The quantum Mpemba effect and entanglement asymmetry

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SISSA-Trieste

Rome, January 2023

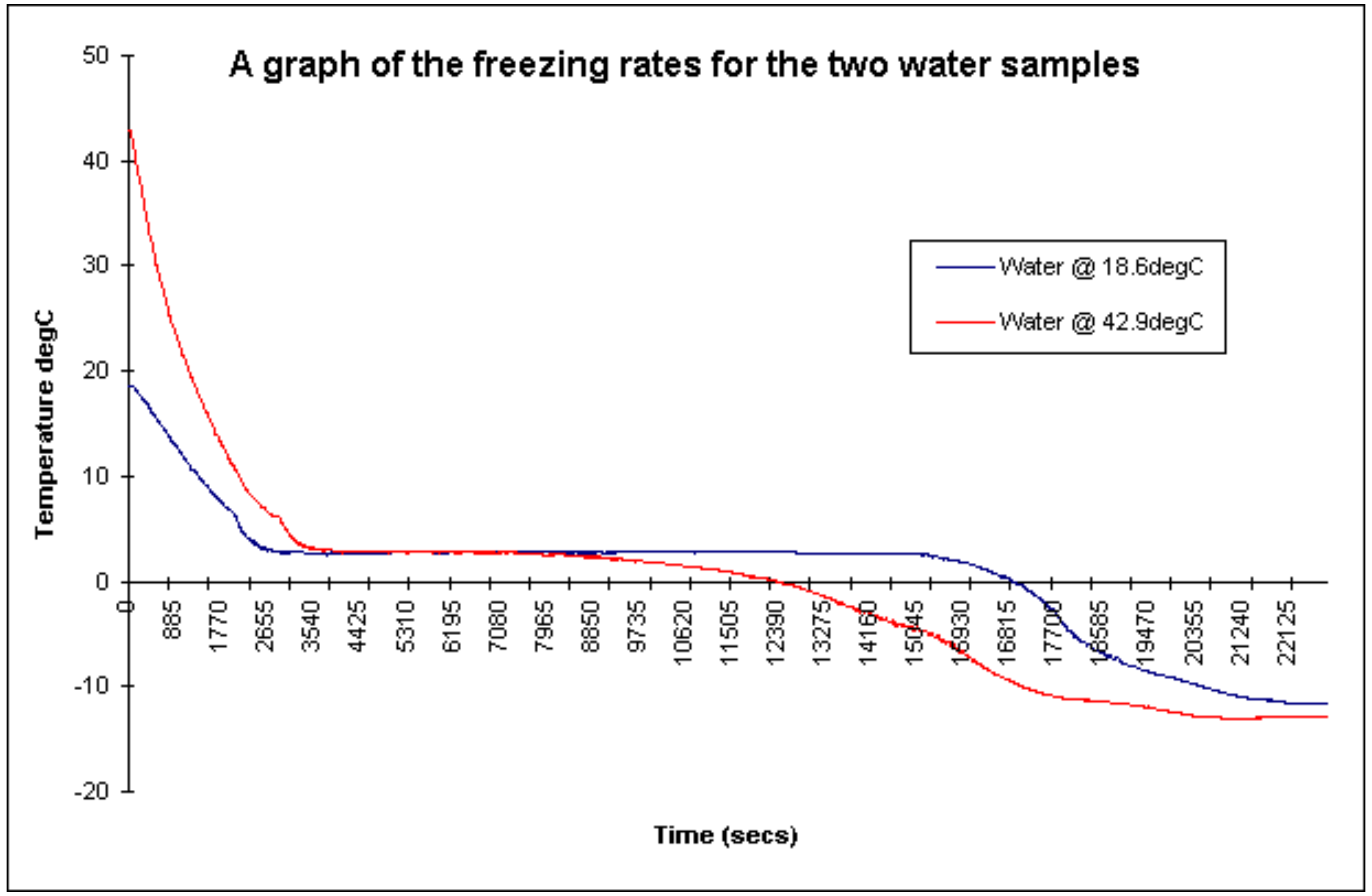
Joint work with Filiberto Ares and Sara Murciano

(Classical) Mpemba effect

(Classical) Mpemba effect

Aristotle wrote "to cool hot water quickly, begin by putting it in the sun"

Rediscovered (thanks to the ice-cream) by Mpemba (and Osborne) in the sixties



Source: picotech.com

(Classical) Mpemba effect

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PHYSICS + MATH

The ice cream that changed physics

Sixty years ago a teenager's homemade ice cream raised a surprisingly complicated question: Can hot liquids freeze faster than cold ones?

