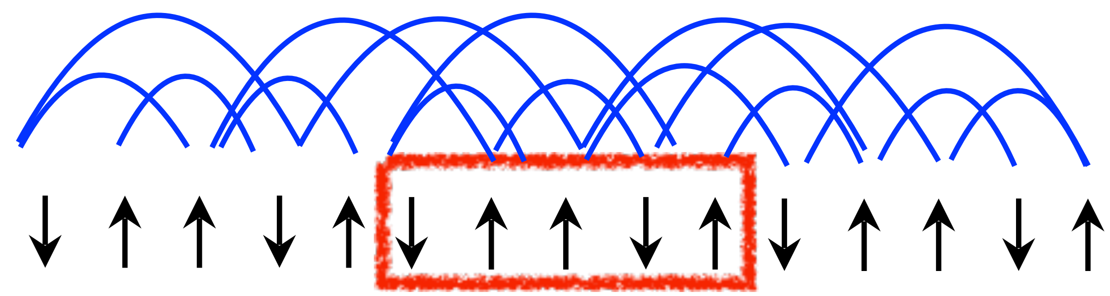
Influence functionals, temporal entanglement, and quantum many-body dynamics



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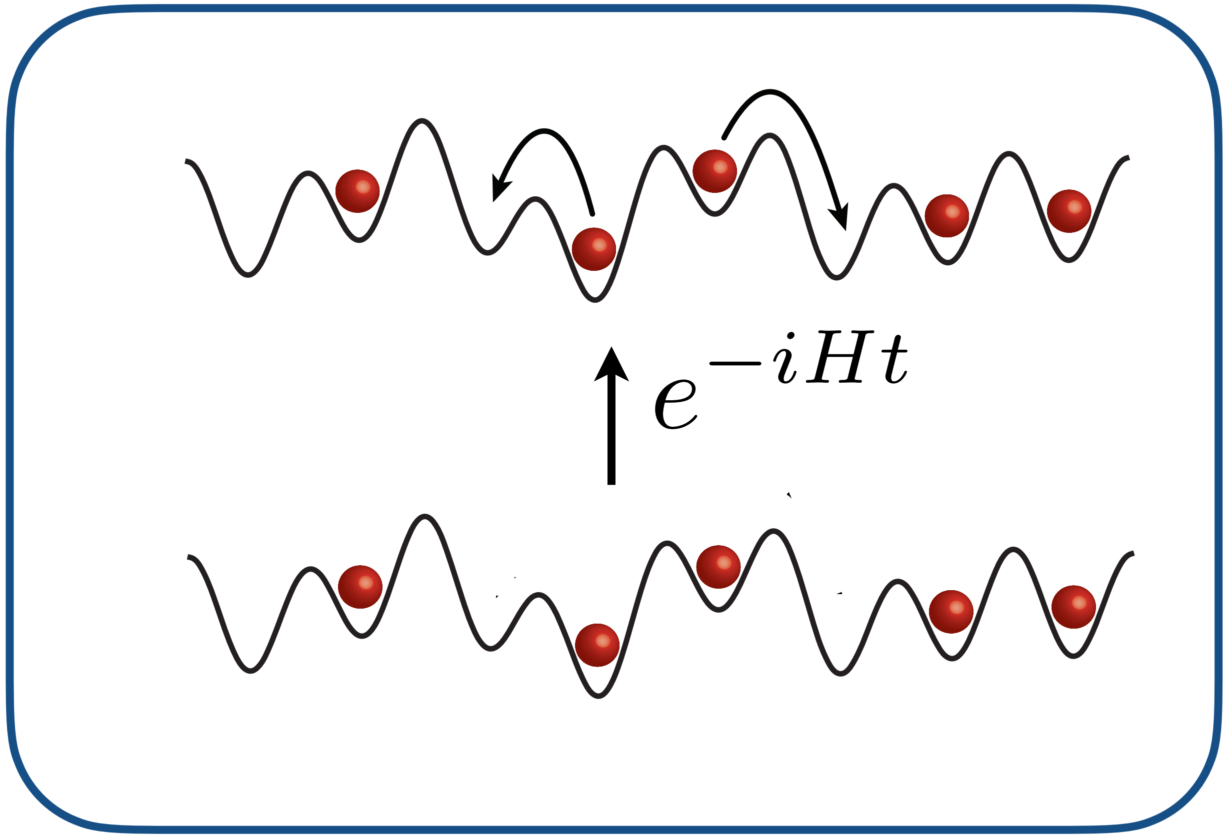
Quantum Transport: Disorder, Interactions, Integrability
Rome
27 January 2023

Introduction

Synthetic quantum systems

Ultracold atoms



Design dimensionality, lattice,interactions,..

Unlike "conventional" materials

- Isolated (no phonon/heat bath)
- Long time scales $\sim 10^{-3}s$ vs $10^{-12}s$

Experiments

- Quantum quench
- Periodic driving (Floquet)

A platform to study non-equilibrium phenomena in interacting quantum systems

Non-equilibrium quantum matter in synthetic systems

Cold atoms

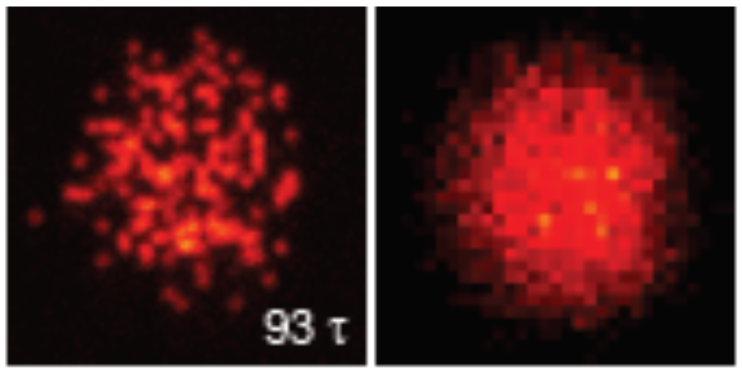


Figure: Bloch group (Munich)

Rydberg atoms

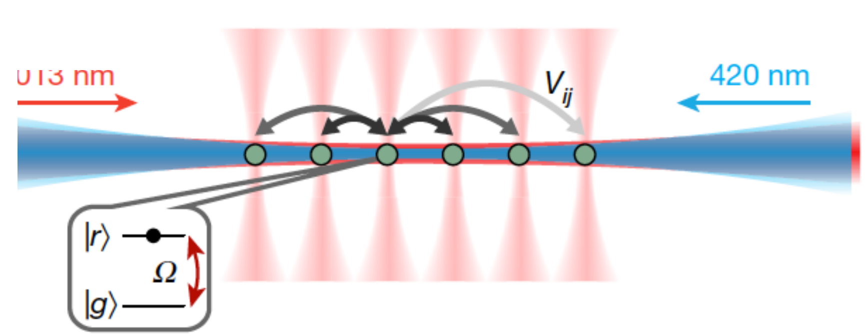


Figure: Bernien et al’17 (Harvard-MIT)

Superconducting qubits

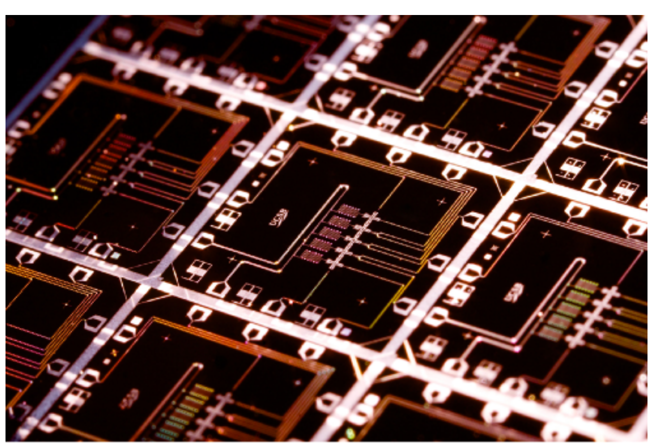


Figure: Google Quantum

Trapped ions

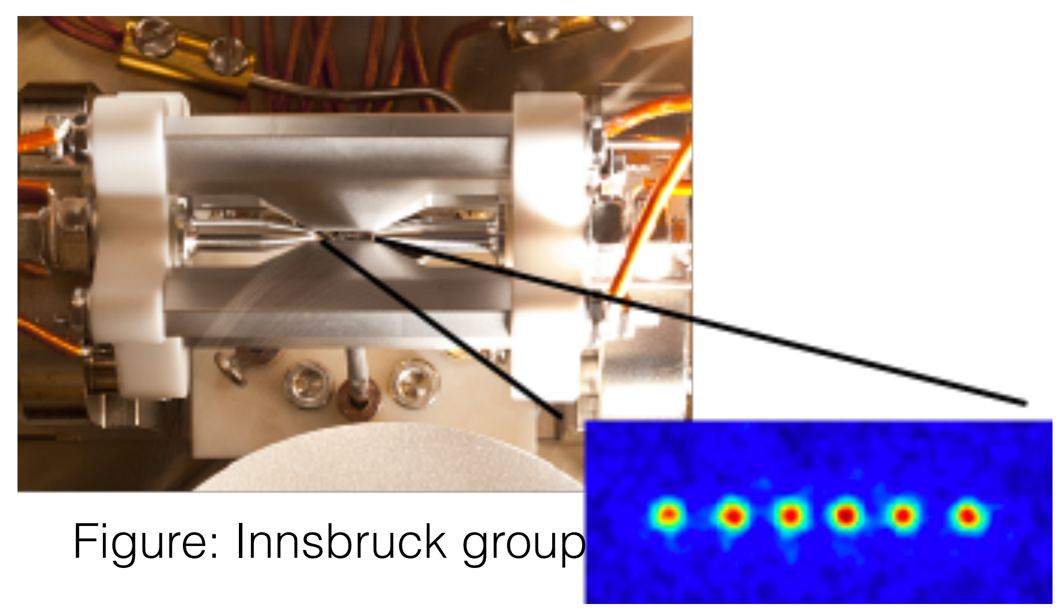
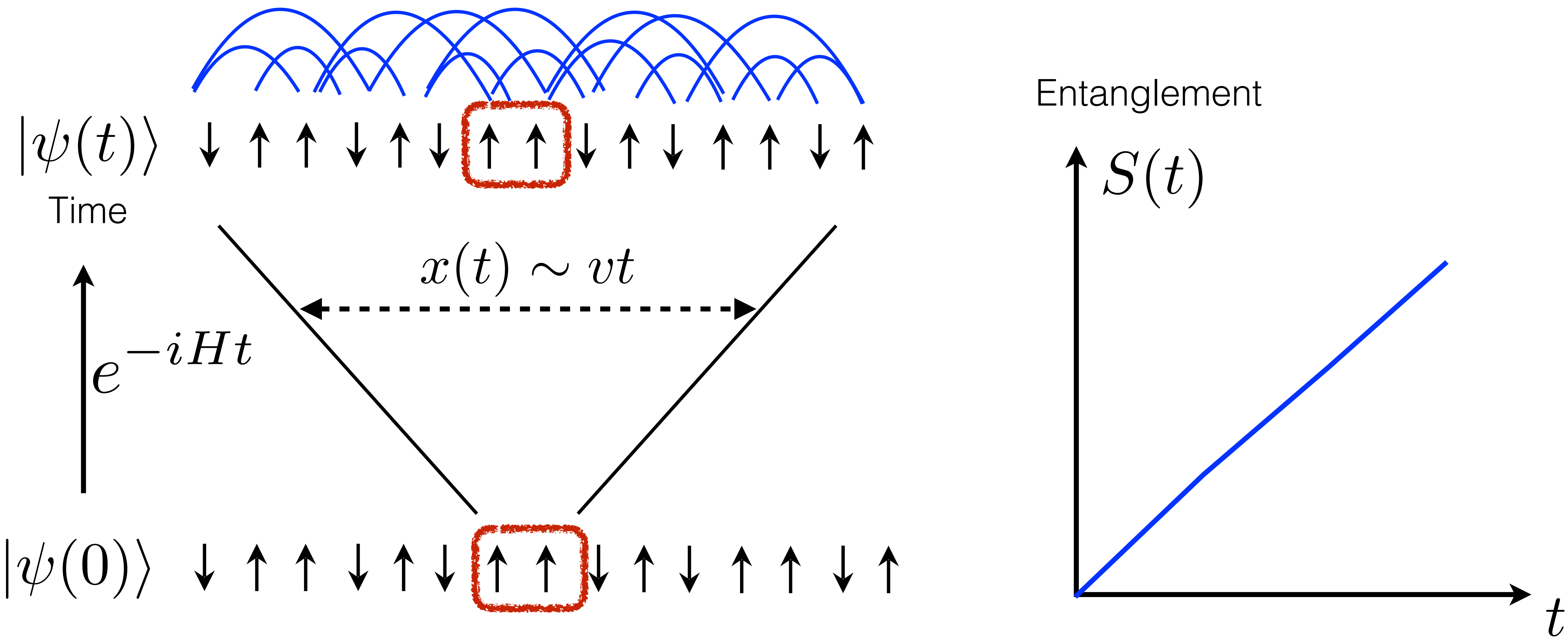


Figure: Innsbruck group

Typically, highly non-equilibrium (e.g. infinite temperature) dynamics

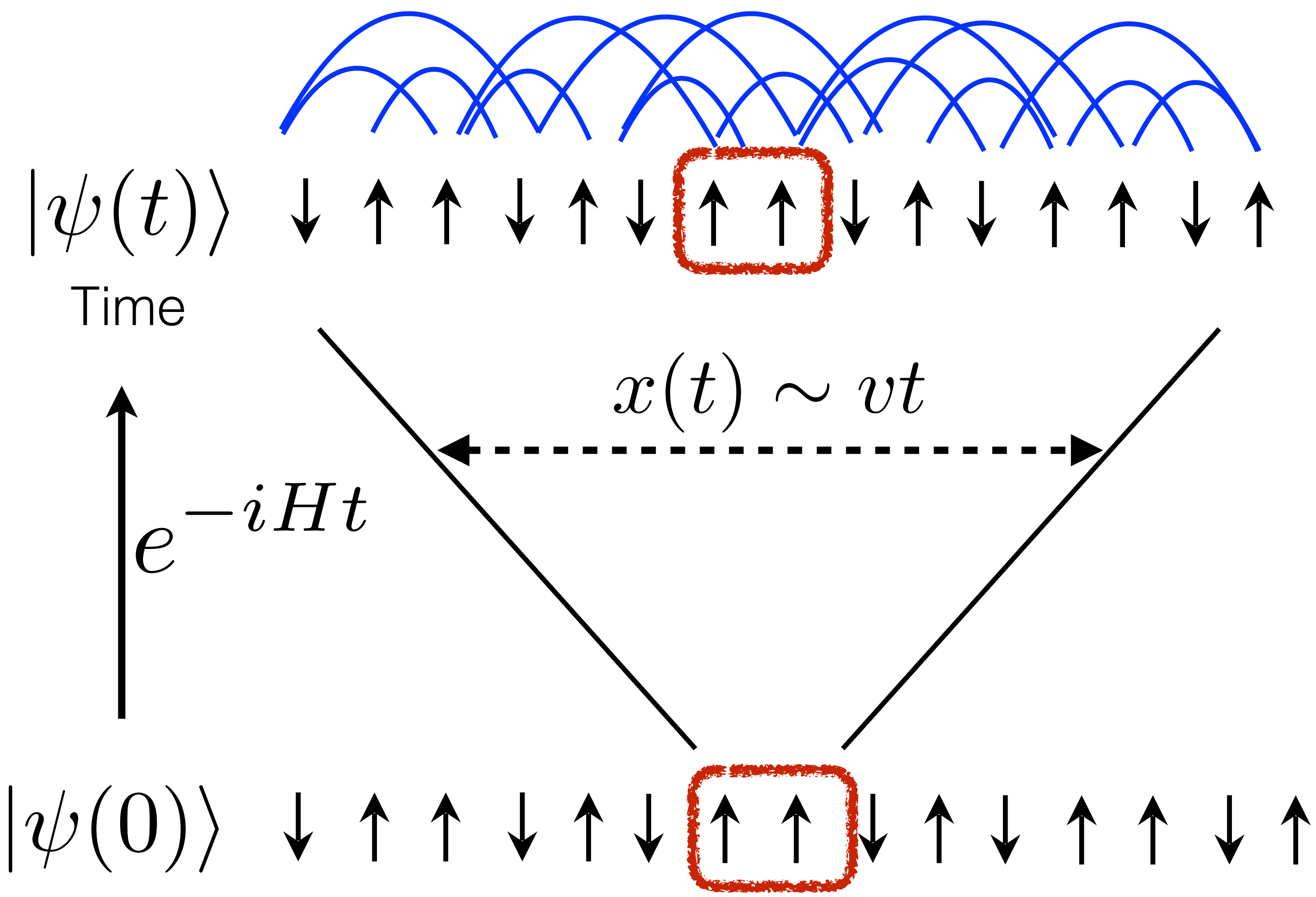
Dynamics, entanglement, and computational complexity

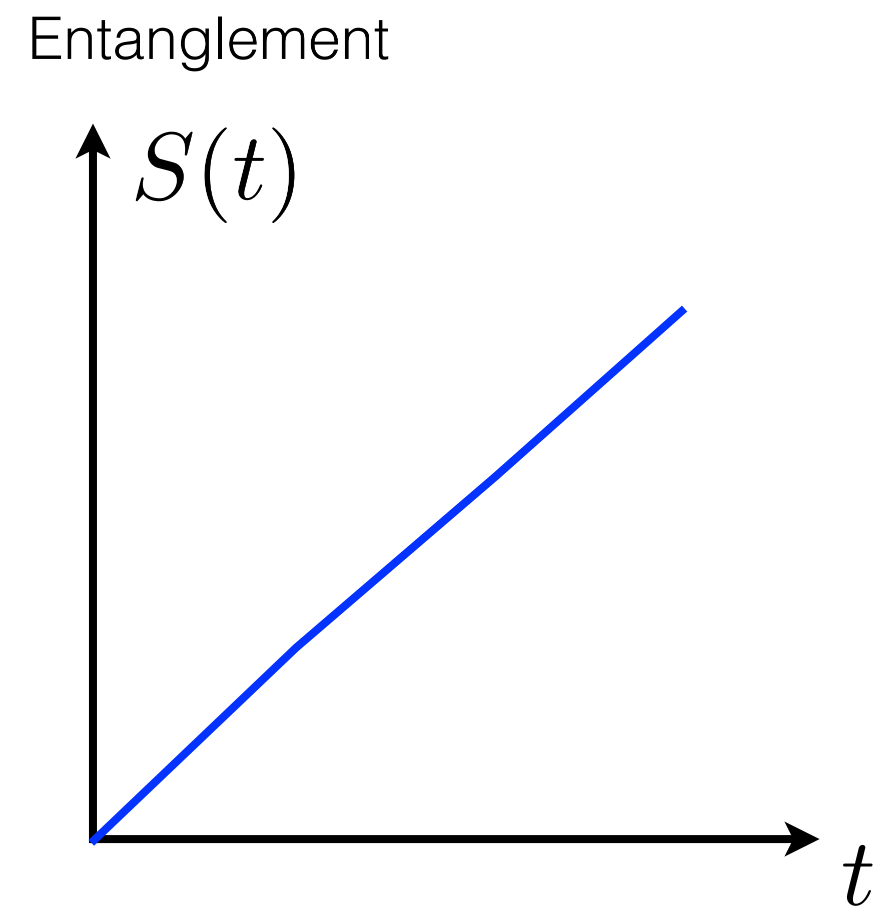


Non-equilibrium dynamics $\longrightarrow$ generation of entanglement

Generally, computationally hard problems

Dynamics, entanglement, and computational complexity





Non-equilibrium dynamics $\longrightarrow$ generation of entanglement

Generally, computationally hard problems

"Corners" of solvability/efficient description?

