Vanishing of the anomaly in lattice chiral gauge theory

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- The $U(1)_Y \times SU(2)$ SM electroweak is renormalizable (Weinberg (1967), t'Hofft (1971)) hence in principle one could reach a cut-off exponentially large in the inverse coupling (higher than experiments).
- Can we construct a non-perturbative EW theory with lattice cut-off (Effective QFT)?

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for (ν, e, u, d) $6(1/3)^3 + 2(-1)^3 - 3(4/3)^3 - 3(-2/3)^3 - (-2)^3 = 0$, $-1 + (1/3)3 = 0$; 3 colors, charges $0, 1, 2/3, -1/3$ $Y_{\nu,L} = Y_{e,L} = -1$, $Y_{u,L} = Y_{d,L} = 1/3$, $Y_{\nu,R} = 0$, $Y_{e,R} = -2$, $Y_{u,R} = 4/3$, $Y_{d,R} = -2/3$

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- Topology (lattice no interaction); Lattice QFT (Luscher ,Neuberger order by order)[Ginspar-Wilson, overlap, extra dimension,...]

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$$+\sum_{i_1}\int dx[\psi^+_{i_1,L,x}\sigma^L_\mu(\partial_\mu + \lambda Y_{i_1}B_\mu)\psi^-_{i_1,L,x} + \psi^+_{i_1,R,x}\sigma^R_\mu\partial_\mu\psi^-_{i_1,L,x}]$$

$$+\sum_{i_2}\int dx[\psi^+_{i_2,R,x}\sigma^R_\mu(\partial_\mu + \lambda Y_{i_2}B_\mu)\psi^-_{i_2,R,x} + \psi^+_{i_2,L,x}\sigma^L_\mu\partial_\mu\psi^-_{i_2,L,x}]$$

with $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$;

$\gamma_0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \quad \gamma_j = \begin{pmatrix} 0 & i\sigma_j \\ -i\sigma_j & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},$

$\sigma^L_\mu = (\sigma_0, i\sigma)$, $\sigma^R_\mu = (\sigma_0, -i\sigma)$.

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- R fermions of kind i_1 and the L fermions of kind i_2 decouple (Testa, Maiani 1990).

Formal conservation laws

- The current coupled to B_μ is

$$j_\mu^T = \sum_{i_1} Y_{i_1} \psi^+_{i_1,L,x} \sigma^L_\mu \psi^-_{i_1,L,x} + \sum_{i_2} Y_{i_2} \psi^+_{i_2,R,x} \sigma^R_\mu \psi^-_{i_2,R,x}$$

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- Formal application of Noether theorem; phase symmetry $\psi \to \psi e^{i\alpha}$ implies $\partial_\mu j^{T,V}_\mu = 0$; Chiral symmetry $\psi \to \psi e^{i\alpha\gamma_5}$ implies implies $\partial_\mu j^{T,A}_\mu = 0$; therefore the current is conserved $\partial_\mu j^T_\mu = 0$

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- $i_1 = (\nu_1, e_1, u_1.d_1)$, $i_2 = (\nu_2, e_2, u_2.d_2)$; $U(1)_Y$ sector with no Higgs and massless fermion.

Lattice model

- $$e^{\mathcal{W}^{\Lambda}(J,J^5,\phi)} = \int P(dB) \int P(d\psi) e^{V(\psi,B,J)+V_c(\psi)+\mathbf{B}(J^5,\psi)+(\psi,\phi)}$$

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$$g^B_{\mu,\nu}(x,y) = \frac{1}{L^4} \sum_k \frac{e^{ik(x-y)}}{|\sigma|^2 + M^2} (\delta_{\mu,\nu} + \frac{\xi \bar{\sigma}_\mu \sigma_\nu}{(1-\xi)|\sigma|^2 + M^2})$$

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- Wilson term proportional to r; with $r=0$ other poles in addition to 0 $(\pi/a,0,0,0$ etc) corresponding to unphysical particles (low energy limit non recovered $r=0$). But $r\neq 0$ breaks chiral symmetry.