CONDENSED MATTER PHYSICS

Controversy Continues Over Whether Hot Water Freezes Faster Than Cold

26

Decades after a Tanzanian teenager initiated study of the "Mpemba effect," the effort to confirm or refute it is leading physicists toward new theories about how substances relax to equilibrium.

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DOI:10.1063/PT.6.1.20220309a

9 Mar 2022 in Research & Technology

Mpemba effect runs in reverse

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The Mpemba Effect: Does Hot Water Really Freeze Faster Than Cold Water?

By: Dylan Ris | Jan 19, 2023

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SCIENZA | GIOVEDÌ 21 LUGLIO 2022

L'acqua calda congela prima di quella fredda?

È un fenomeno osservato da tempo e un rompicapo per i fisici, nacque tutto da un gelato in Tanzania

(Classical) Mpemba effect

It is noways clear that it is not related to freezing:
the general statement is "hot systems cool faster",
as shown in many different experiments and
theoretical calculations

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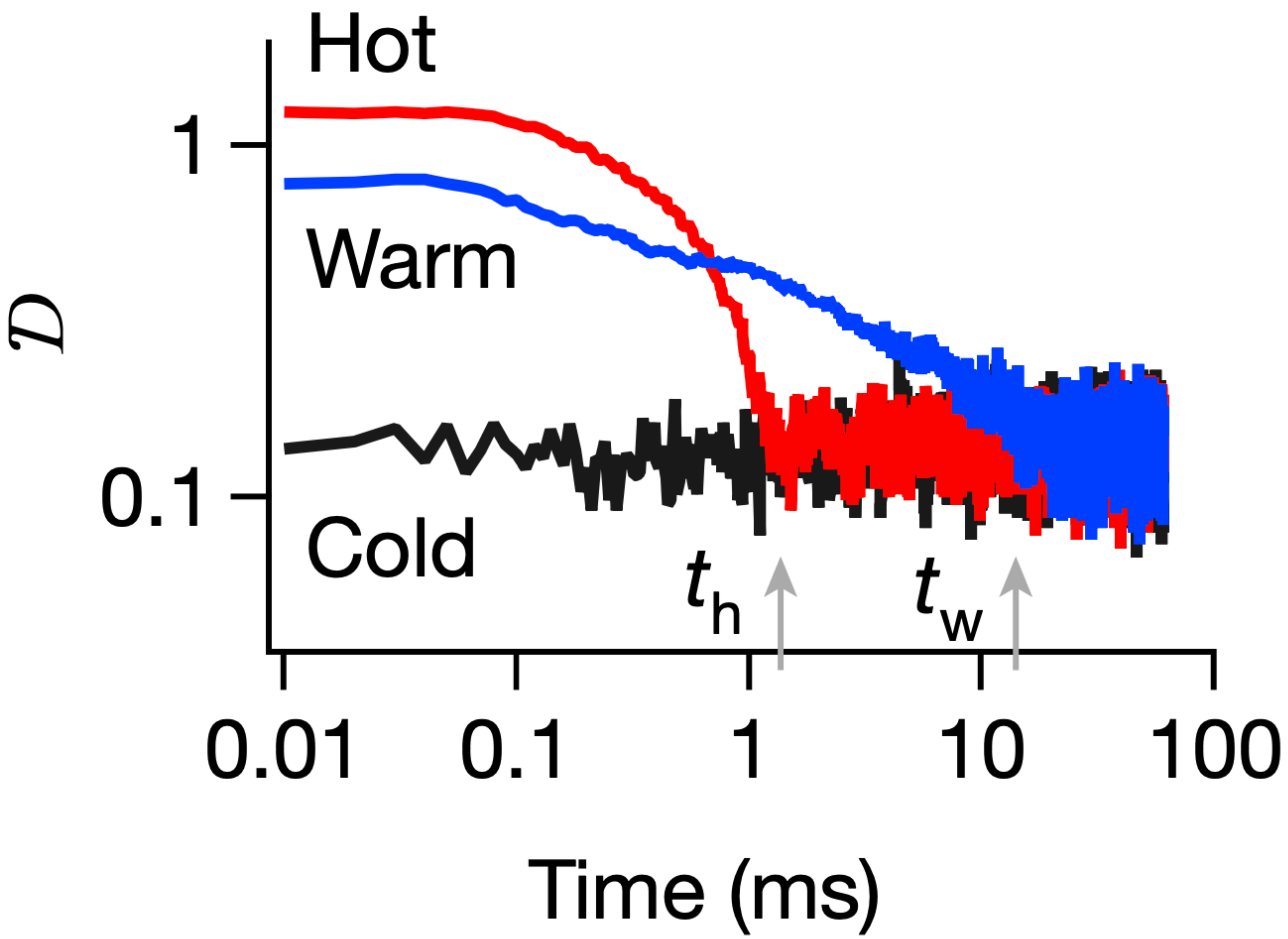
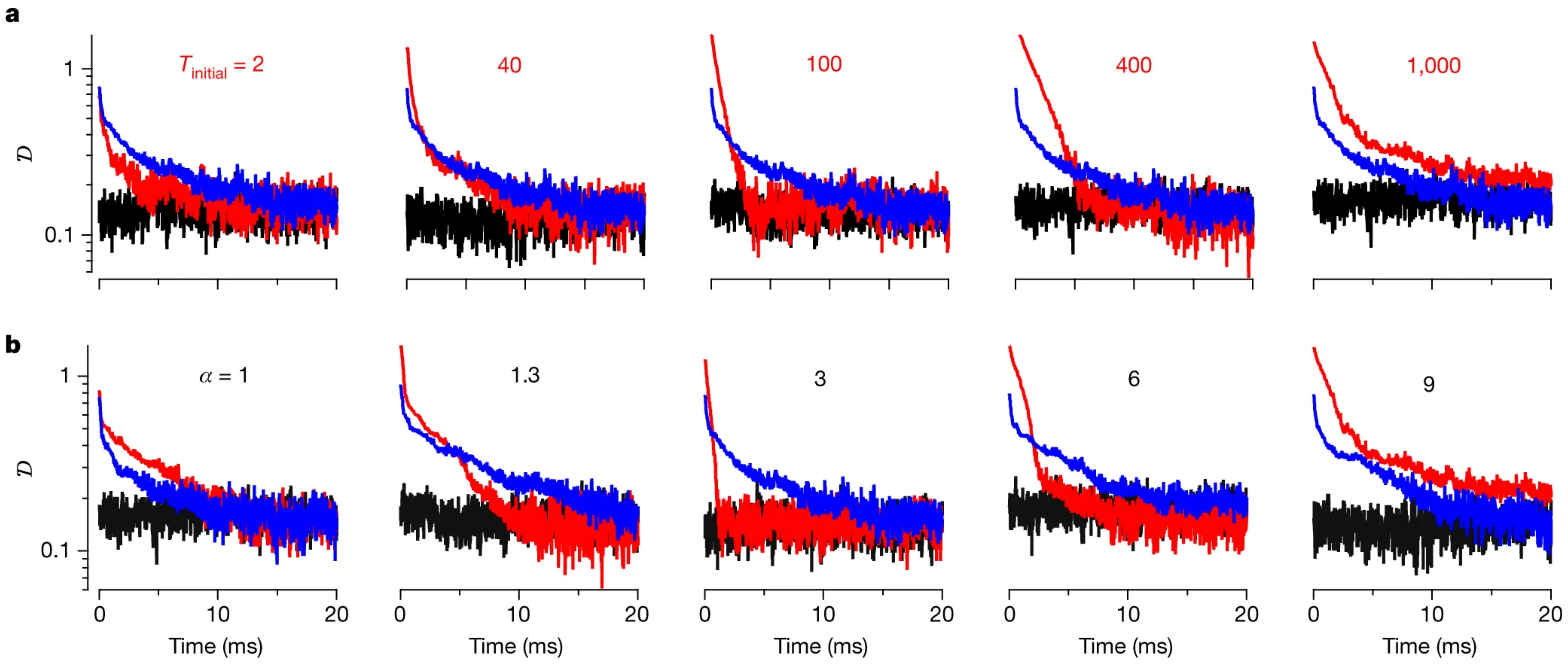


