi \quad k \in \mathbb{Z}.$$