Natural task for quantum computers — useful quantum advantage?

Correlated materials out-of-equilibrium

Photo-induced charge and spin phase transitions

Cavalleri et al'01, Iwai et al'03, Zong et al'19,…

Control superconductivity?

Fausti et al'11, Mitrano et al'16,..

Ultra-fast dynamics of electrons and phonons (hard)

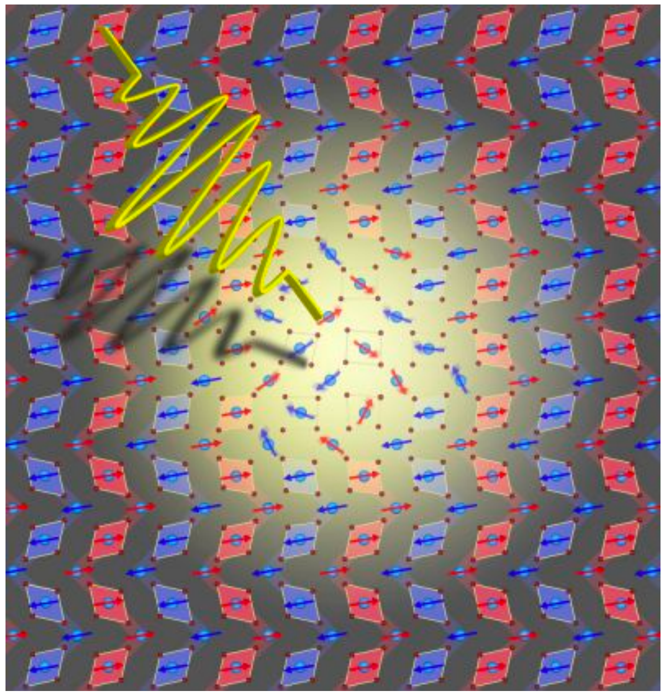
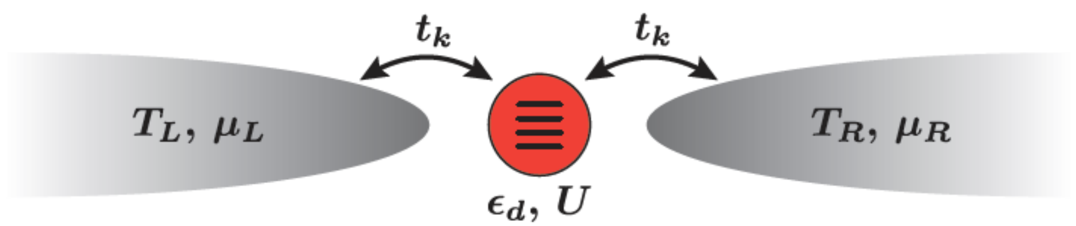


Figure: Cavalleri group

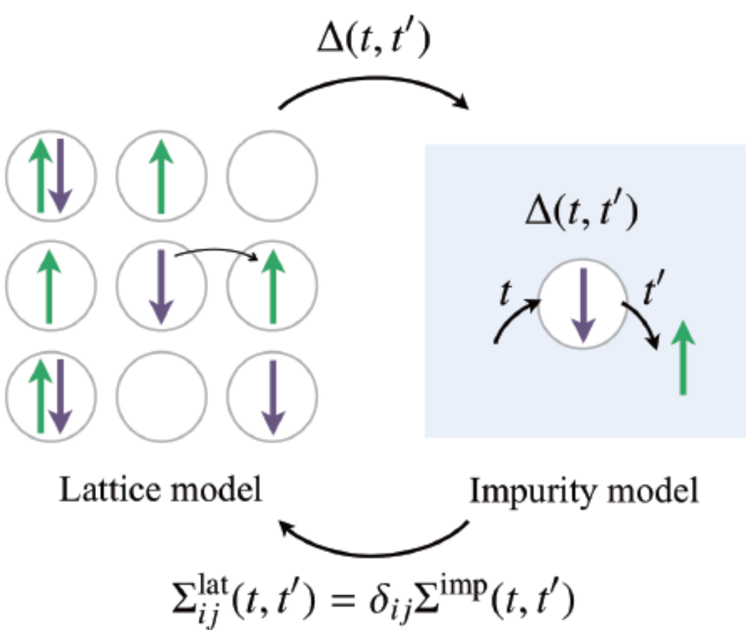
Quantum impurity problems



-Transport in mesoscopic devices

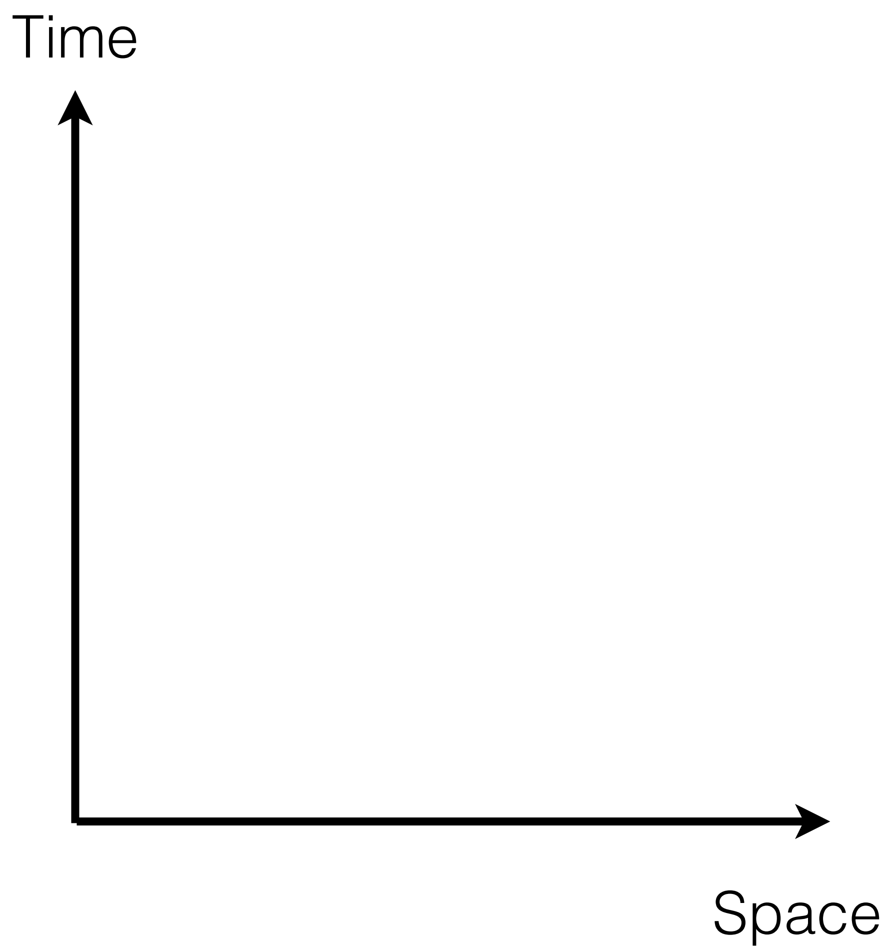
-Model correlated materials (embedding methods)

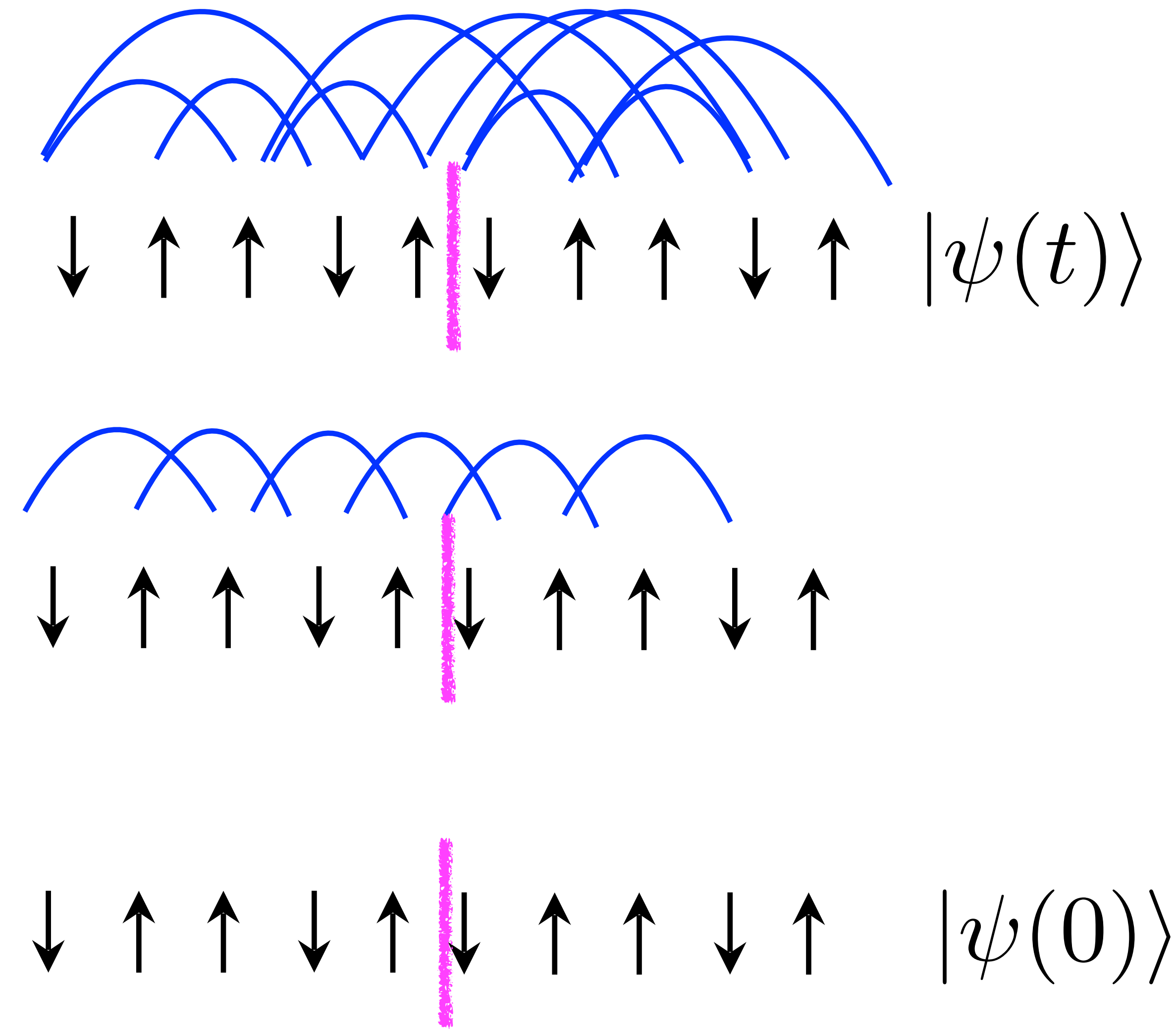
Review: Georges et al'96, Aoki et al'14



Many-body dynamics at low temperature

Preview





Time evolution of a many-body state $|\psi(t)\rangle$

Generally, high complexity

Low spatial entanglement — efficient compression by **tensor networks**

Complexity and entanglement

$$|\psi\rangle = \sum_{s_i=\uparrow,\downarrow} A_{s_1...s_N} |s_1 ... s_N\rangle$$

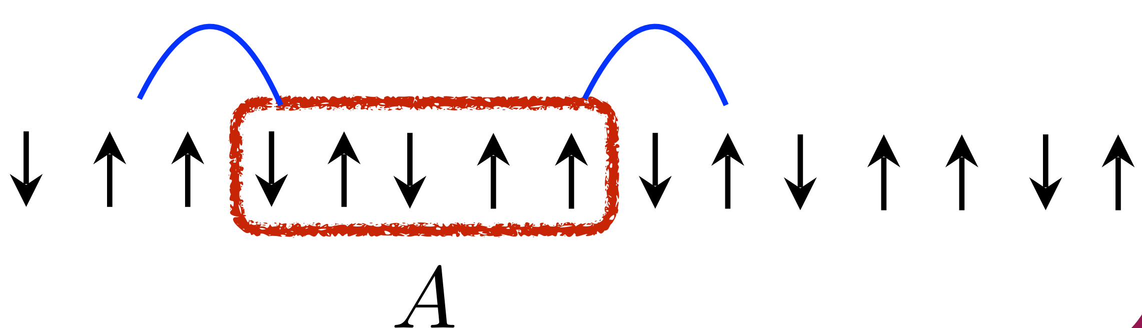
$\downarrow \uparrow \uparrow \downarrow \uparrow \downarrow \uparrow \uparrow \downarrow \uparrow$

How difficult to represent?

Naively $\sim 2^N$ parameters

Weakly entangled states can be efficiently "compressed" $\sim N^a$

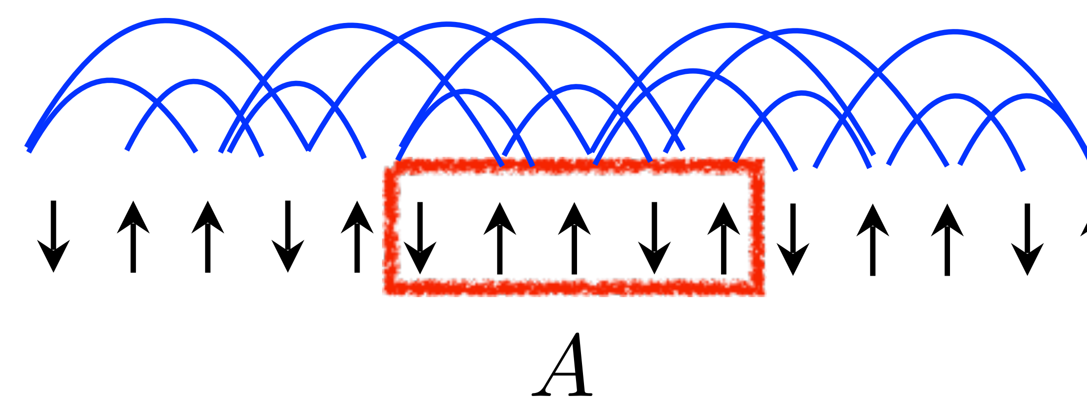
$S_{\text{ent}}(A) \propto \text{vol}(\partial A)$ "area-law"



Ground states, "easy"

Highly entangled states "hard"

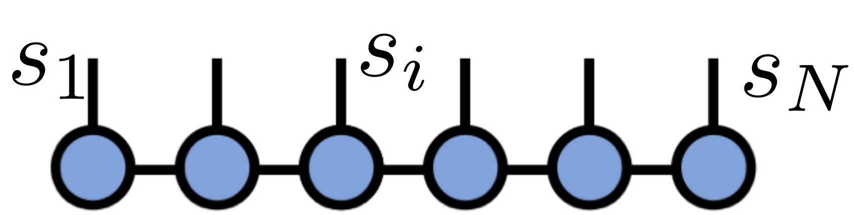
$S_{\text{ent}}(A) \propto \text{vol}(A)$ "volume-law"



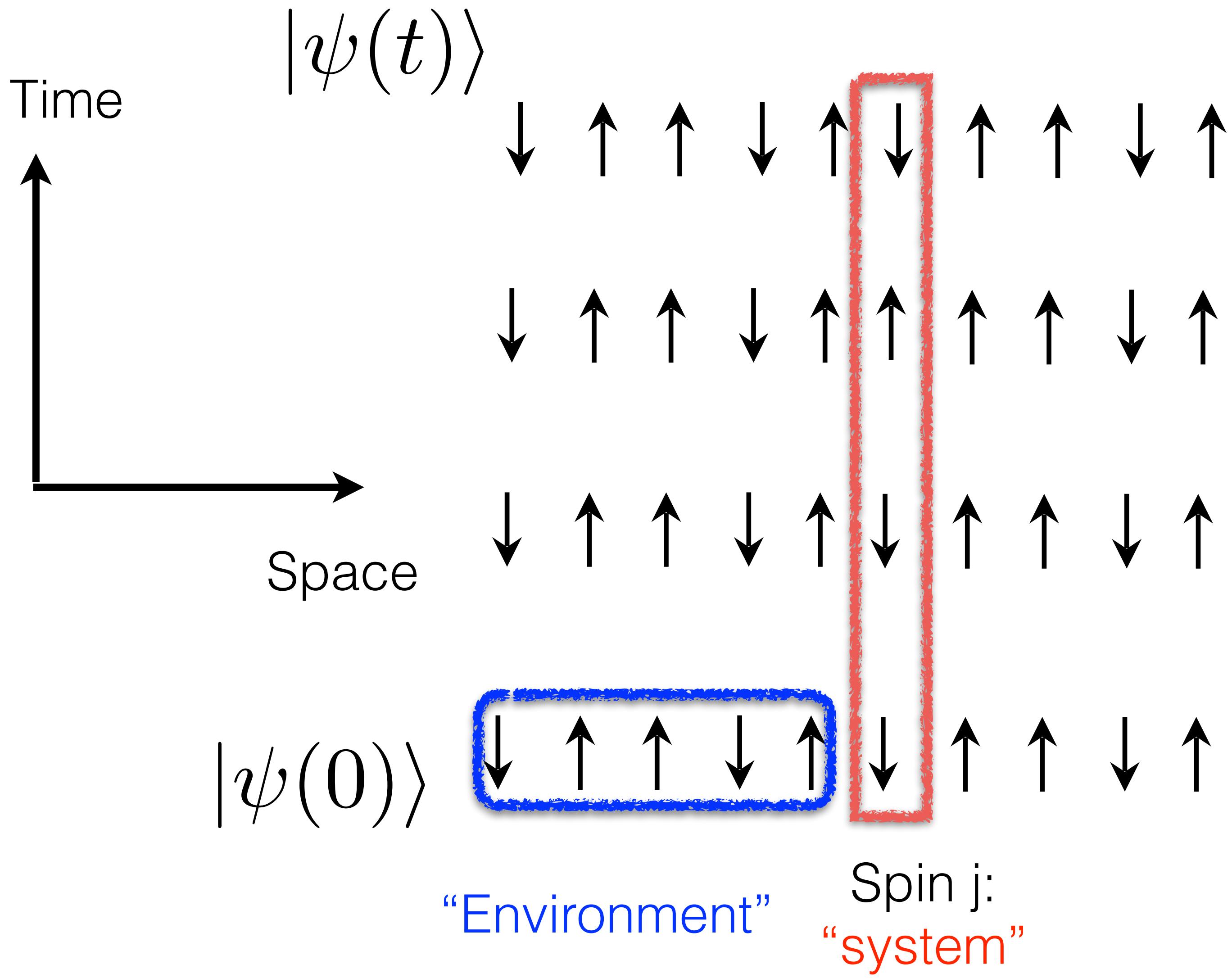
Non-equilibrium states, "hard"