Colloidal systems provide a controlled setting in which the Mpemba effect can be quantitatively described with the help of an experimentally measurable "distance" from the equilibrium state

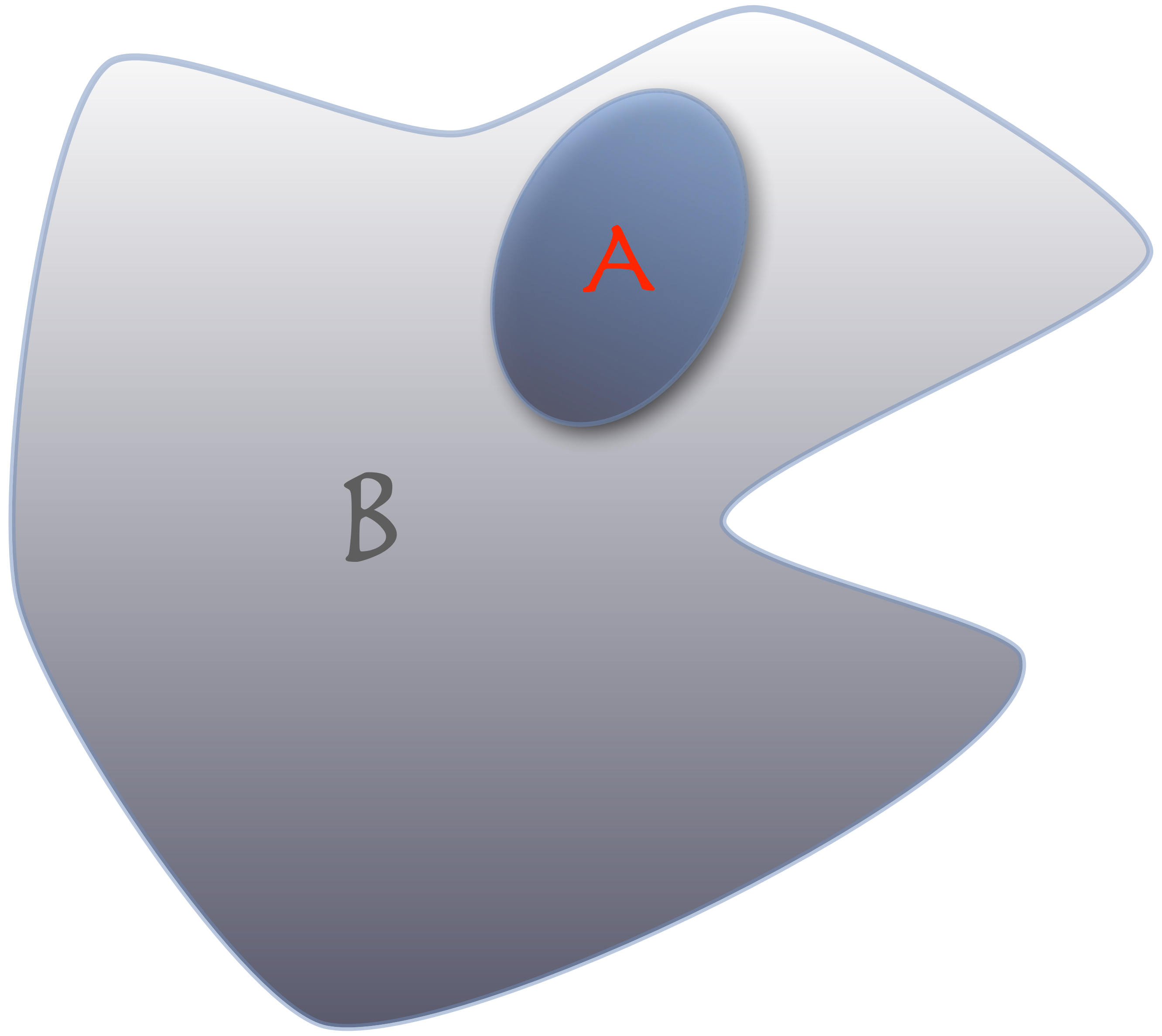
A. Kumar, J. Bechhoefer, Nature 584, 64 (2020)



A. Kumar, J. Bechhoefer, Nature 584, 64 (2020)

What about quantum systems?