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- Interaction with gauge field $V = V_1 + V_2$ with
  $V_1 = a^4 \sum_{i,s,x}[O^+_{\mu,i,s,x} G^+_{\mu,i,s,x} + O^-_{\mu,i,s,x} G^-_{\mu,i,s}]$

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- V_2 is obtained by the Wilson term with the perierls substitution with $H^{\pm}_{\mu,i,x} = a^{-1}(e^{\mp iaY_i J_{\mu,x}} - 1)$
- The mass counterterm is $V_c = \sum_i a^{-1}\nu_i a^4 \sum_x(\psi^+_{i,L,x}\psi^-_{i,R,x} + \psi^+_{i,R,x}\psi^-_{i,L,x})$ (fixed to get vanishing dressed fermionic mass)

Observables

- $\mathbf{B} = a^4 \sum_{\mu,x} J^5_{\mu,x} j^5_{\mu,x}$ $\quad \mathsf{j}^5_{\mu,x} = \sum_{i,s} \tilde{\varepsilon}_i \varepsilon_s Y_i Z^5_{i,s} \psi^+_{x,i,s} \sigma^s_\mu \psi^+_{x,i,s}$ with $\tilde{\varepsilon}_{i_1} = -\tilde{\varepsilon}_{i_2} = 1$ and $\varepsilon_L = -\varepsilon_R = 1$.

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- The 2-point function $S^\Lambda_{i,s,s'}(x,y) = \frac{\partial^2}{\partial \phi^+_{i,s,x} \partial \phi^-_{i,s',y}} \mathcal{W}^\Lambda(J, J^5, \phi)|_0$;

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- The vertex functions are $\Gamma^\Lambda_{\mu,i',s}(z,x,y) = \frac{\partial^3}{\partial J_{\mu,z} \partial \phi^+_{i',s,x} \partial \phi^-_{i',s,y}} \mathcal{W}^\Lambda(J, J^5, \phi)|_0$ and similar $\Gamma^{5,\Lambda}_{\mu,i's}(z,x,y)$

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- They are the lattice analogue of $< j^{T,V}_\mu ; j^{T,V}_{\nu_1} ; . j^{T,V}_{\nu_n} >_T$ and $< j^{T,A}_\mu ; j^{T,V}_{\nu_1} ; . j^{T,V}_{\nu_n} >_T$

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- Battle-Bridges-Federbush formula $X = 1, .., n$

$$\mathcal{E}_B^T = \sum_{T \in \mathcal{T}} \prod_{i,j \in T} V_{i,j} \int dp_T(s) e^{-V(s)}$$

with T trees $\int dp_T(s) = 1$, $V(s)$ convex combination of of $V(Y) = \sum_{i,j \in Y} V_{i,j}$, Y subsets of X, $V_{i,j} = \mathcal{E}(aB_{\mu_i}(x_i) aB_{\mu_j}(x_j))$. $V(Y) \geq 0$.

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- We can integrate the B_μ; $e^{V^0(\psi)} = \int P(dB) e^{V(\psi,B)}$ with $V^0 = \sum \int d\underline{x} H_n(\underline{x}) \prod_i \psi^{\varepsilon_i x_i}$,$H$ given by Truncated expectations $\mathcal{E}^T_B(e^{i\varepsilon_1 e Y_1 a B_{\mu_1}}; ...; e^{i\varepsilon_n e a Y_n B_{\mu n})})$.
- Battle-Bridges-Federbush formula $X = 1, .., n$

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with T trees $\int dp_T(s) = 1$, $V(s)$ convex combination of of $V(Y) = \sum_{i,j\in Y} V_{i,j}$, Y subsets of X, $V_{i,j} = \mathcal{E}(aB_{\mu_i}(x_i) aB_{\mu_j}(x_j))$. $V(Y) \geq 0$.

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- Fermionic part determinant bounds (L finite). To get $L \to \infty$ multiscale (see below)

Vector current conservation

- By the change of variables $\psi^{\pm}_{i,s,x} \to \psi^{\pm}_{i,s,x} e^{\pm i Y_i \alpha_x}$ we get $W(J, J^5, \phi) = W(J + d_\mu \alpha, J^5, e^{iY\alpha}\phi)$ hence Ward Identites

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- Do all other terms cancel under the same condition?

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- **Theorem** *Let us fix* $r=1$ *and* $Ma \geq 1$. *For* $|\lambda| \leq \lambda_0(Ma)$ *and suitable* ν_i, $Z^5_{i,s}$
  1)the limits $L \to \infty$ *of correlations exist*
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- Convergence if the lattice cut-off is smaller than the boson mass.
- The vanishing of the anomaly is proved with a finite lattice cut-off at a non perturbative level with the same condition in the continuum perturbative
- Prerequisite for construction at higher cut-off.