Tensor networks methods

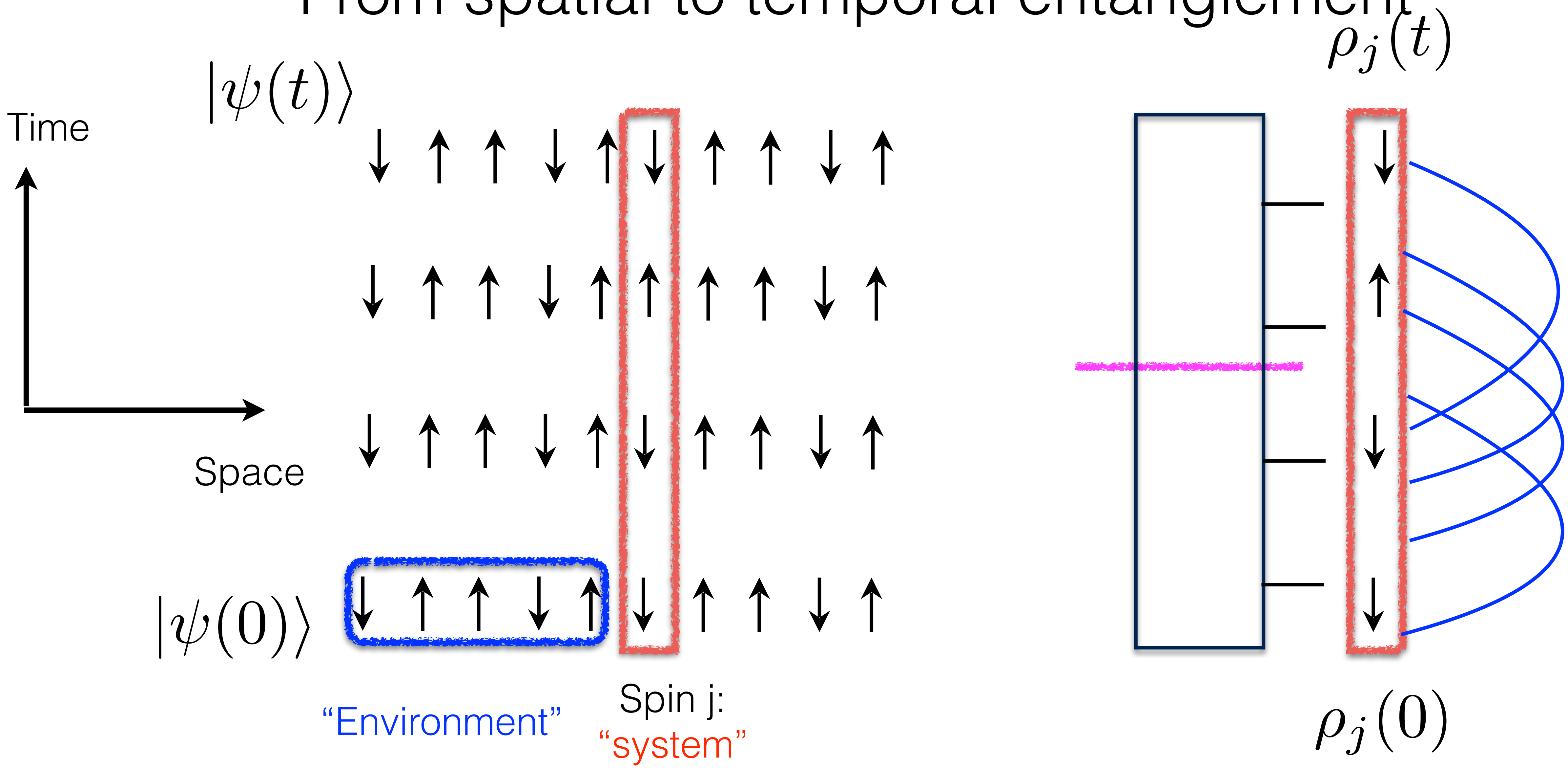
$A_{s_1...s_N} =$



From spatial to temporal entanglement



From spatial to temporal entanglement



Dynamics of local observables: view part of system as <u>environment</u>

-**Exact solutions**

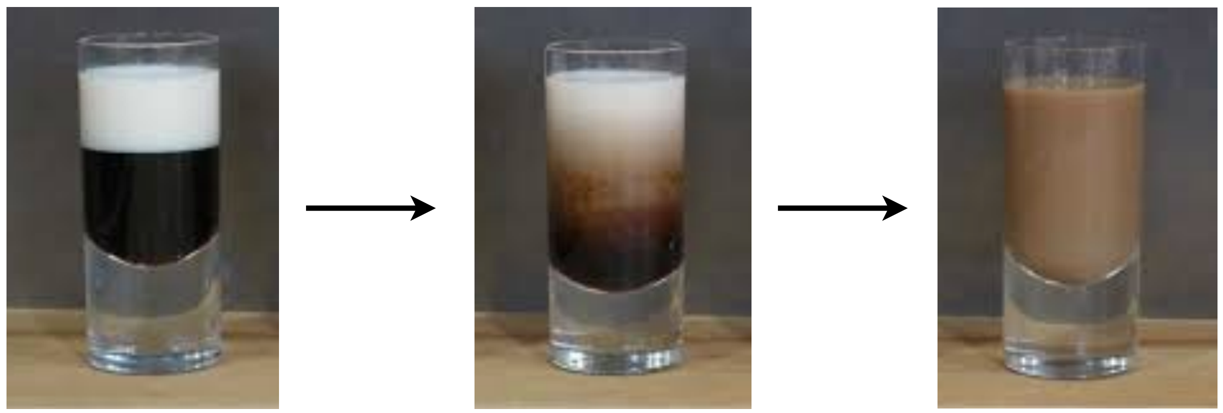
-New efficient methods based on low **temporal entanglement**

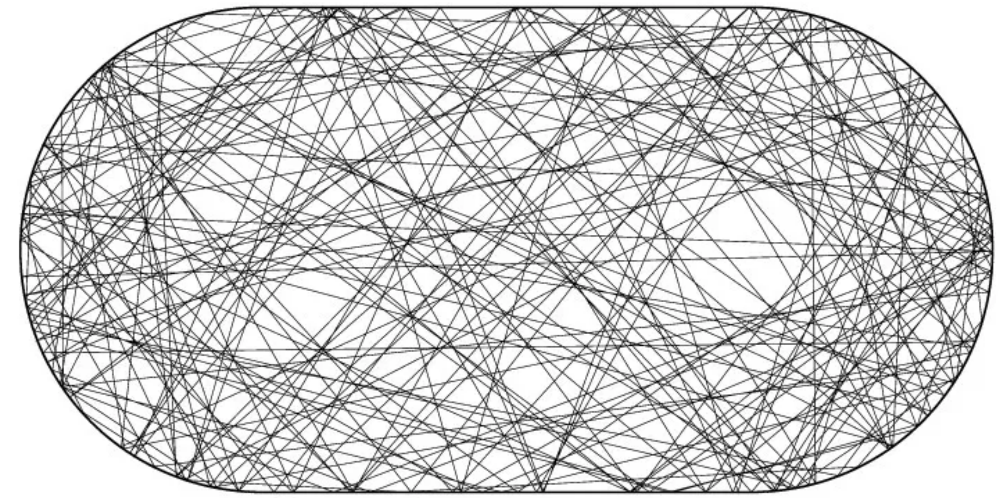
-**Quantum control** of many-body systems…

Thermalisation and mechanisms to avoid it

Quantum ergodicity and eigenstate thermalisation hypothesis (ETH)

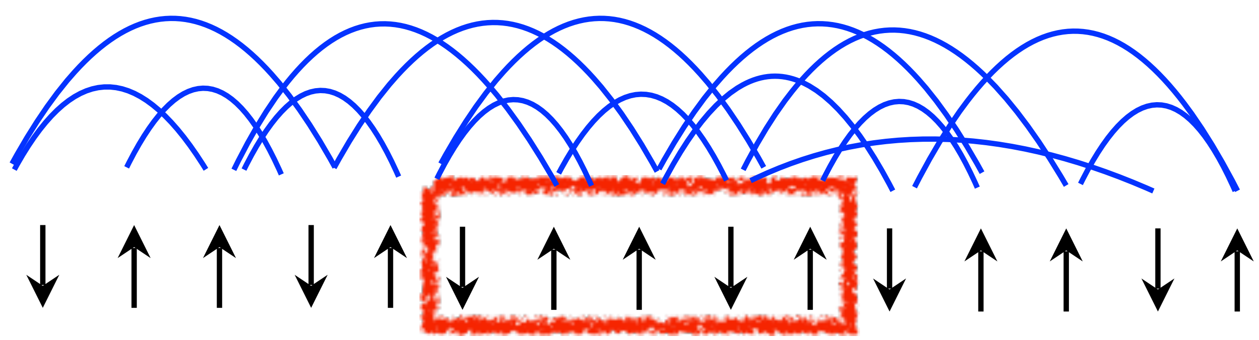
Deutch'91, Srednicki'94,..





-Quantum-ergodic Hamiltonian H

-Eigenstates have thermal observables, are highly entangled



Eigenstate $|n\rangle$ A

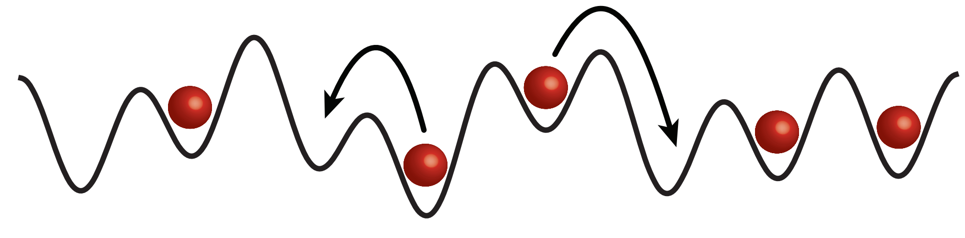
$$\rho(A) \approx \rho_{\text{Thermal}}$$

$$S_{\text{ent}}(A) \approx S_{\text{Thermal}}(A) \propto \text{Vol}(A)$$

-System acts as a "good thermal bath" for its subsystems

Many-body localization: integrability and entanglement

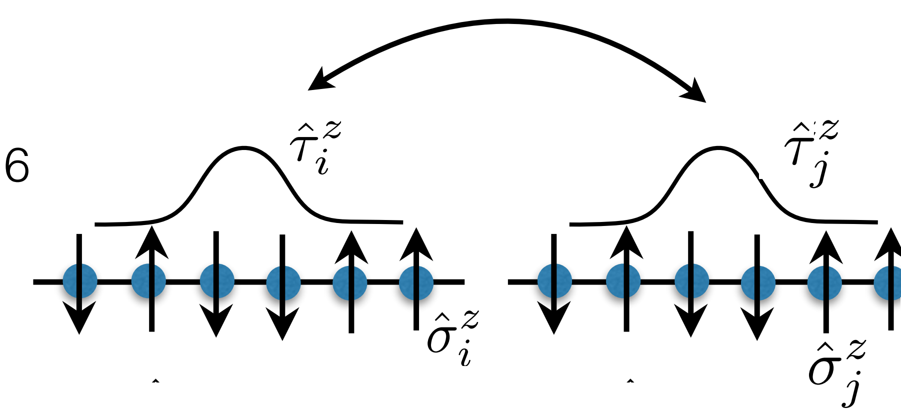
Avoiding thermalisation by disorder (spins, fermions) Anderson'58, Altshuler et al'97, Gornyi et al'05, Basko et al'05, Oganesyan, Huse'07



Quasi-local integrals of motion $[\hat{\tau}^z_i, \hat{H}] = 0$

$$H = \sum_i \tilde{h}_i \tau^z_i + \sum_{i,j} J_{ij} \tau^z_i \tau^z_j + \sum J_{ijk} \tau^z_i \tau^z_j \tau^z_k + \ldots$$

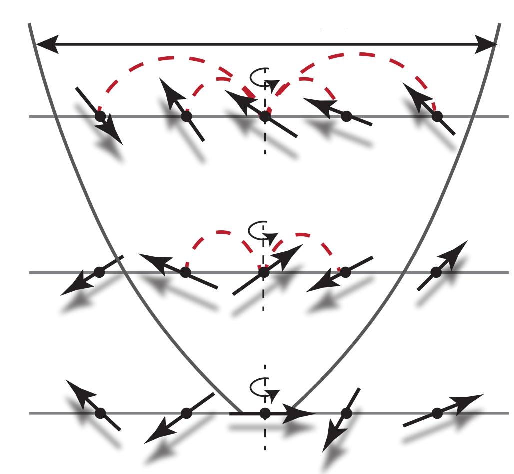
Serbyn, Papic, DA'13; Oganesyan, Huse'13 Imbrie'16



Eigenstates: **area-law entangled**

Tensor-network algorithms Chandran et al'15, Serbyn et al'16, Khemani et al'16, Wahl et al'17,'19,…

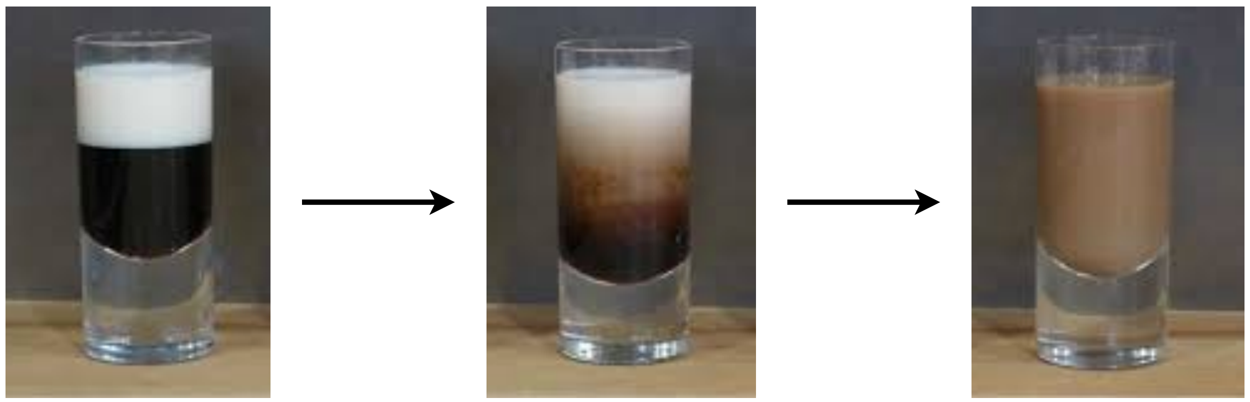
$$x_{\mathrm{ent}}(t) \sim \log t$$



Quantum quench

$$S_{\mathrm{ent}}(t) \sim \log(t)$$

Thermalisation

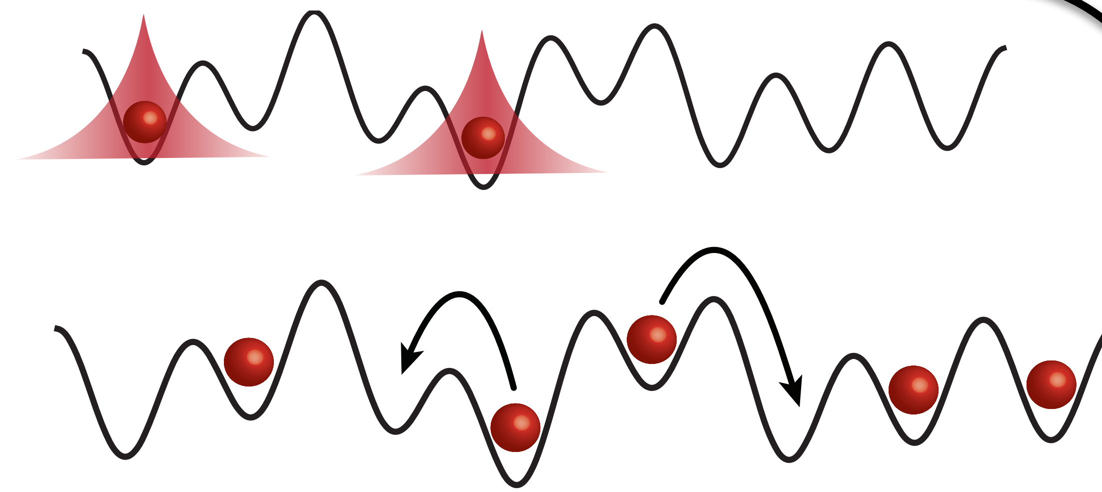


-Forget initial state

-Quantum information scrambled

"Good thermal bath"

Many-body localisation



$U\hat{c}^{\dagger}_{\alpha}\hat{c}^{\dagger}_{\beta}\hat{c}_{\gamma}\hat{c}_{\delta}$

-Memory of initial state preserved

-Quantum coherence protected

"Bad bath"

Prethermalization

-Parameterically slow thermalisation

-Approximate integrals of motion

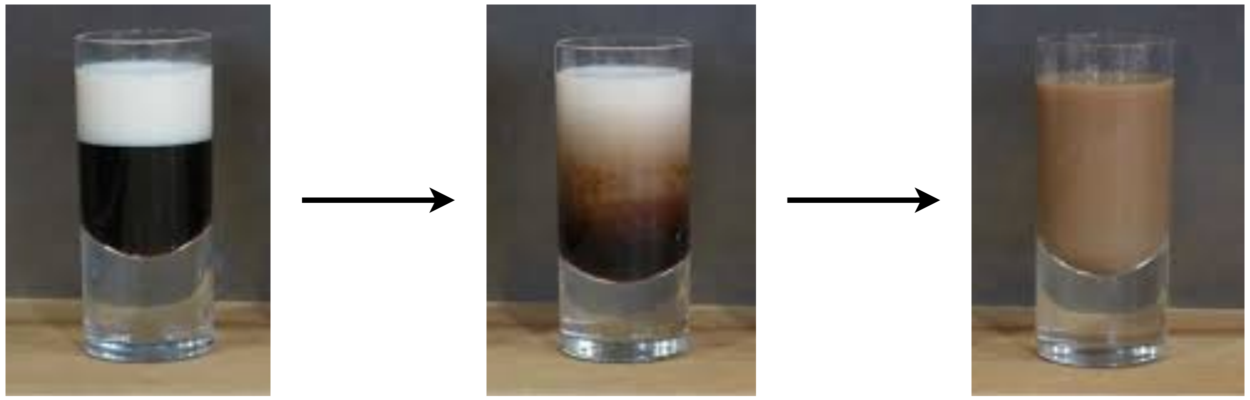
-High-frequency driving

Weak ergodicity breaking

-Quantum scars: special non-thermal states

-Fracturing..

Thermalisation

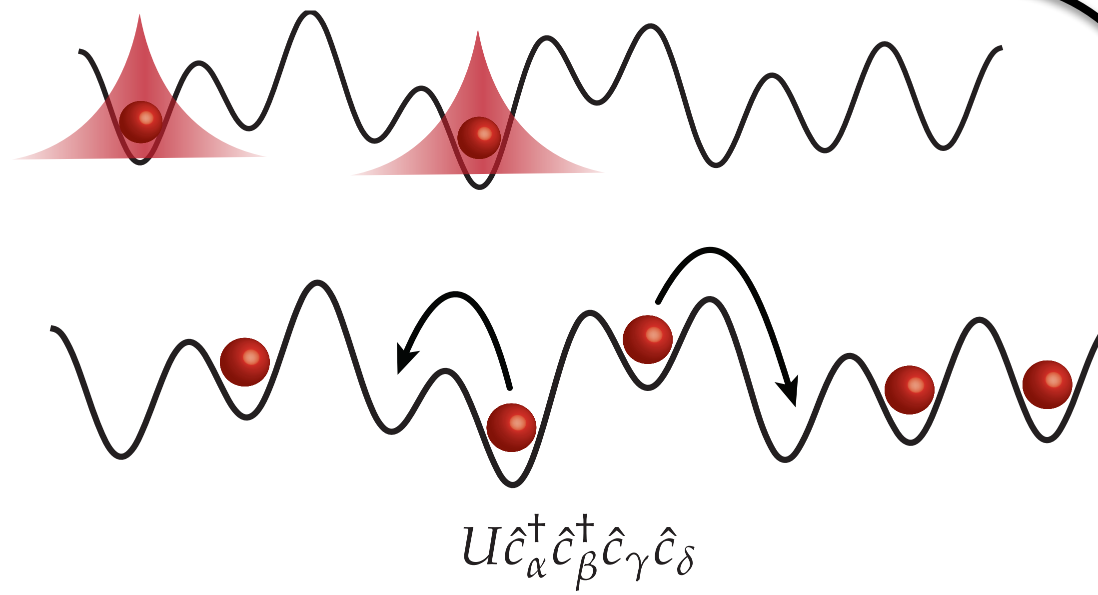


-Forget i

-Quantu

“Good t

Many-body localisation



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Describe *quantum-bath* properties of a many-body system?