We have in mind an **isolated** quantum system at zero temperature prepared in a non-equilibrium pure state (like in a quantum quench)



Such a system relaxes locally to a (generalized) Gibbs ensemble

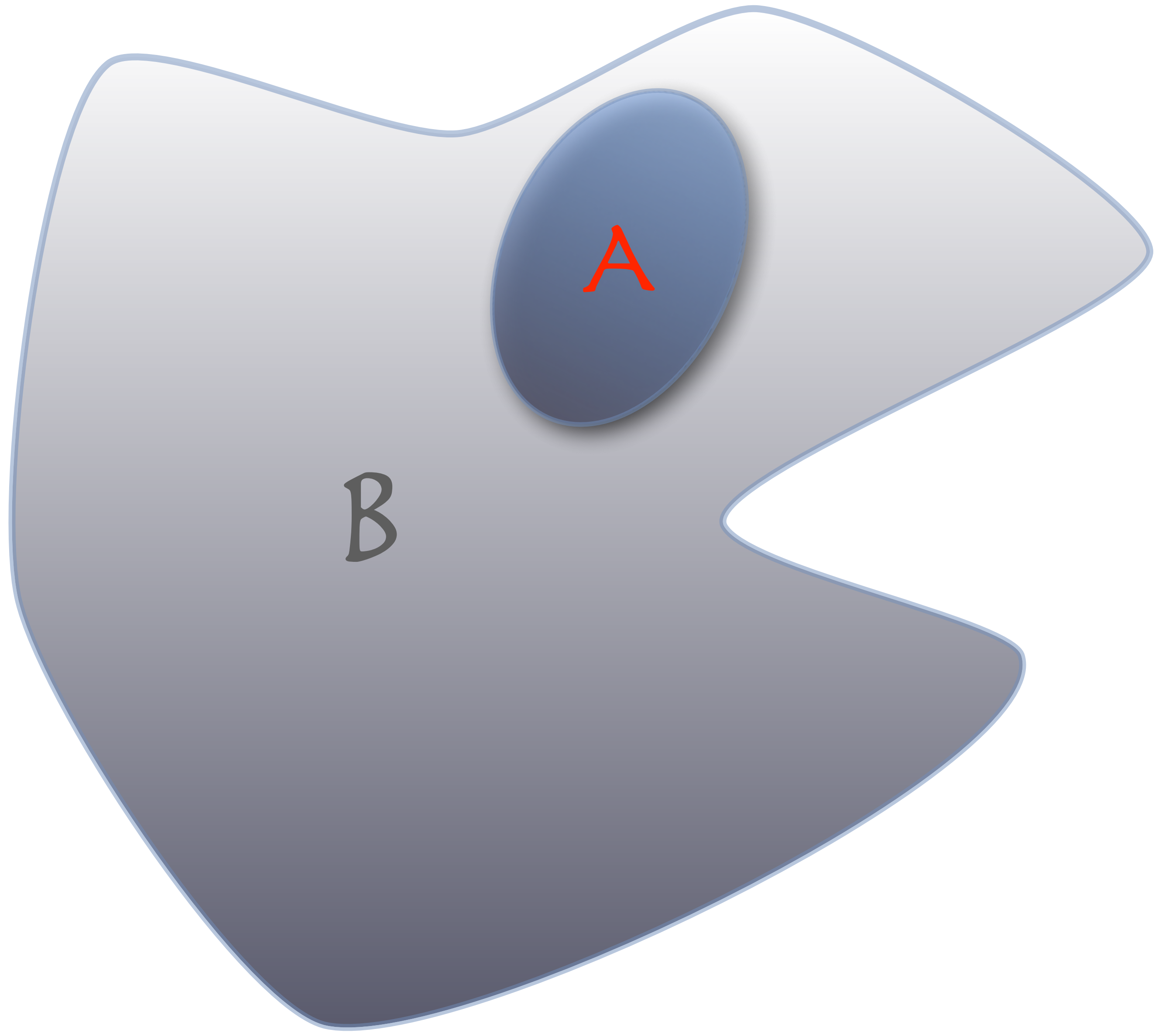
in the sense that the reduced density matrix $\rho_A(t)$ tends to the equilibrium one

$$\lim_{t\to\infty} \rho_A(t) \equiv \rho_A(\infty) = \mathrm{Tr}_B\left(\frac{e^{-\beta H}}{Z}\right)$$