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- Renormalization; convergent expansion in the rcc $\mathcal{Z}_{h,i,s}$ (Gallavotti trees). $Z_{h,i,s}$ (wave function renormalization) $\mathrm{Z}^J_{h,i,s}$ (vector current renormalization), $\mathrm{Z}^5_{h,i,s}$ (axial current renormalization), $\nu_{h,i}$ (mass renormalization)

Cancellations

- rcc have a finite imit $|\mathcal{Z}_{h,i,s} - \mathcal{Z}_{-\infty,i,s}| \leq C\varepsilon\gamma^{\theta(h-N)}$; the $\mathcal{Z}_{-\infty,i,s}$ depends on i, s and all lattice details.

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- Crucial step ; the 3 current correlation can be decomposed as

$$\hat{\Pi}^5_{\mu,\rho,\sigma}(p_1,p_2) = \hat{\Pi}^a_{\mu,\rho,\sigma}(p_1,p_2) + \hat{R}_{\mu,\rho,\sigma}(p_1,p_2)$$

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- Renormalized triangle graphs [f_h momentum cut-off in the RG]

$$\hat{\Pi}^a_{\mu,\rho,\sigma}(p_1, p_2) = \sum_{\substack{h_1 \\ h_2,h_3}} \sum_{i,s} \tilde{\varepsilon}_i \varepsilon_s Y_i^3 \frac{Z^5_{-\infty,i,s}}{Z_{-\infty,i,s}} \frac{Z^J_{-\infty,i,,s}}{Z_{-\infty,i,s}} \frac{Z^J_{-\infty,i,s}}{Z_{-\infty,i,s}}$$

$$\int \frac{dk}{(2\pi)^4} \mathrm{Tr} \frac{f_{h_1}(k)}{i\sigma^s_\mu k_\mu} i\sigma^s_\mu \frac{f_{h_2}}{i\sigma^s_\mu (k_\mu + p_\mu)} i\sigma^s_\nu \frac{f_{h_3}}{i\sigma^s_\mu (k_\mu + p^2_\mu)} (i\sigma^s_\rho)$$

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- The lattice WI implies exact relations between cancellation $\frac{Z^J_{-\infty,i,s}}{Z_{-\infty,i,s}} = 1$ [because of the fact that correlation are equal to free with reormalized parameters up to subleading]; moreover by the choice of $Z^5_{i,s}$ we have $\frac{Z^5_{-\infty,i,s}}{Z_{-\infty,i,s}} = 1$.

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- It does not conserve current

$$\sum_\mu (p_{1,\mu} + p_{2,\mu}) \hat{I}_{\mu,\nu,\sigma} = \frac{(\sum_i \tilde{\varepsilon}_i Y_i^3)}{6\pi^2} p_{1,\alpha} p_{2,\beta} \varepsilon_{\alpha\beta\nu\sigma}$$

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- Note the resummation; from the naive perturbation expansion one extracts (in the renormalizaed one) a class of dominant terms renormalizing the triangle, up to more regular ones

Renormalizability

- The boson propagator is composed by two terms; one is $O(1/k^2)$ for $k^2 >> M^2$ and the other is $O(1)$ for $k^2 >> M^2$. If the second term does not contribute $D = 4 - 3n^\psi/2 - n^A$ (renormalizable) so in principle one can get $|\lambda^2 \log a| \leq \varepsilon_0$ (if contribute $D = 4 + 2n - 3n^\psi/2$ non renormalizable).

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- Averages of invariant observables are ξ independent hence only first part contribute $\partial_\xi \int P(dB) \int d\psi O = \frac{1}{L^4} \sum_p \partial_\xi \hat{g}^{-1}_{\mu,\nu}(p) \int P(dB) \hat{B}_{\mu,p} \hat{B}_{\nu,-p} \int d\psi O = 0$
- With $r \neq 0$ one needs the anomaly cancellation to reach exponential cut-off; the result is a necessary prerequiste as it holds at lower scales.
- Note that M breaks gauge invariance in the B but for renormalizability one needs invariance in the external fields or WI (see massive QED, e.g. Okun book)

Conclusions

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- $SU(2)$ (even in the low energy regime); mass of fermions requires Higgs; infrared problem for A_μ needs aymptotic freedom; and so on....