Prethermalization

-Parameterically slow thermalisation

-Approximate integrals of motion

-High-frequency driving

Weak ergodicity breaking

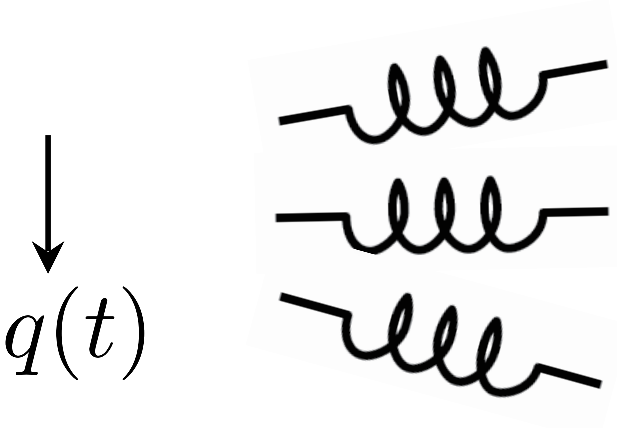
-Quantum scars: special non-thermal states

-Fracturing..

Influence matrix and temporal entanglement: A connection with open quantum systems

Characterising a quantum bath by influence functional

System $\mathcal{S}$



$q(t)$

$Q(t)$

Bath $\mathcal{E}$

ANNALS OF PHYSICS: **24**, 118–173 (1963)

The Theory of a General Quantum System Interacting with a Linear Dissipative System

R. P. FEYNMAN

California Institute of Technology, Pasadena, California

AND

F. L. VERNON, JR.*

Integrate out bath → **influence functional**

$$\mathcal{I}[q(t),\bar{q}(t)] = \int e^{iS_{\mathcal{E}}[Q(t)]-iS_{\mathcal{E}}[\bar{Q}(t)]} e^{iS_{\mathcal{S}E}[q(t),Q(t)]-iS_{\mathcal{S}E}[\bar{q}(t),\bar{Q}(t)]}\mathcal{D}Q(t)\mathcal{D}\bar{Q}(t)$$

forward trajectory   backward trajectory

Nonlocal in time functional of system trajectory

Encodes **memory of the bath**

Previous work: bath — many independent oscillators

Dynamics of the dissipative two-state system

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system:spin

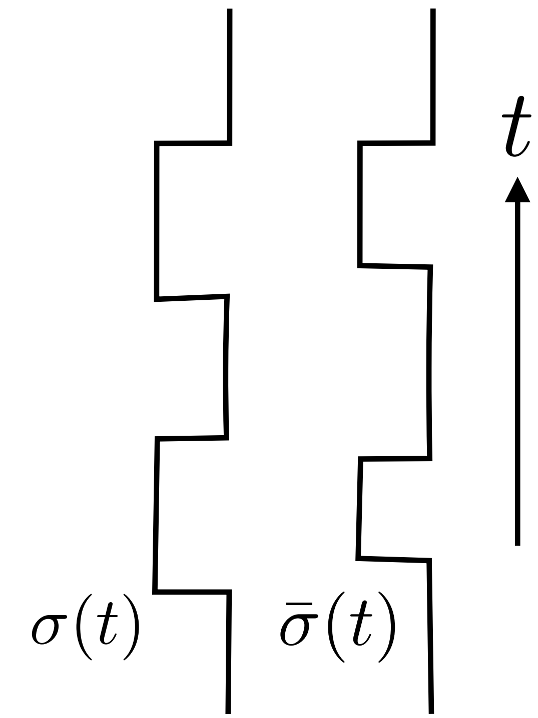
bath: oscillators

$$H_{\mathrm{SB}} = -\tfrac{1}{2}\hbar\Delta\sigma_x + \tfrac{1}{2}\varepsilon\sigma_z + \sum_\alpha \left(\tfrac{1}{2}m_\alpha\omega_\alpha x_\alpha^2 + p_\alpha^2/2m_\alpha\right) + \tfrac{1}{2}q_0\sigma_z\sum_\alpha C_\alpha x_\alpha \; . \tag{1.4}$$

interaction

This talk: many-body system itself viewed as bath

Influence functional (IF): intuition



Keldysh: forward, backward trajectories

Classical trajectories: $\sigma(t) = \bar{\sigma}(t)$

Quantum trajectories (interference contributions) $\sigma(t) \neq \bar{\sigma}(t)$

Example: Interaction with a classical noise

$$\delta H = \xi(t)\sigma^z(t)$$

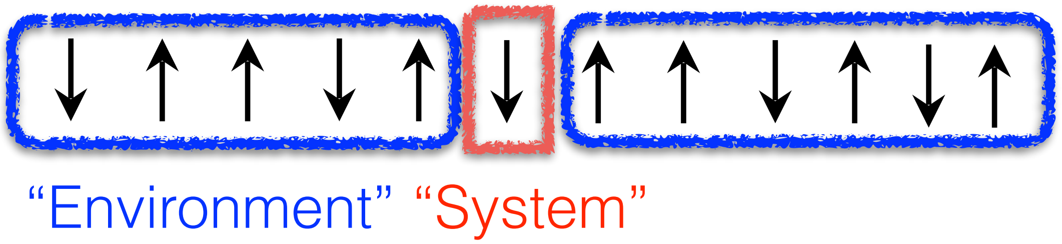
$$\langle \xi(t)\xi(t')\rangle \propto F(t-t')$$

Influence functional:

$$\mathcal{I}(\sigma(t), \bar{\sigma}(t')) = e^{-\int dt dt' (\sigma(t)-\bar{\sigma}(t))\boxed{G(t-t')}(\sigma(t')-\bar{\sigma}(t'))}$$

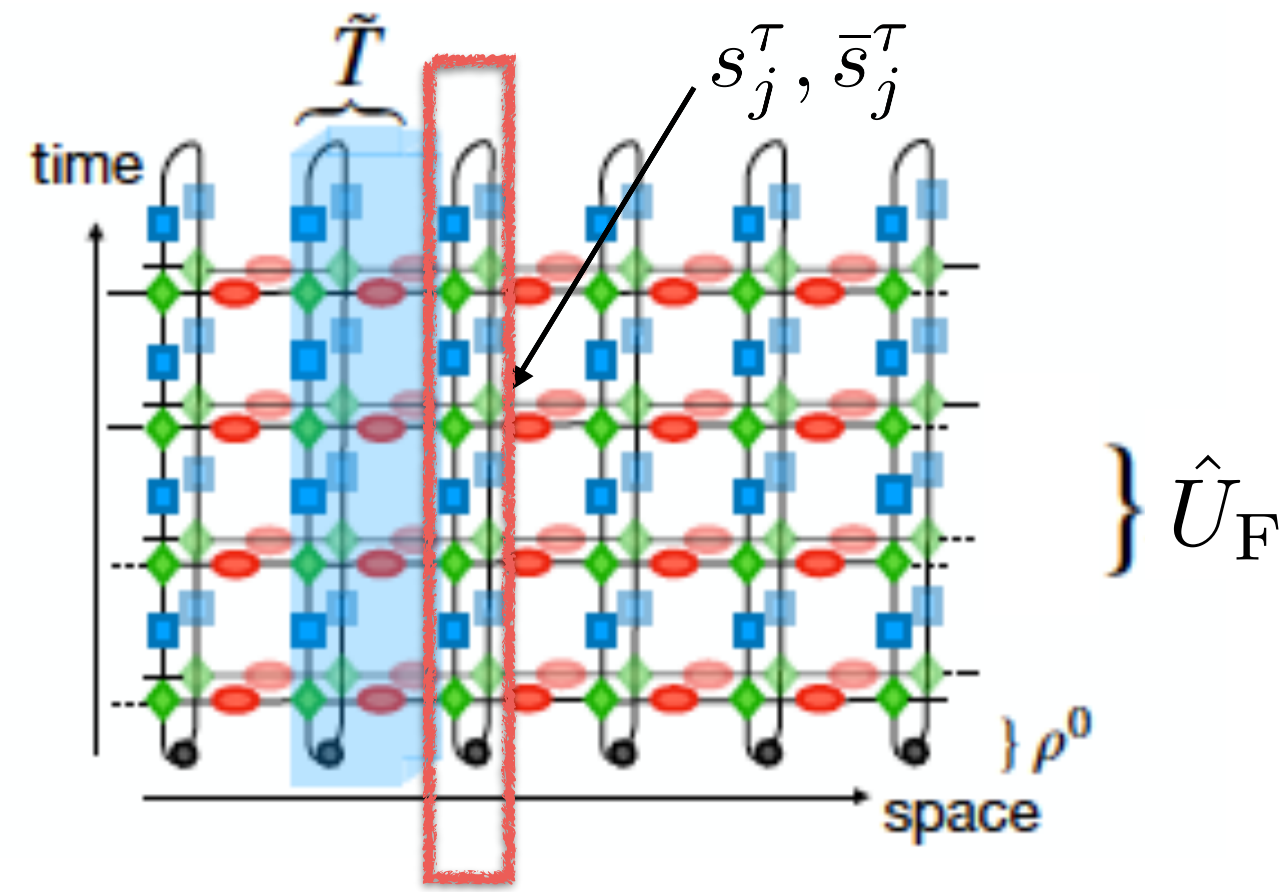
"Measurement" (by classical noise, or q-bath) suppresses quantum trajectories

Influence matrix: tensor-network representation



Floquet spin chain

$$\hat{U}_{\mathrm{F}} = e^{iJ\sum_i Z_i Z_{i+1}} e^{ih\sum_i Z_i} e^{ig\sum_i X_i}$$

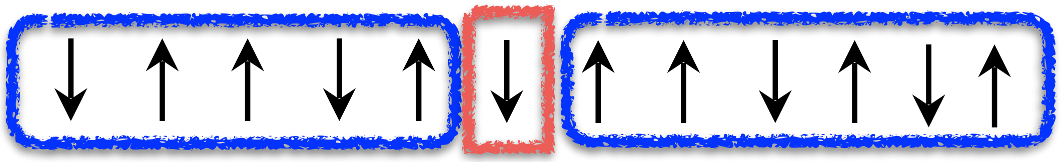


Keldysh evolution tensor-network

Lerose, Sonner, DA, PRX’21

Transverse TN contraction: Banuls et al’10

Influence matrix: tensor-network representation

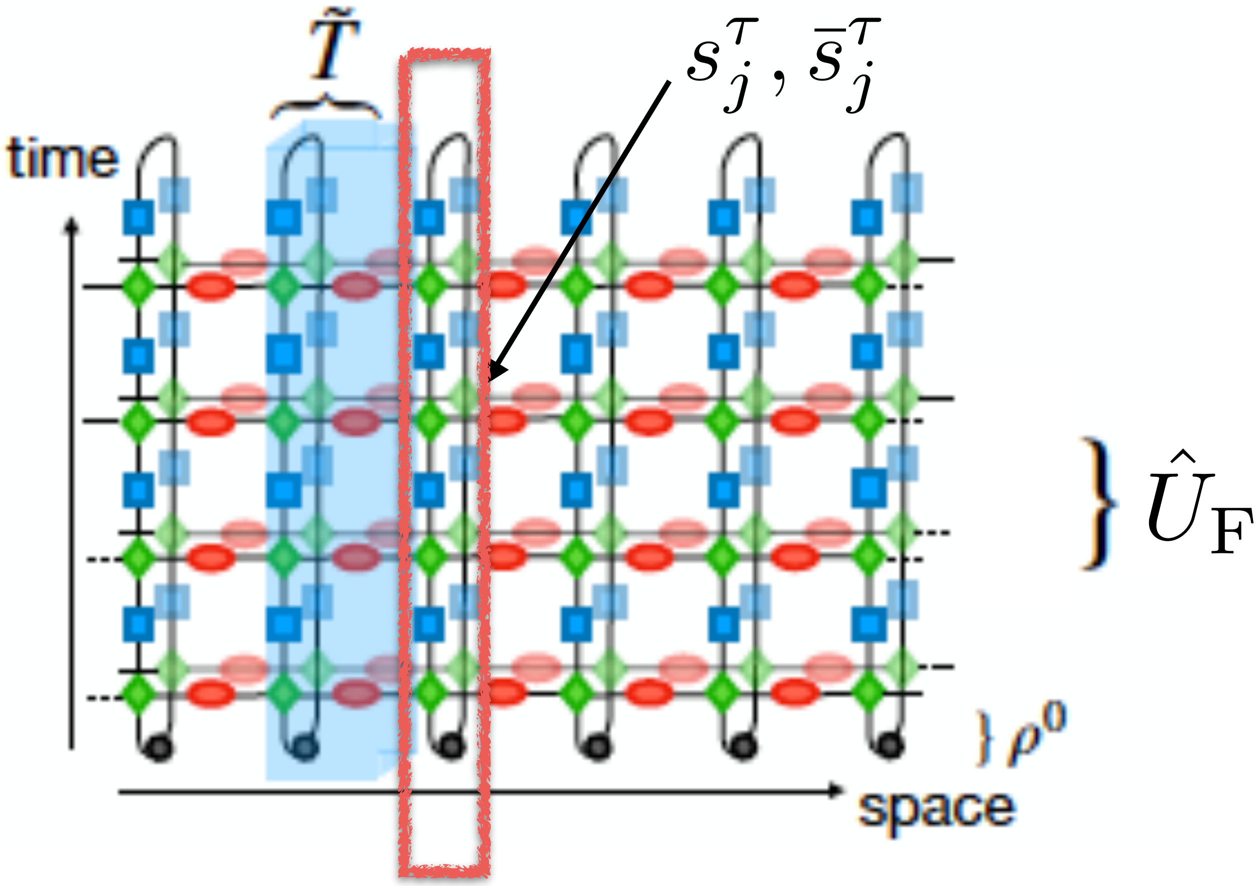


"Environment" "System"

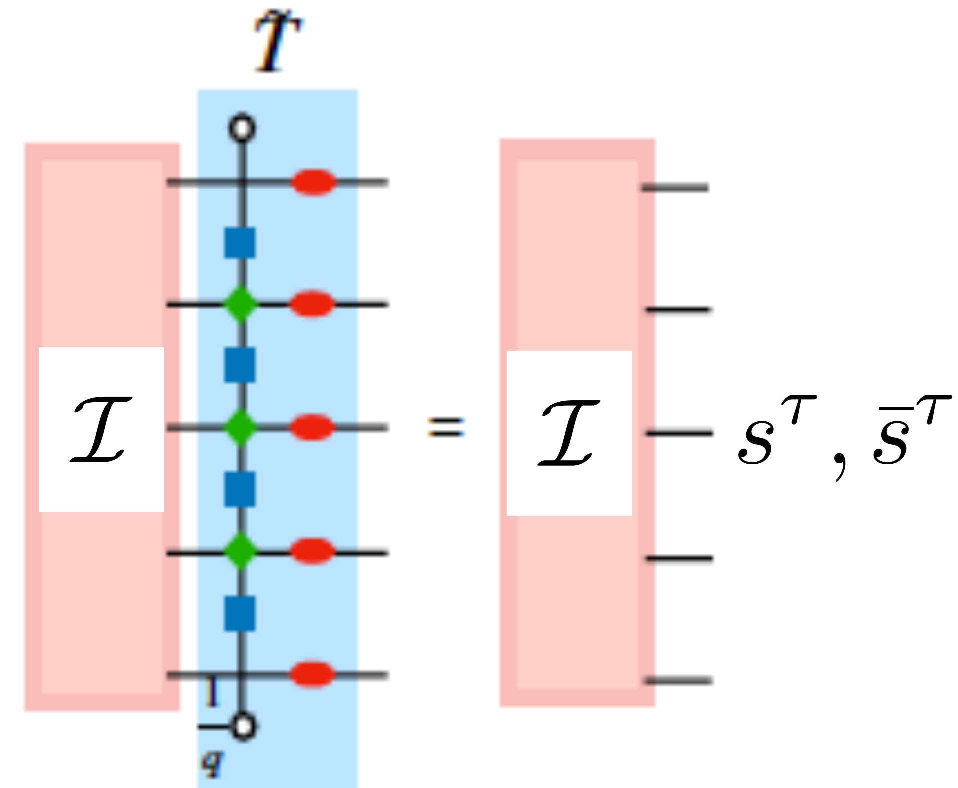
Floquet spin chain

$$\hat{U}_{\mathrm{F}} = e^{iJ\sum_i Z_i Z_{i+1}} e^{ih\sum_i Z_i} e^{ig\sum_i X_i}$$

X_i, Y_i, Z_i Pauli matrices for spin *i*



Keldysh evolution tensor-network



Influence matrix is an eigenvector of a dual transfer matrix

"wave function" of a temporal spin chain

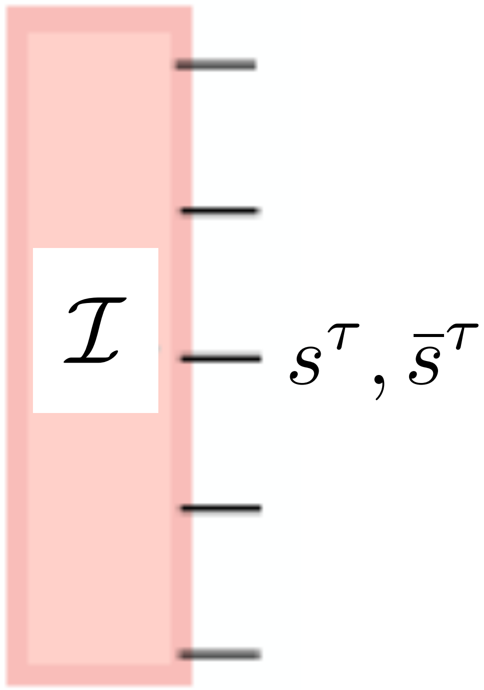
$$|\mathcal{I}\rangle = \sum_{\{s^\tau, \bar{s}^\tau\}} \mathcal{I}(s^1, \bar{s}^1, \ldots, s^t, \bar{s}^t) |s^1, \bar{s}^1, \ldots s^t, \bar{s}^t\rangle$$

Lerose, Sonner, DA, PRX'21

Transverse TN contraction: Banuls et al'10

Complexity of the influence matrix and temporal entanglement

Influence matrix is a **"wave function" (in time)**



Minimal complexity for computing dynamics of local observables
**can be smaller than full wave function complexity*

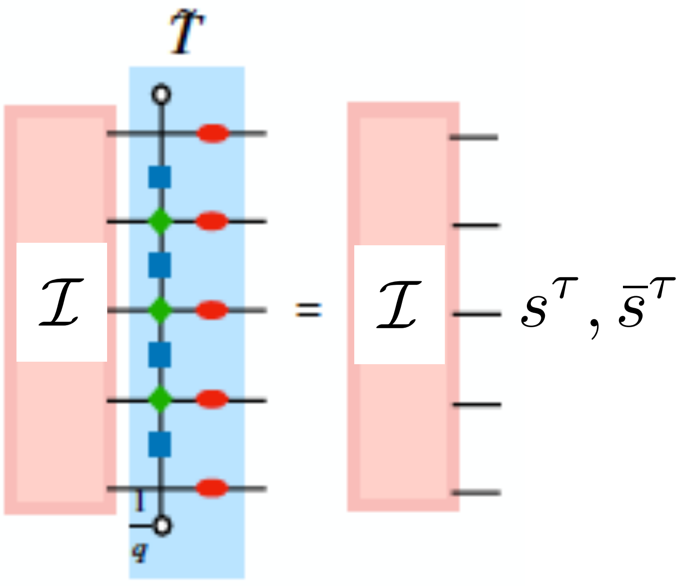
Temporal entanglement structure and scaling with *t* ?