➡ A "quantum Mpemba effect" should be defined at the level of subsystems

What about quantum systems? (II)

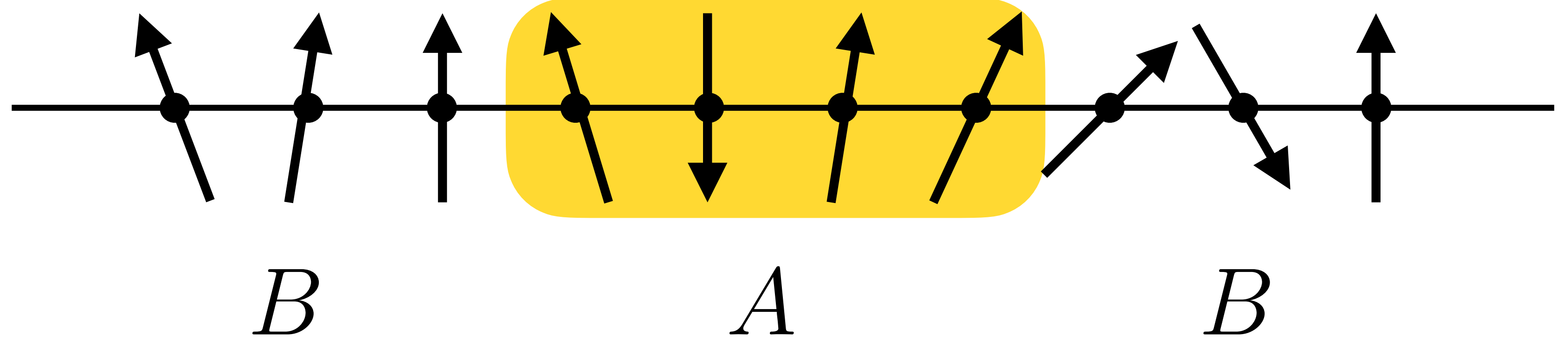
We focus here on initial states that break a symmetry of the Hamiltonian which is eventually restored by the time evolution (Mermin-Wagner type of argument)



We then want an (experimentally measurable) quantity that tells as how much a symmetry is broken at the level of an (arbitrary) subsystem