If area law, $S_{\rm temp}(t) < C$ $\longrightarrow$ describe efficiently with tensor-networks

Exact solutions

Perfect dephaser: exactly solvable chaotic dynamics

$$\hat{U}_{\mathrm{F}} = e^{iJ\sum_i Z_i Z_{i+1}} e^{ih\sum_i Z_i} e^{ig\sum_i X_i}$$



For $|g| = |J| = \pi/4, \, \forall \, h$, solve eigenvalue eqn exactly

"Perfect dephaser" influence matrix

$$\mathcal{I}(\{s,\bar{s}\}) = \prod_{\tau=1}^{T-1} \delta_{s^\tau \bar{s}^\tau} \qquad S_{\mathrm{TE}}(t) = 0$$

Lerose, Sonner, DA, PRX'21

This model is chaotic if $h \neq 0$ (level repulsion)

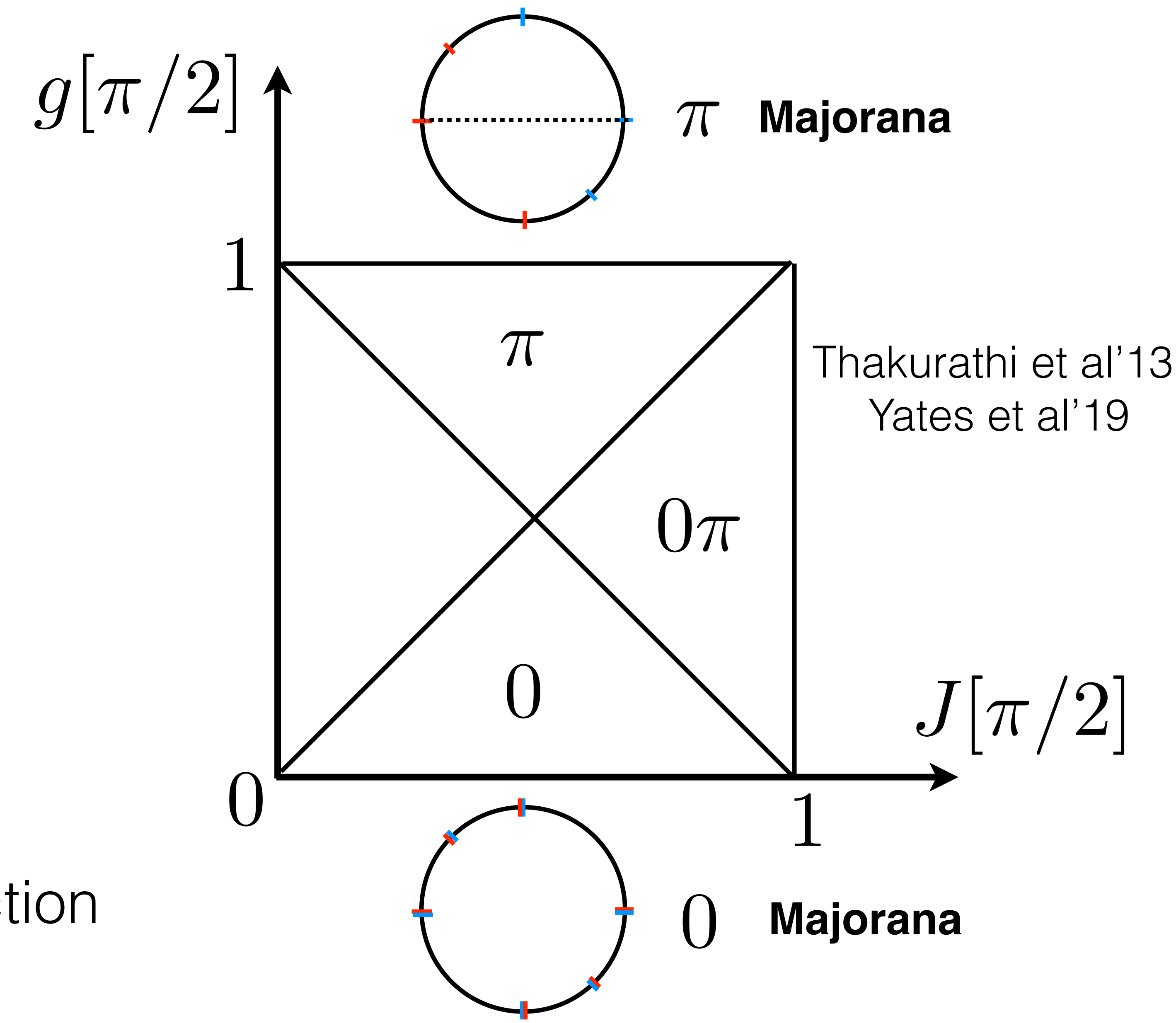
Relation to dual-unitary circuits Akila et al'16; Bertini et al'19; Piroli et al'20

A difficult problem (chaotic dynamics, high entanglement) becomes easy

Influence matrix (IM) of an integrable model

$$\hat{U}_{\mathrm{F},0} = e^{iJ\sum_i Z_i Z_{i+1}} e^{ig\sum_i X_i}$$

Solve by mapping to free fermions



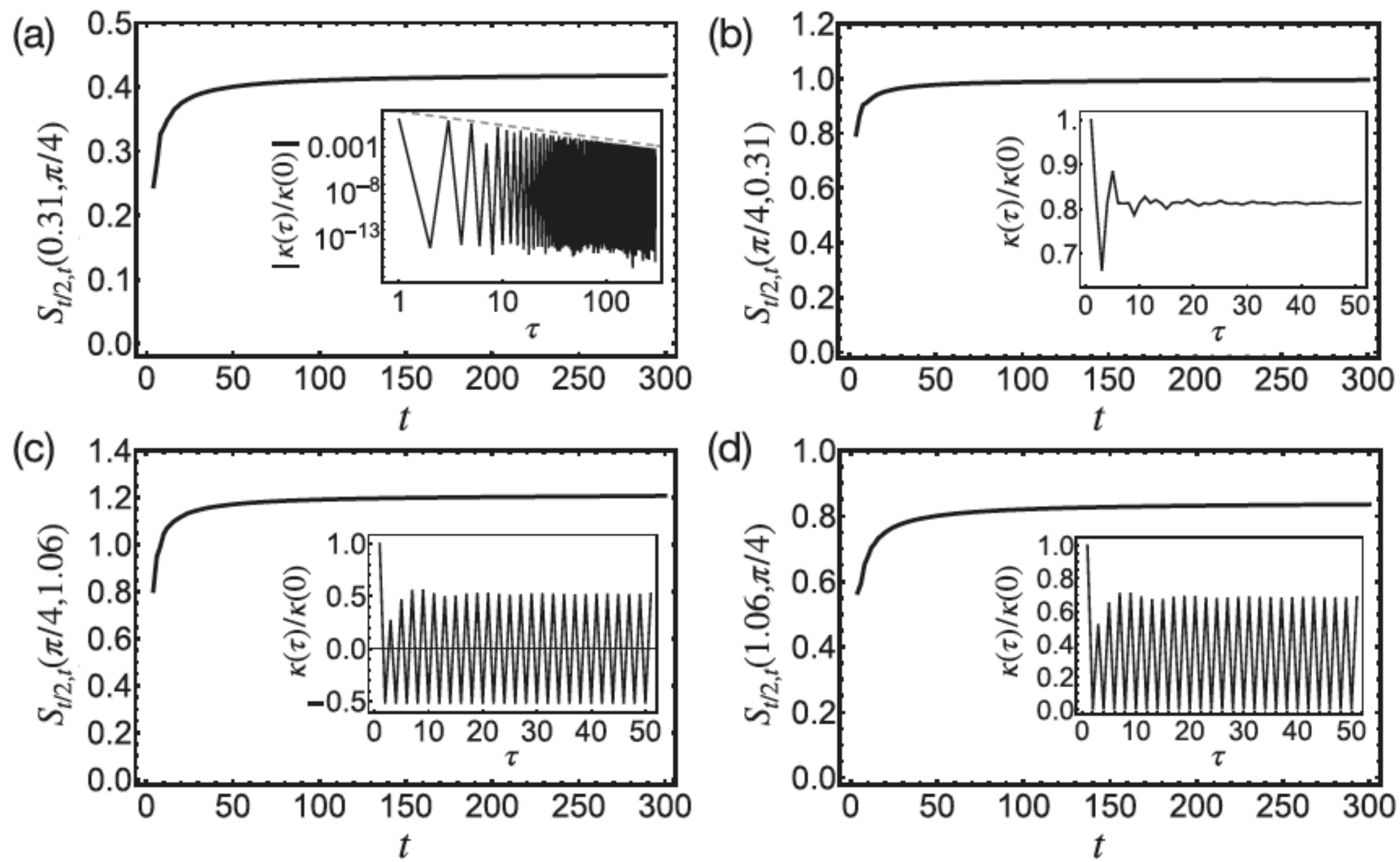
Influence matrix — BCS-like wave function
(in the space of trajectories)

$$|\mathscr{I}\rangle \propto e^{\sum_\tau (f^\dagger_{\uparrow+,\tau} f^\dagger_{\downarrow+,\tau} - f^\dagger_{\uparrow-,\tau} f^\dagger_{\downarrow-,\tau}) + \sum_{\tau,\tau'} \kappa(\tau'-\tau)[f^\dagger_{\uparrow+,\tau} f^\dagger_{\uparrow-,\tau'} + \Theta(\tau'-\tau)(f^\dagger_{\uparrow+,\tau} f^\dagger_{\uparrow+,\tau'} - f^\dagger_{\uparrow-,\tau} f^\dagger_{\uparrow-,\tau'})]} |\emptyset\rangle.$$

Pairing term depends on the spectral properties of **quasiparticles**

Lerose, Sonner, DA, arXiv:2104.07607

Area-law temporal entanglement of influence matrix "wave function"



Additionally, IM detects phase transitions, edge modes

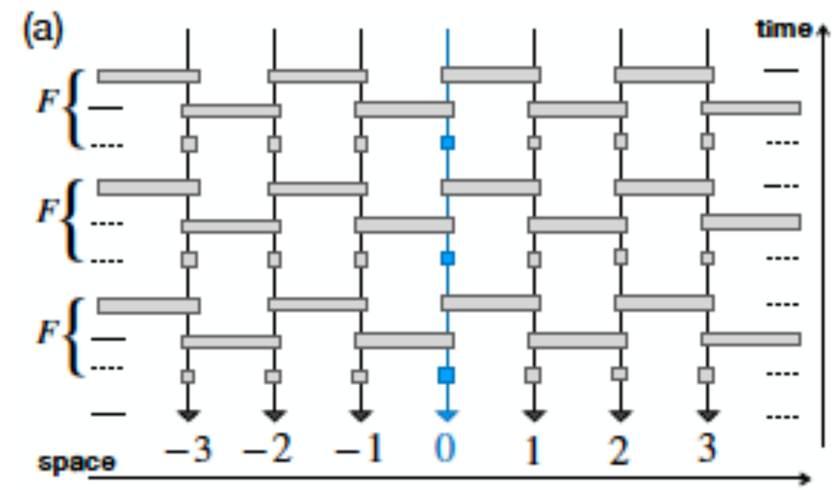
Interactions in the temporal chain decay as $\kappa(\tau) \propto |\tau|^{-3/2}$

Implies area-law

Temporal entanglement of a discretised XXZ chain

Collaboration with Piroli, Giudice, Giudici, arXiv:2112.14264, PRL'22

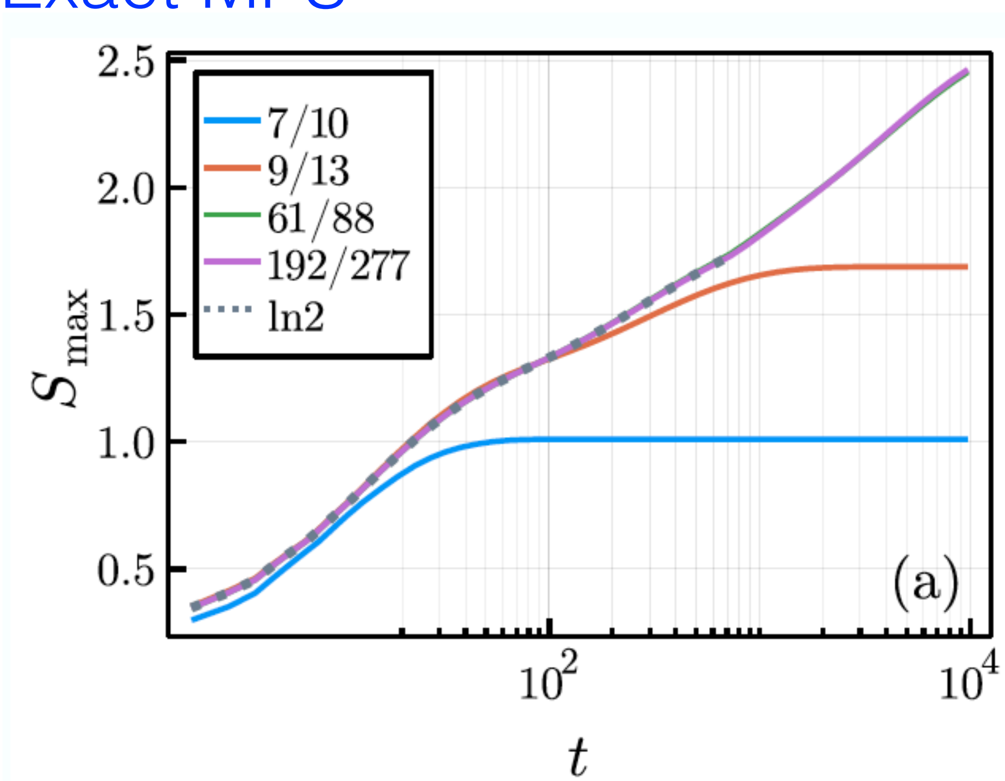
Floquet operator (integrable by Bethe ansatz)

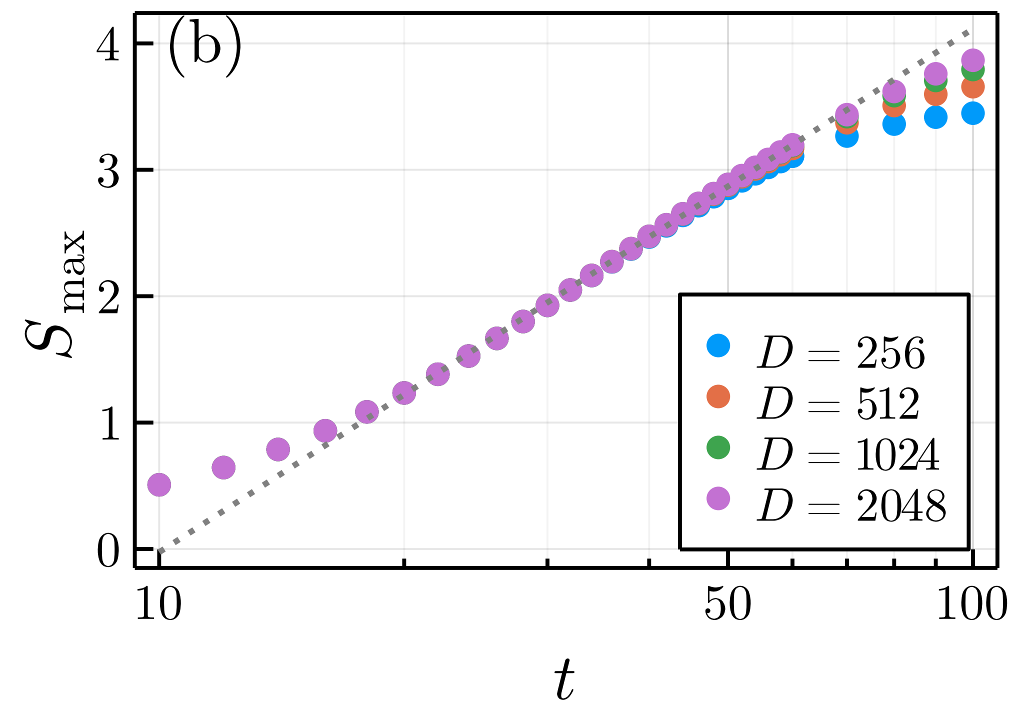


$$F = e^{iJ(X_i X_{i+1} + Y_i Y_{i+1}) + iJ' Z_i Z_{i+1}}$$

New exact solutions $\quad J = \pi/4, J' = \pi/4 + K$

Exact MPS





Product initial state $\quad \otimes_j |+\rangle_j$

Infinite-T initial state

Temporal entanglement scaling $\quad S_{\mathrm{TE}} \propto \log t$

Ballistic quasiparticles $\longrightarrow$ *low temporal entanglement*

Thermalisation and transport

Temporal entanglement in thermalising systems

Low temporal entanglement — efficient algorithms for broken integrability

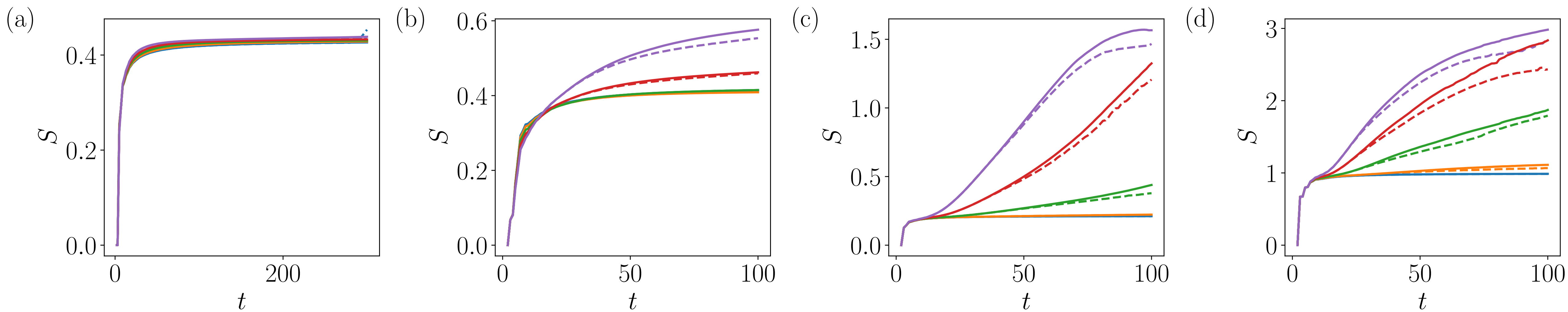
$$\hat{U}_{\mathrm{F}} = e^{iJ\sum_i Z_i Z_{i+1}} e^{ih\sum_i Z_i} e^{ig\sum_i X_i}$$

MPS algorithms for IM Lerose, Sonner, DA, arXiv:2103.13741, 2201.04150; Frias-Perez, Banuls, arXiv:2201.08402

Different curves: $h = 0.02...0.18$

——— $\chi = 128$

- - - - - $\chi = 64$



$J = 0.31, g = \pi/4$ $J = 0.31, g = 0.5$ $J = 0.31, g = 0.31$ $J = \pi/4, g = 0.31$

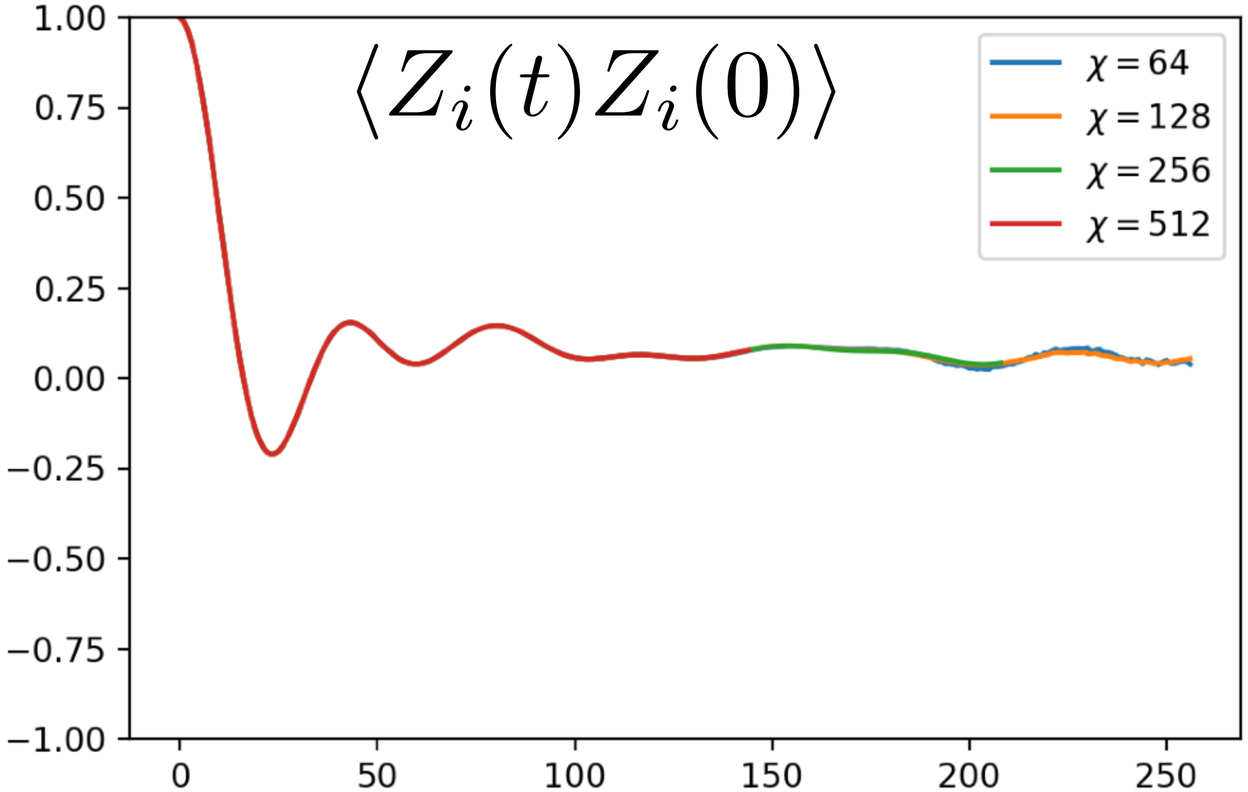
Intuitively, fast thermalisation $\longrightarrow$ short memory time of the bath

Thermalisation and transport via influence matrix

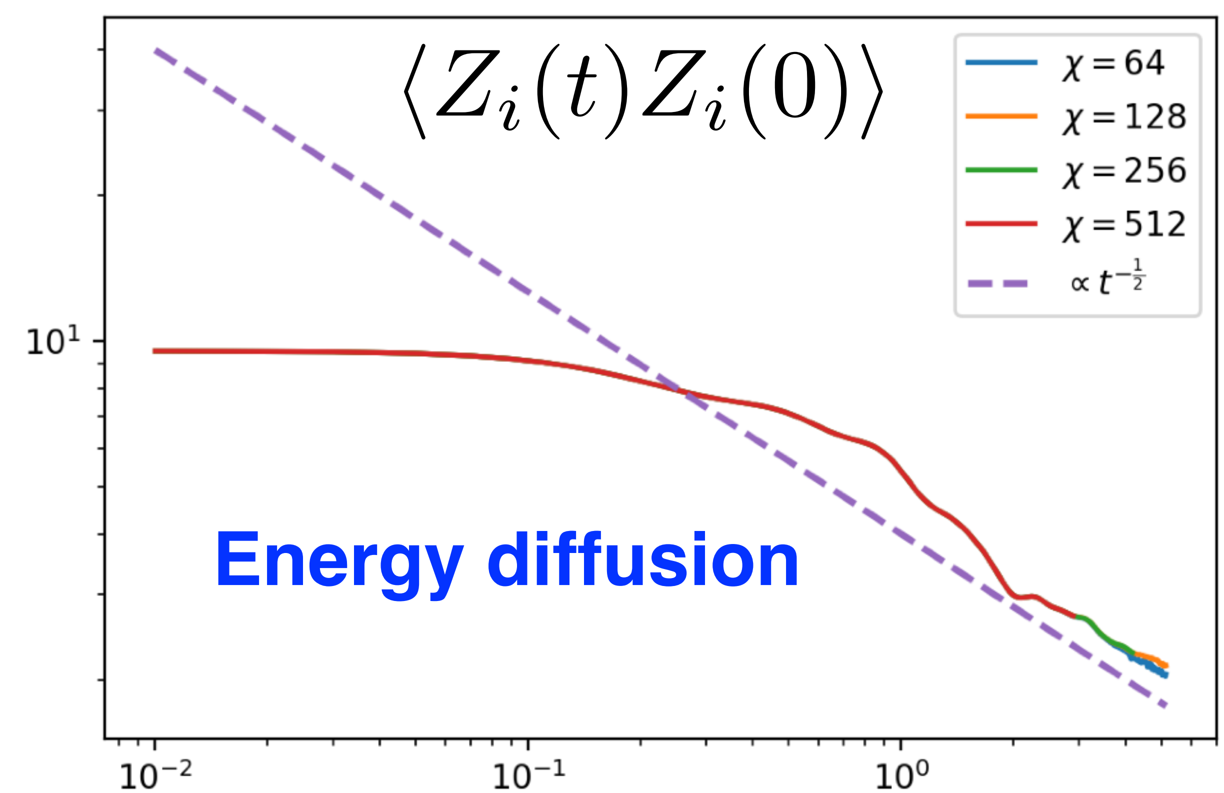
Energy transport in a thermalising model

$$H = \sum (gX_i + hZ_i + JZ_iZ_{i+1})$$

$$(g, h, J) = (0.9045, 0.8090, 1)$$ Kim, Ikeda, Huse'14



Number of cycles $N = t/\Delta t$



Evolution time

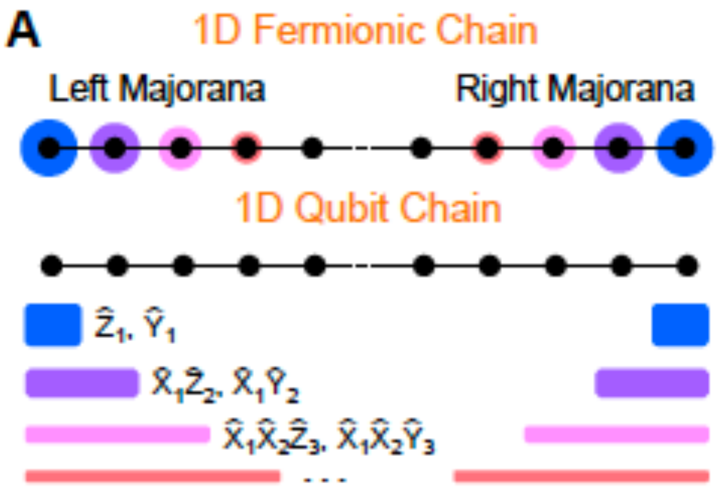
Sonner et al., in preparation

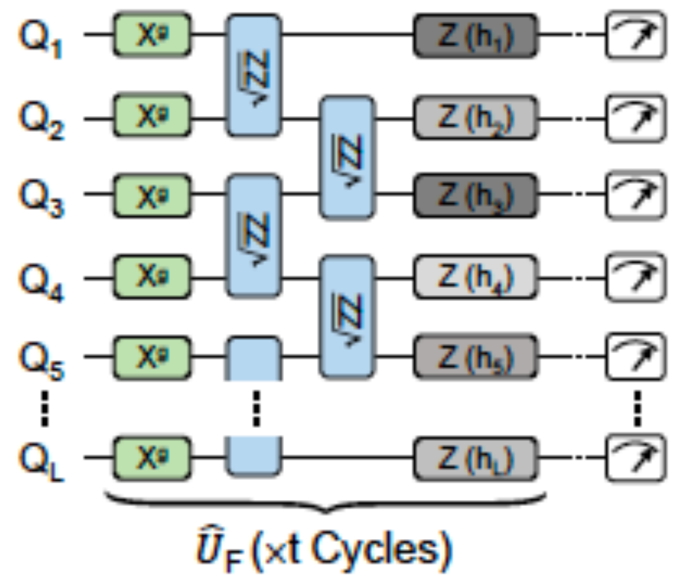
Modeling quantum circuits with dissipation:
Application in a recent Google experiment

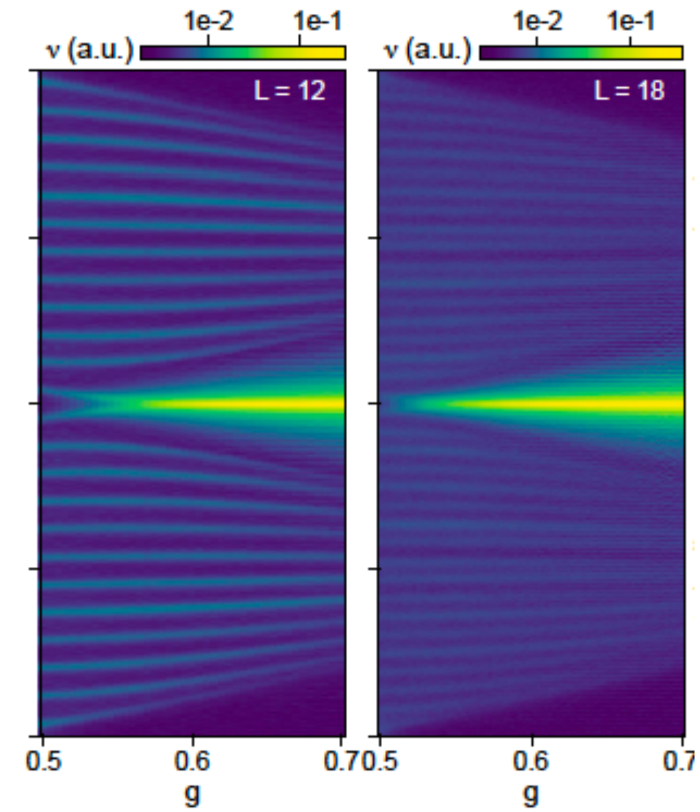
Observation of Majorana edge modes with superconducting qubits

Xiao Mi, M. Sonner et al., Science'22

Experimental quantum circuit





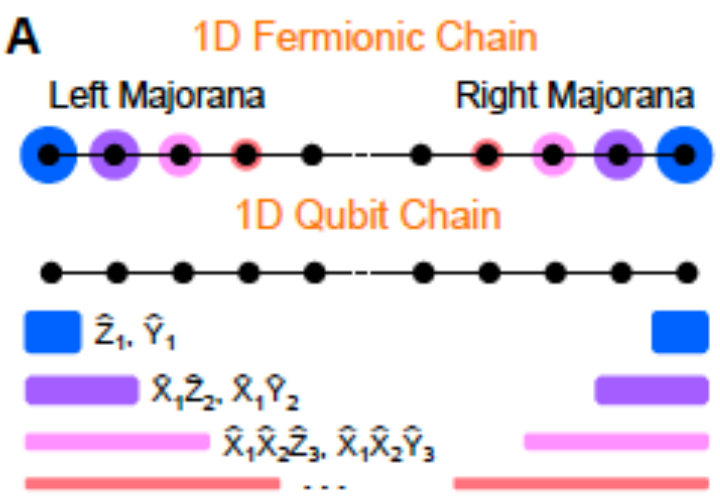


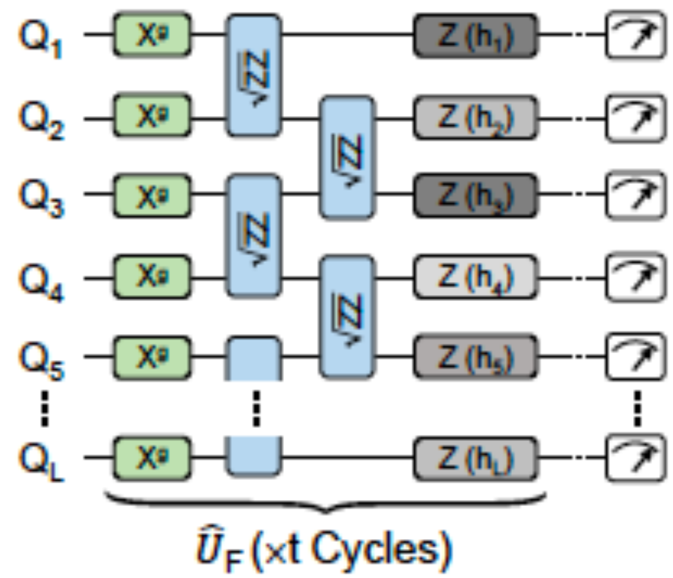
Long-lived edge correlators signal robust edge modes

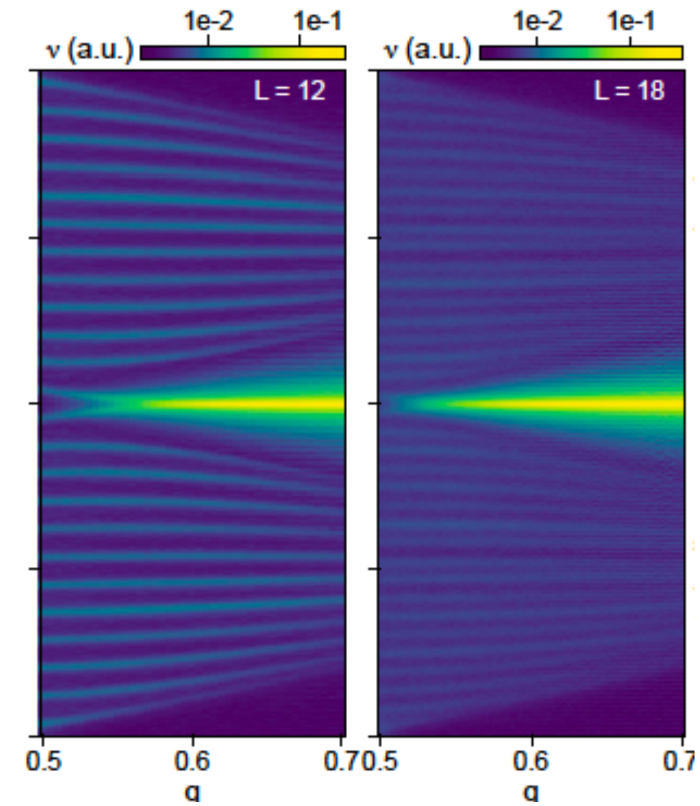
Observation of Majorana edge modes with superconducting qubits