→ Entanglement asymmetry

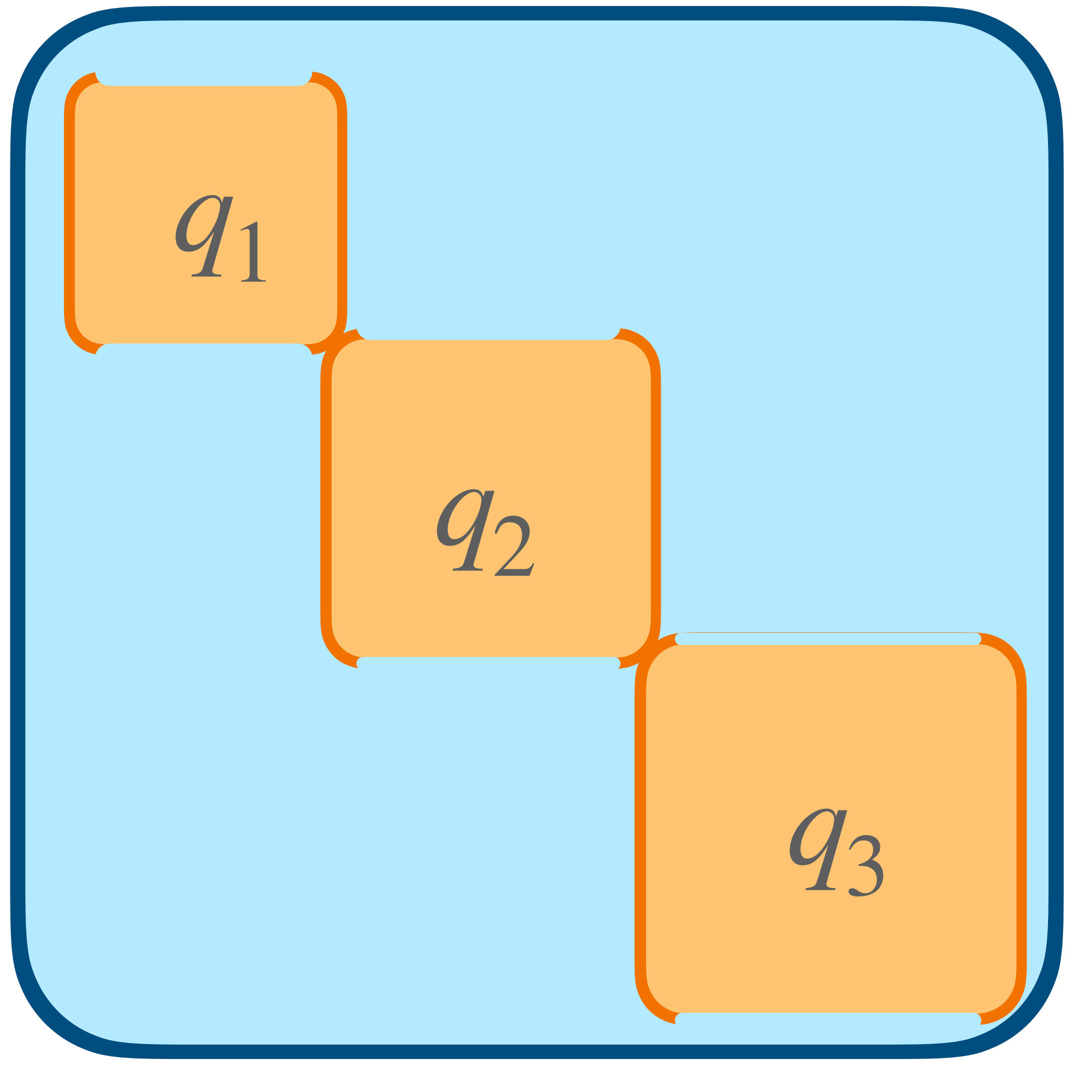
Basic setup

$|\Psi\rangle =$ 

$\rho = |\Psi\rangle \langle\Psi|$

Let Q be a local charge generating a *U(1)* symmetry $Q = Q_A + Q_B = \frac{1}{2}\sum_j \sigma_j^z$

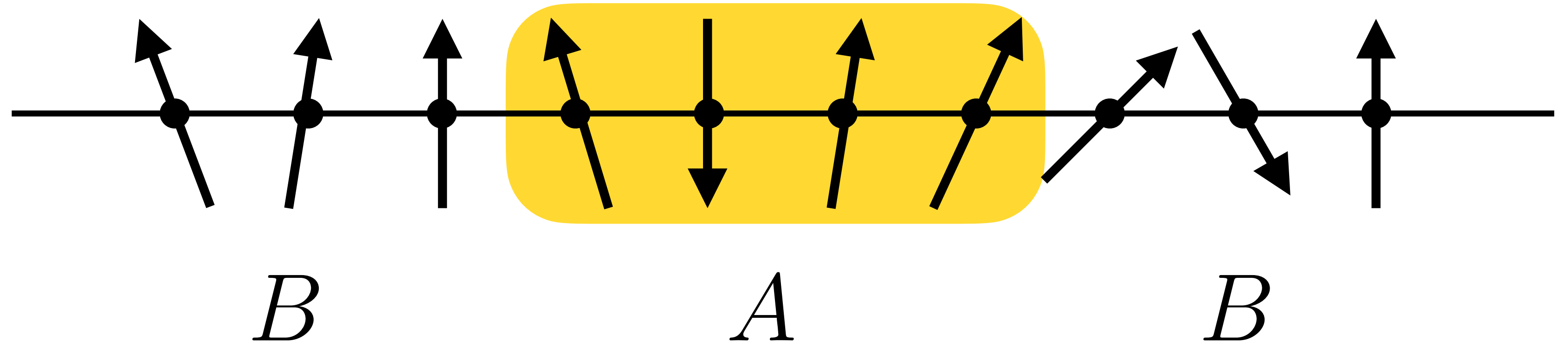
If the state is symmetric $[\rho, Q] = 0 \implies [\rho_A, Q_A] = 0$