Xiao Mi, M. Sonner et al., Science'22

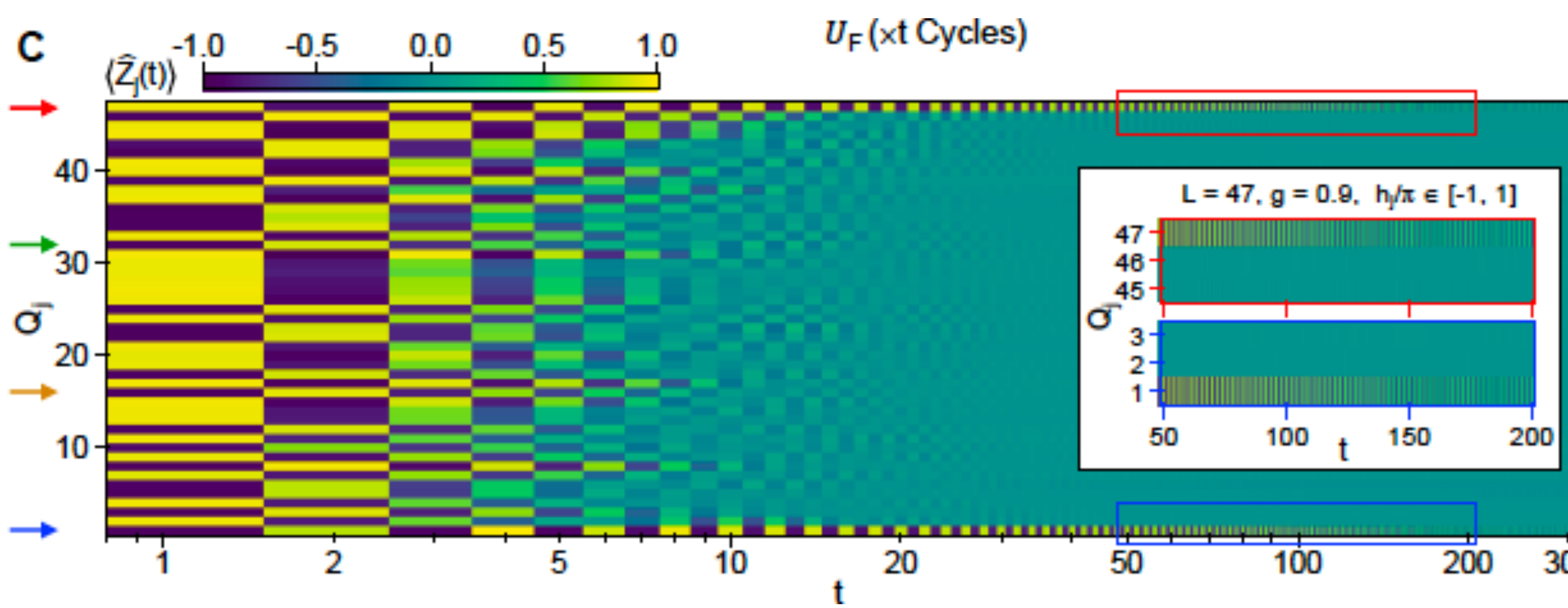
Experimental quantum circuit

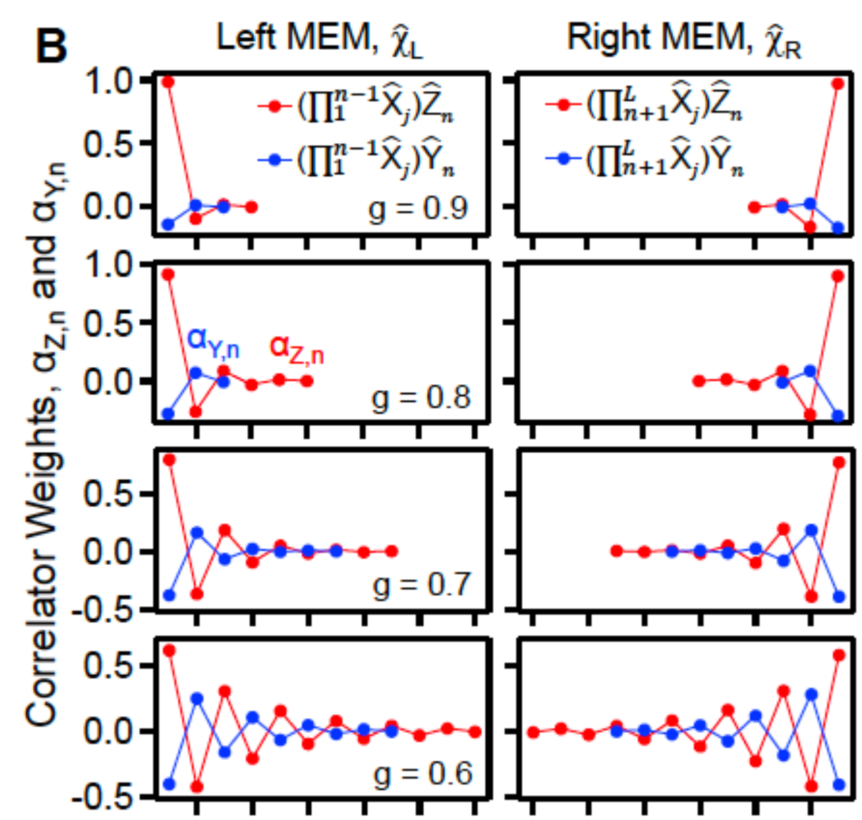






Long-lived edge correlators signal robust edge modes



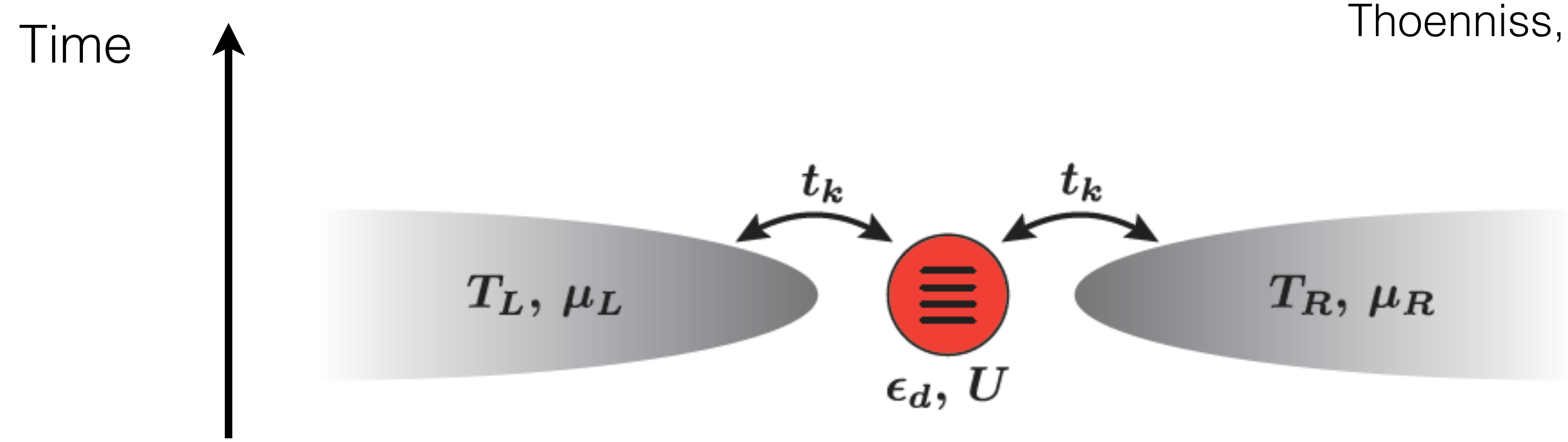


Influence matrix to describe edge modes with weak dissipation (ask for more)

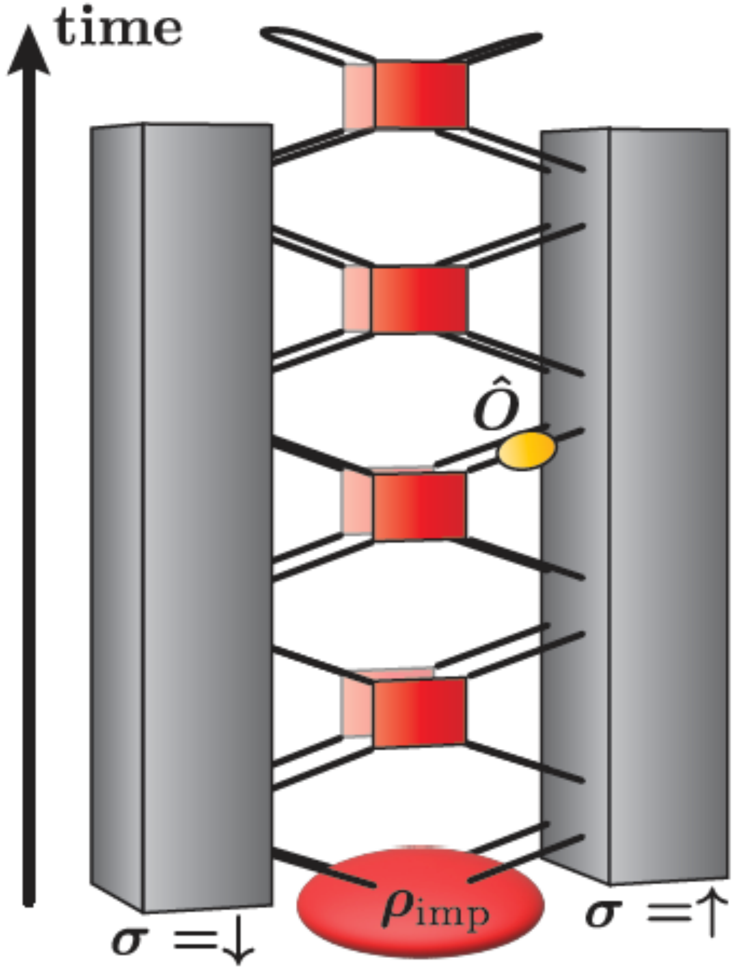
Efficient method for quantum impurity problems

Non-equilibrium quantum impurity problems

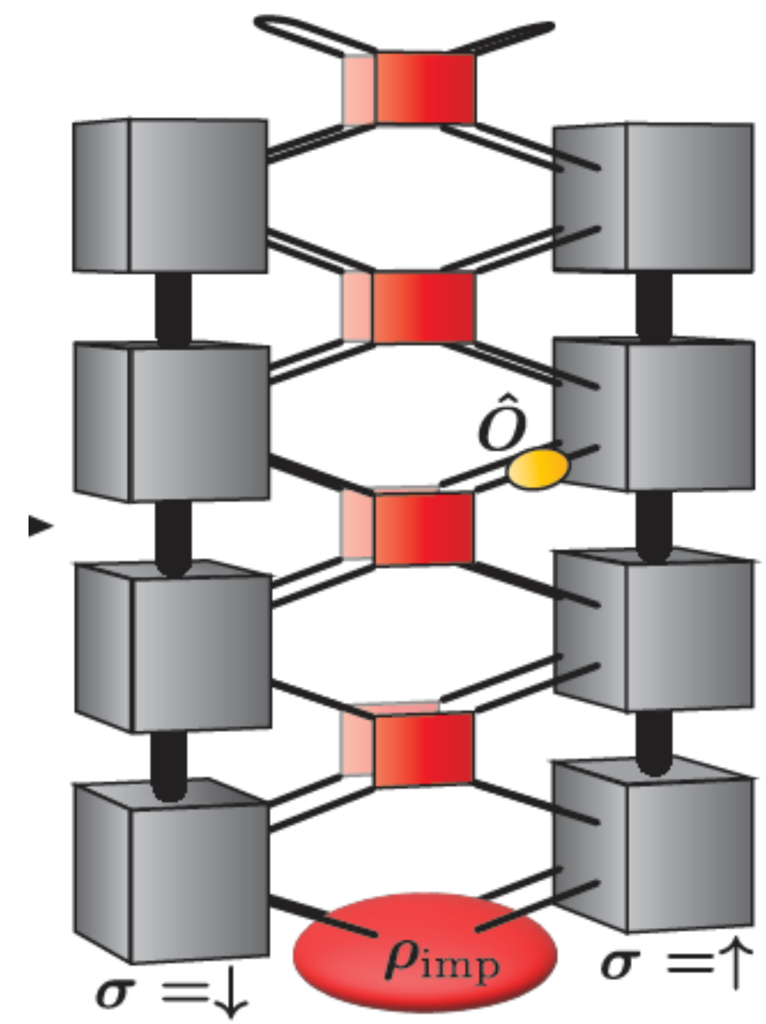
Thoenniss, Lerose, DA,arXiv:2205.04995



Grassman path integral



Extend Fishman-White algorithm (2015) to convert Gaussian IM to MPS



Quench in an Anderson impurity model

Thoenniss, Sonner, Lerose, DA,arXiv:2205.04995+2211.10272

For $T = 0$ Fermi-sea initial states

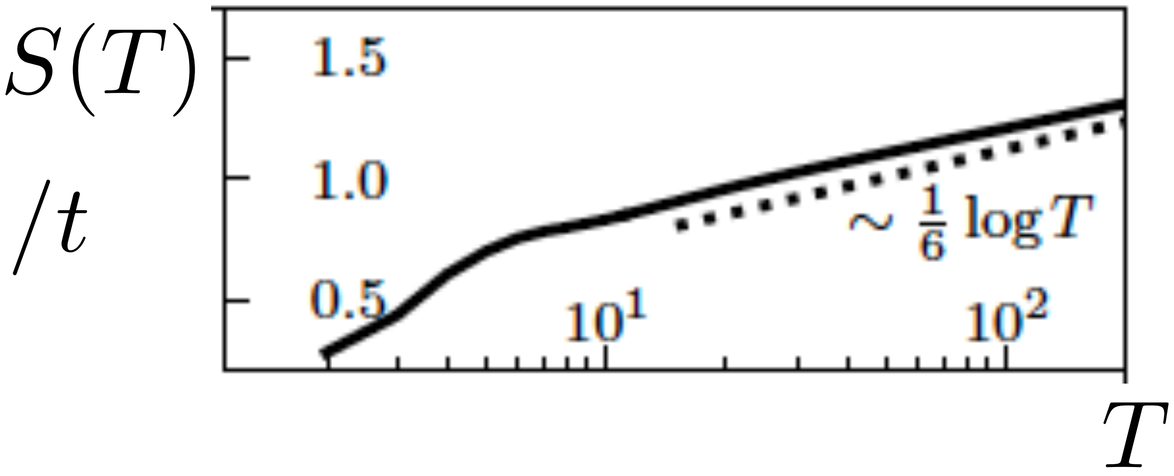
$$S_{\mathrm{TE}}(T) \sim \log t$$

Temporal correlations of influence matrix $\sim 1/t$

For finite-temperature initial states

$$S_{\mathrm{TE}}(T) \sim C$$

1d nearest-neighbour fermions,
Fermi sea state



(Most) non-equilibrium impurity problems can be efficiently solved

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Thoenniss, Sonner, Lerose, DA,arXiv:2205.04995+2211.10272

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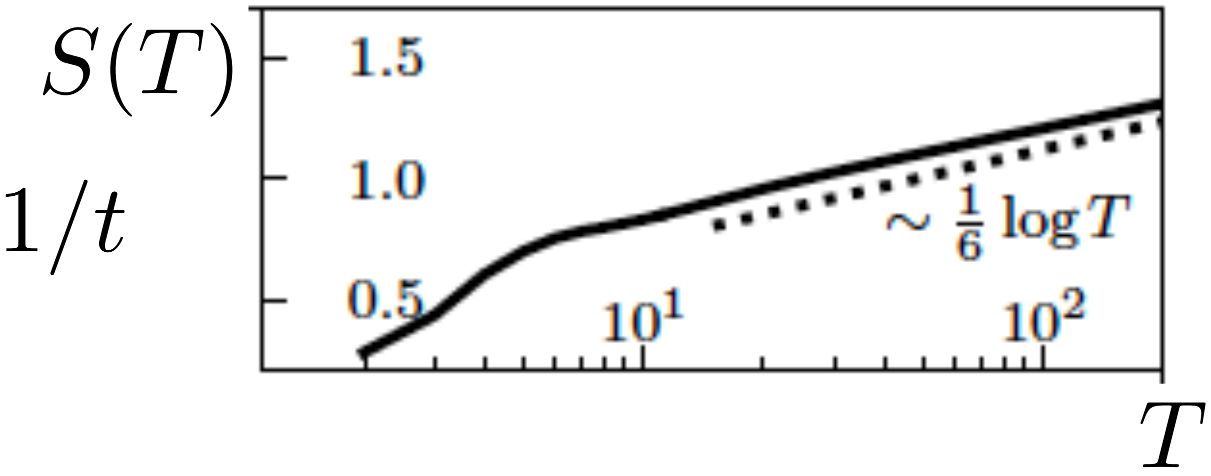


Temporal correlations of influence matrix $\sim 1/t$

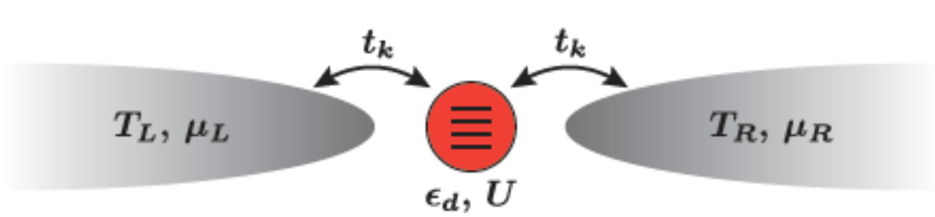
For finite-temperature initial states

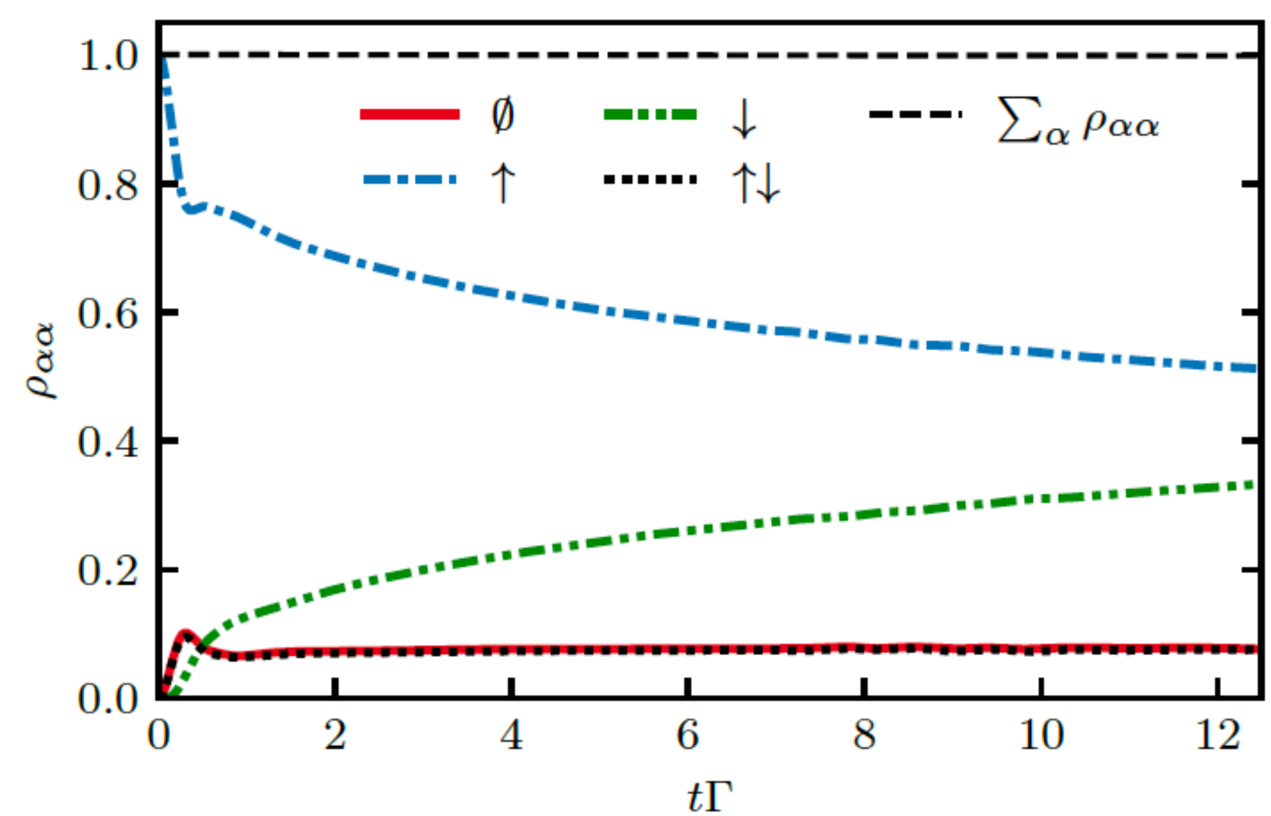
$$S_{\mathrm{TE}}(T) \sim C$$

(Most) non-equilibrium impurity problems can be efficiently solved

Benchmark: quench in Anderson model

Slow dynamics — Kondo singlet formation



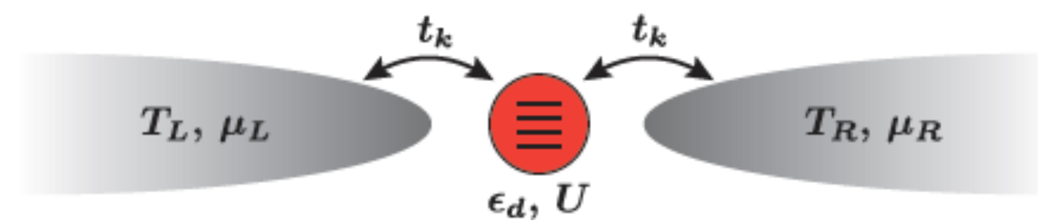


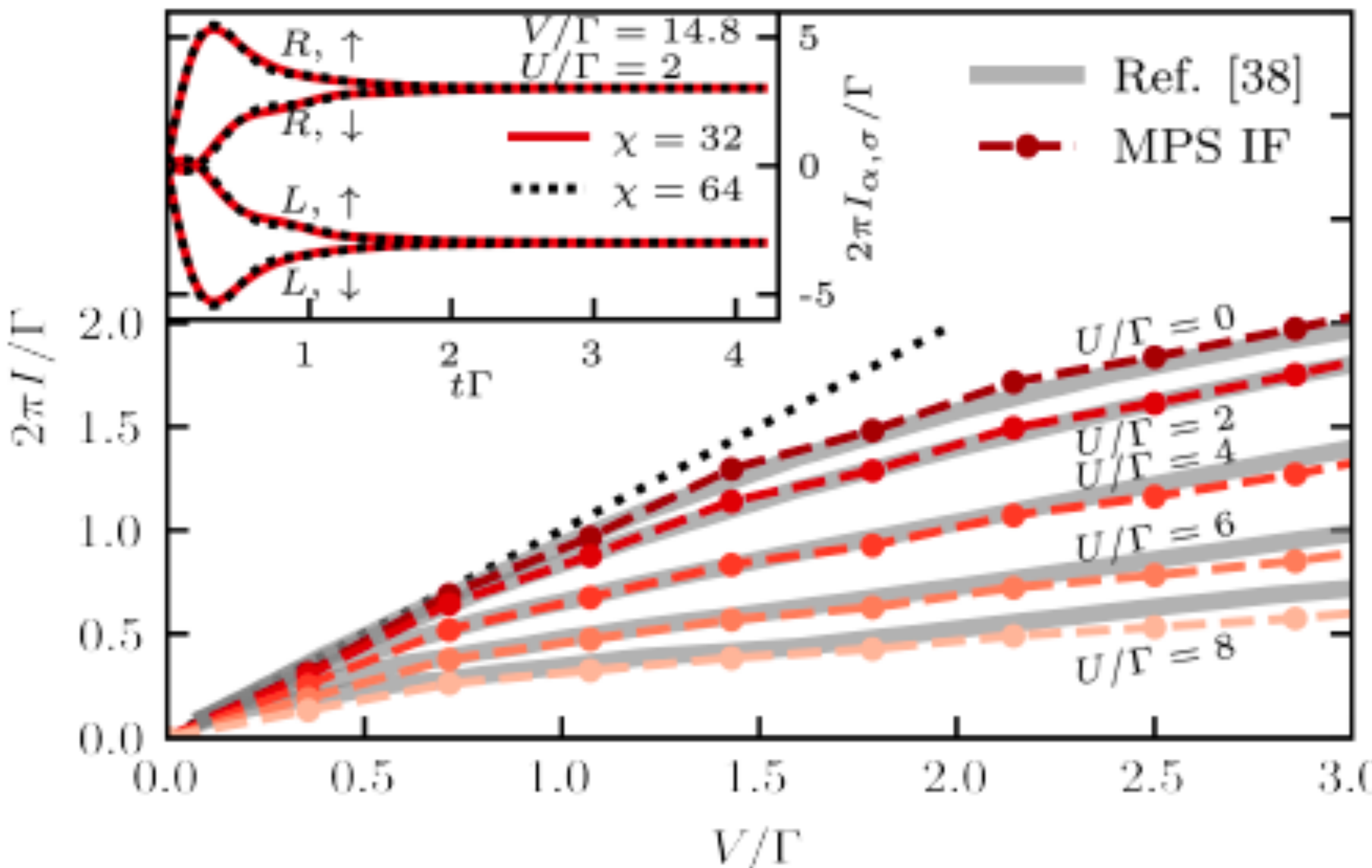
Current in an Anderson impurity model

Steady state current at large voltage bias

Approach to steady state

Low bond dimension sufficient





Next: modeling correlated materials by non-equilibrium DMFT

Disorder-averaged influence matrix and many-body localization

Exact disorder averaging

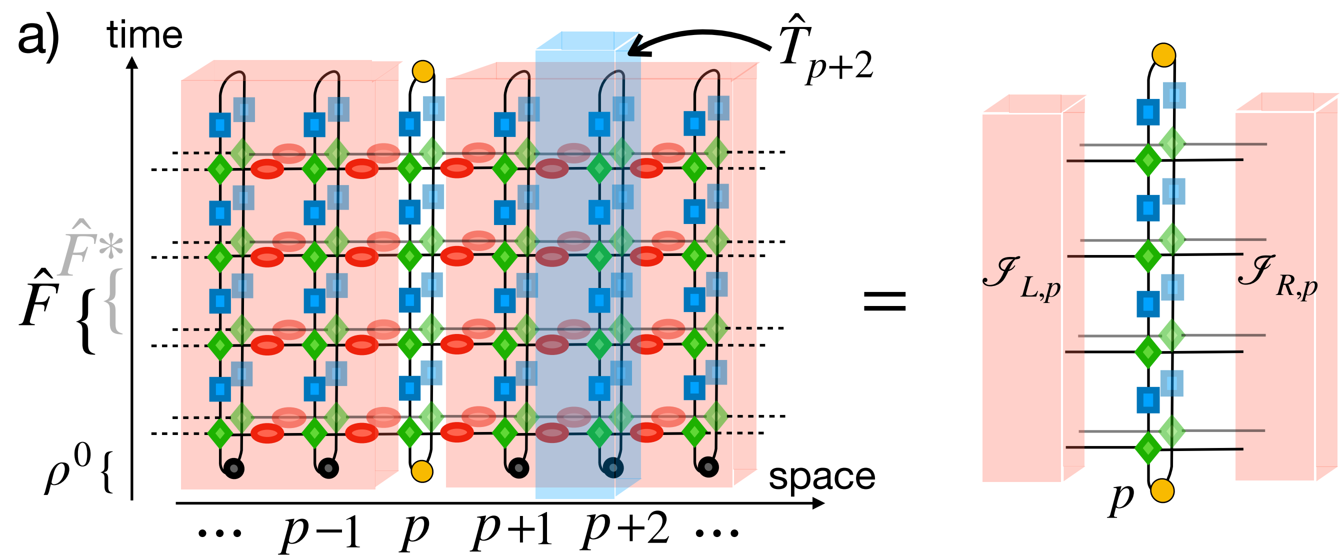
arXiv:2012.00777, PRB'21

Disorder-averaged transfer matrix:
couples forward&backward trajectory

$$e^{ih_j(s_j^\tau - \bar{s}_j^\tau)}$$

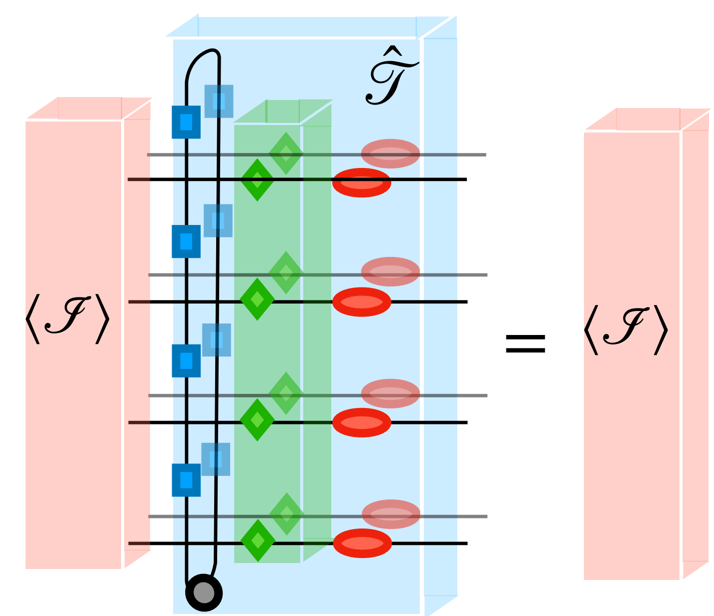
$$h_j \in [0; 2\pi)$$

$$\delta\left(\sum_{\tau=0}^{t-1}(s^\tau - \bar{s}^\tau)\right) \equiv \delta_{M,\overline{M}}$$



Self-consistency equation for
disorder-averaged influence matrix

$$\hat{\mathcal{T}}\langle\mathscr{I}\rangle = \langle\mathscr{I}\rangle$$



nonlocal constraint

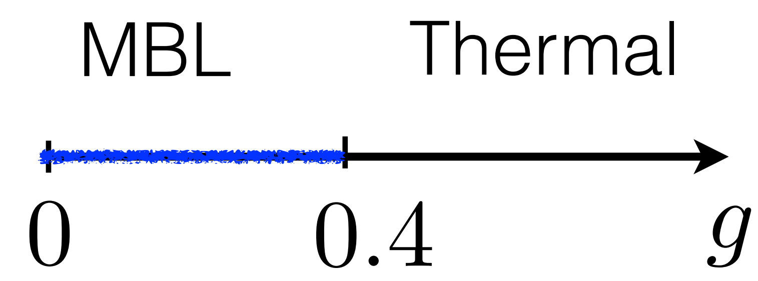
Includes all rare-region effects

Temporal entanglement in the MBL phase

arXiv:2012.00777, PRB'21

Phase diagram

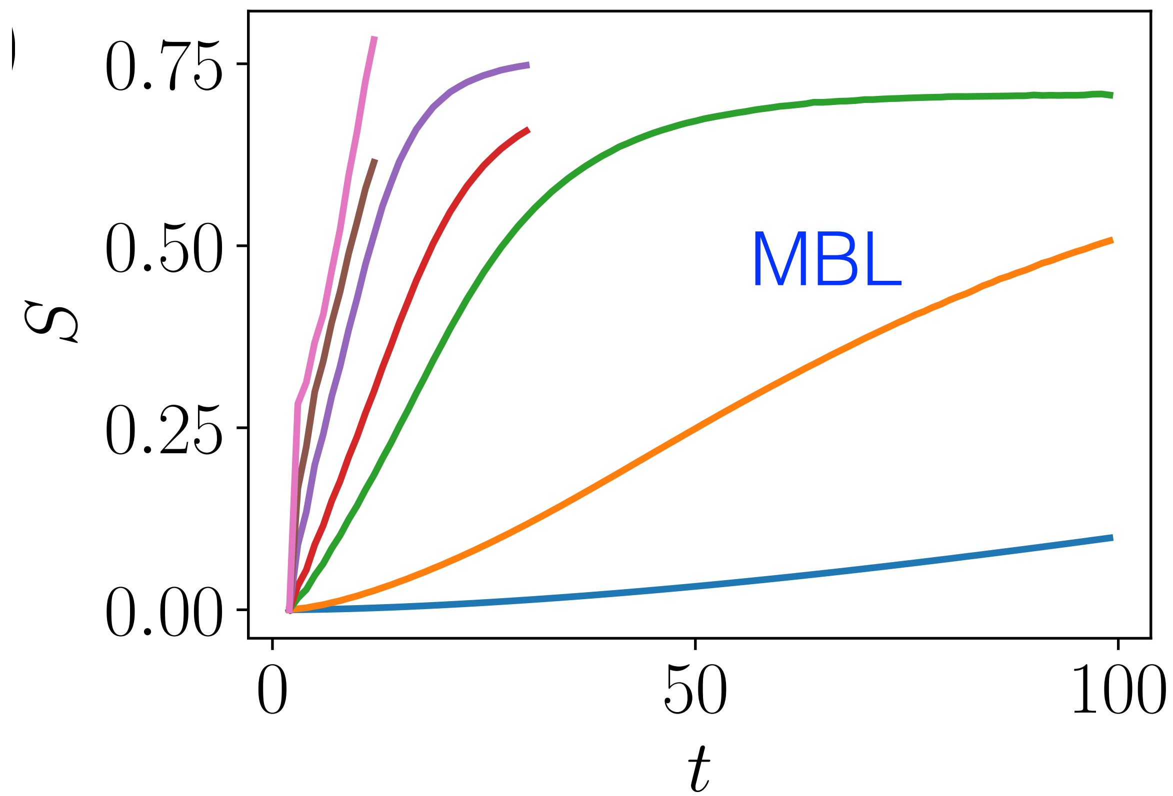
$$\hat{F} = \exp\left(i\sum_j g\hat{\sigma}_j^x\right)\exp\left(i\sum_j J\hat{\sigma}_j^z\hat{\sigma}_{j+1}^z + h_j\hat{\sigma}_j^z\right)$$



Temporal ent. (MPS+exact diag.)

Strongly sub-linear growth

contrast to ergodic phase (linear growth)

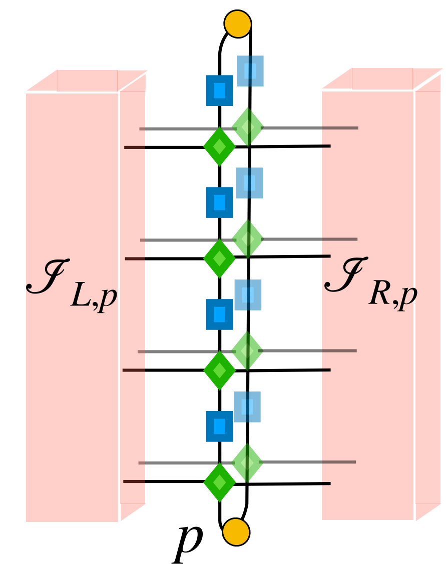


Note: ent. of an unfolded influence matrix also low

Disorder-averaged correlation functions

arXiv:2012.00777, PRB'21

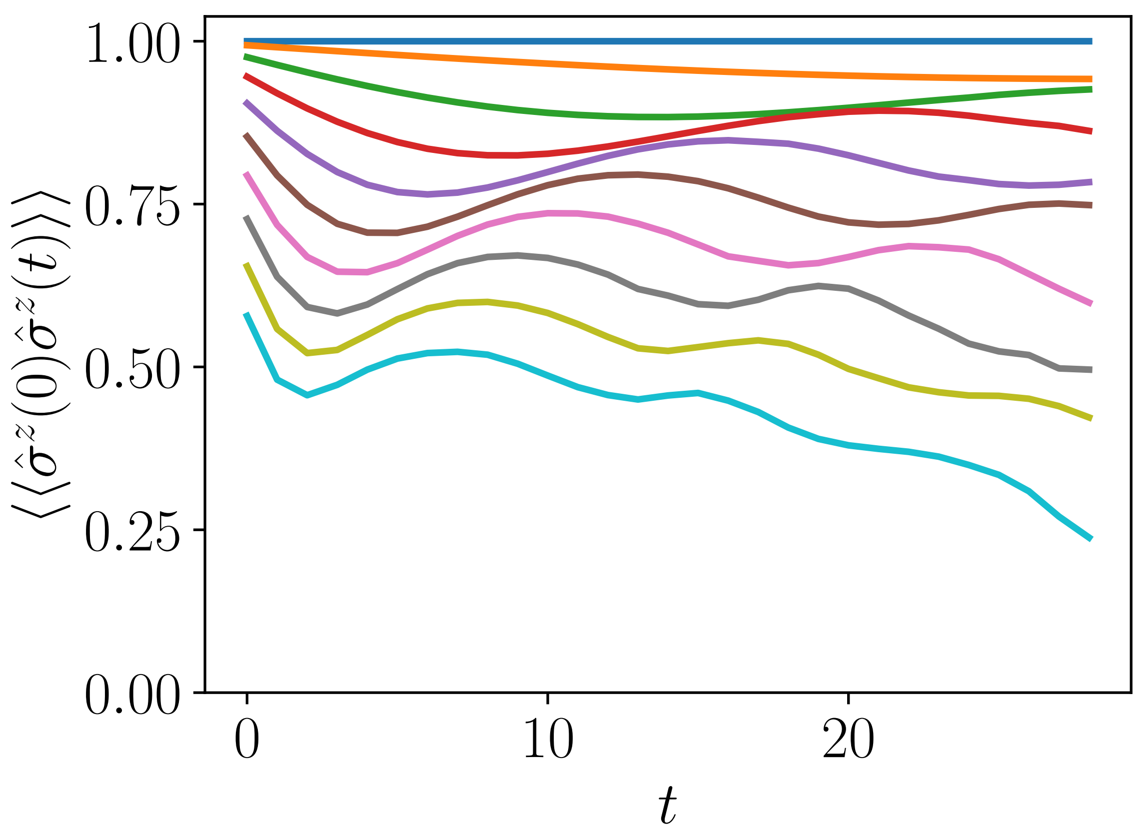
Influence matrix —> all temporal correlators



Magnetisation decay

Access to longer times in MBL phase

Disorder averaging accounted for exactly



Structure of the non-ergodic influence matrix

arXiv:2012.00777, PRB'21

Blip interactions are non-local, non-decaying (interfence persists)
[vs weakly interacting blip gas in ergodic phase]

Influence matrix linked to temporal long-range order

$$\langle\langle Z \rangle\rangle = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=1}^{t} \langle\langle \sigma_j^z(t) \sigma_j^z(0) \rangle\rangle$$

$$\langle\langle Z \rangle\rangle = \lim_{t\to\infty} \frac{1}{2t^2} \sum_{\tau,\tau'=0}^{t} \sum_{\{\sigma^s\},\{\bar\sigma^s\}} \sigma^\tau \sigma^{\tau'} \times \delta\Big(\sum_s (\sigma^s - \bar\sigma^s)\Big) W_{\sigma^{s+1},\sigma^s} W^*_{\bar\sigma^{s+1},\bar\sigma^s} \langle \mathscr{I} \rangle^2 [\{\sigma, \bar\sigma\}],$$

$$\langle\langle Z \rangle\rangle = \lim_{t\to\infty} \sum_{M=-t}^{t} (M/t)^2 P(M) = \int_{-1}^{1} dm\, m^2 p(m)$$

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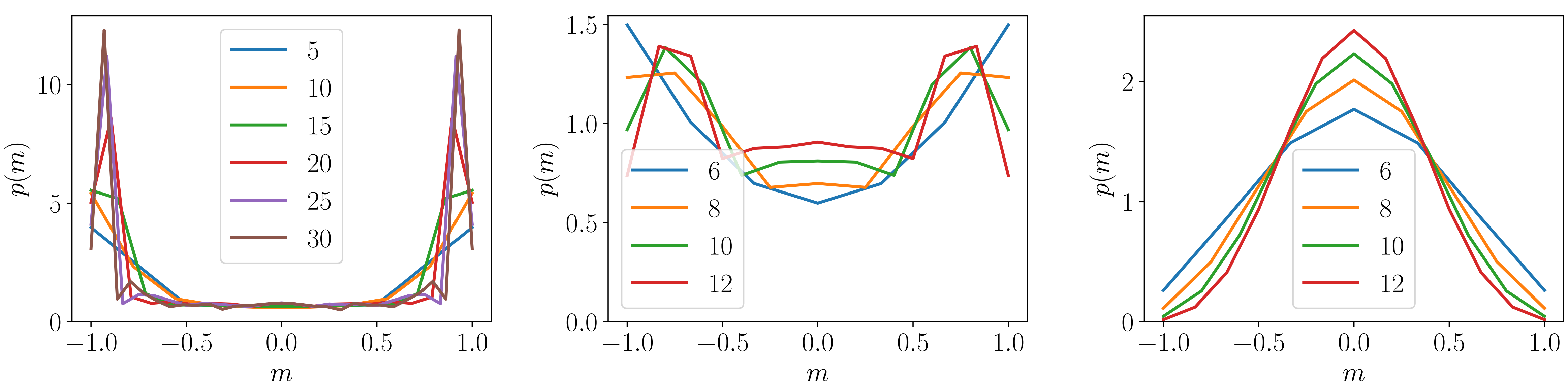
$P(M)$ Contribution of sector with “magnetisation” *M*



MBL Transition Ergodic

Summary

Influence matrix: many-body system as a bath

Low temporal entanglement $\longrightarrow$ efficient tensor-network algorithms

-Generally, lower complexity than full wave function

Applications: dynamics, transport, impurity models, noisy circuits, q-control,…

Versatile approach for quantum dynamics, simulation & control

Further steps/questions

Theory:

-Scaling of $S_{\mathrm{temp}}(t)$ for thermalising dynamics?

-Structure of IM in the MBL phase?

-Approximate truncation schemes for 2d

-Renormalization on the influence functional? Route to universality classes?

Experiments:

-Measure IM via multi-time correlation functions?

-Construct effective representation of (noisy) environment $\longrightarrow$ quantum control

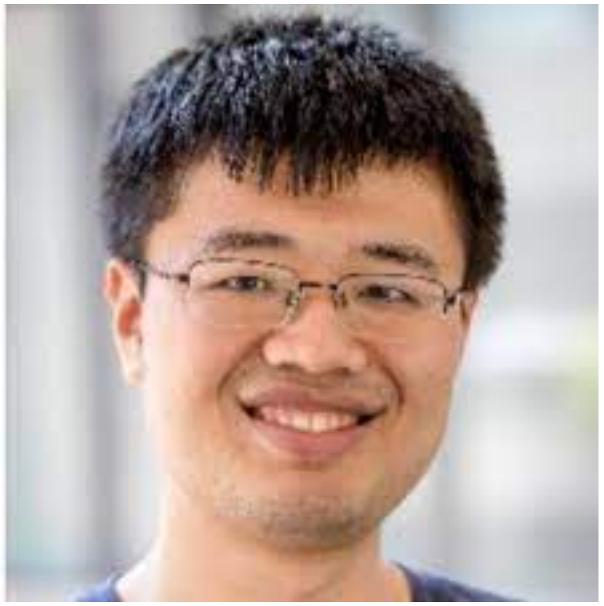
-Realistic models of noise, dissipation

Acknowledgements

Alessio Lerose

Michael Sonner

Julian Thoenniss



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