$\rho_A =$ 

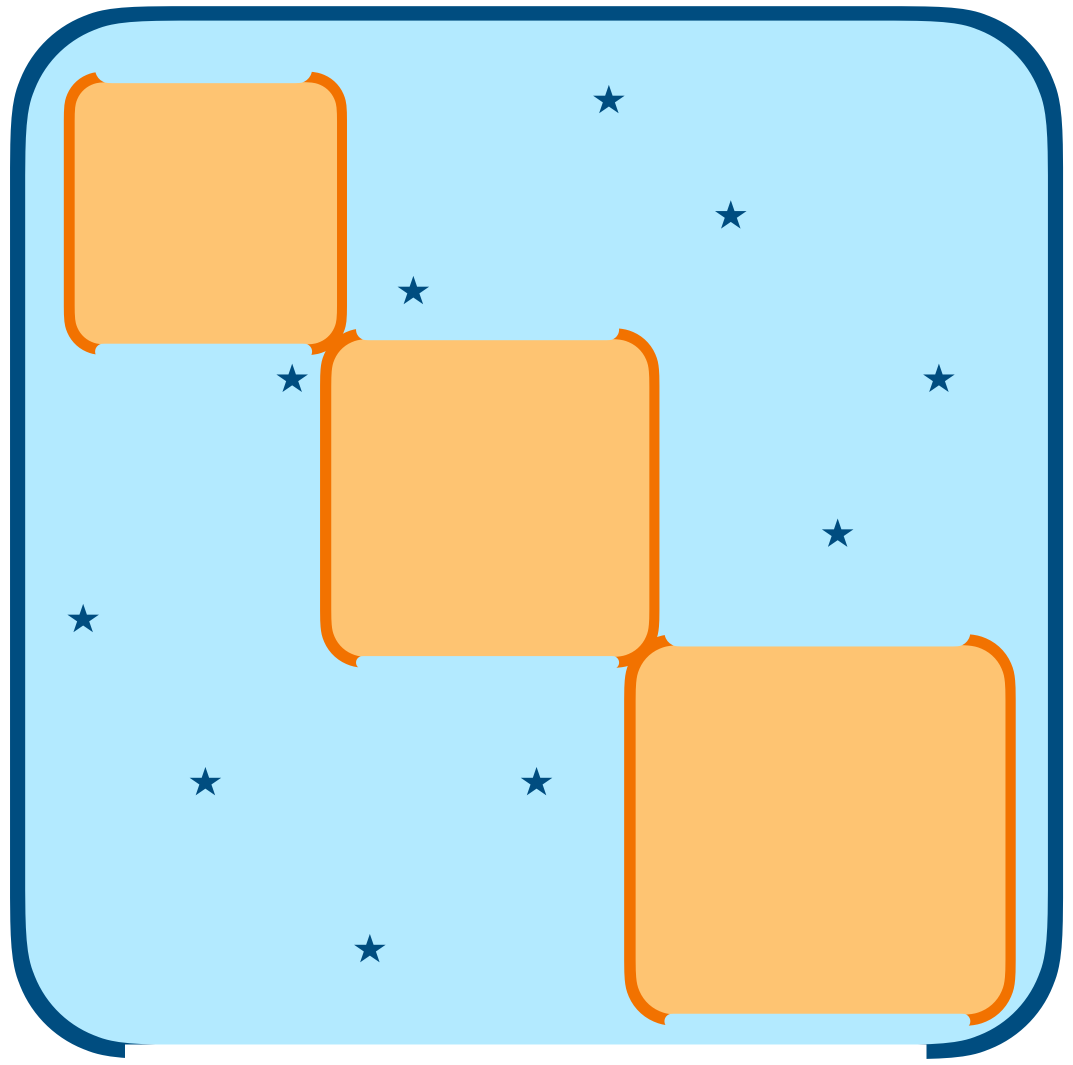
$= \bigoplus_q p(q)\rho_A(q)$

symmetry resolved entanglement

What happens is the symmetry is broken?

$|\Psi\rangle =$  $\rho = |\Psi\rangle \langle\Psi|$

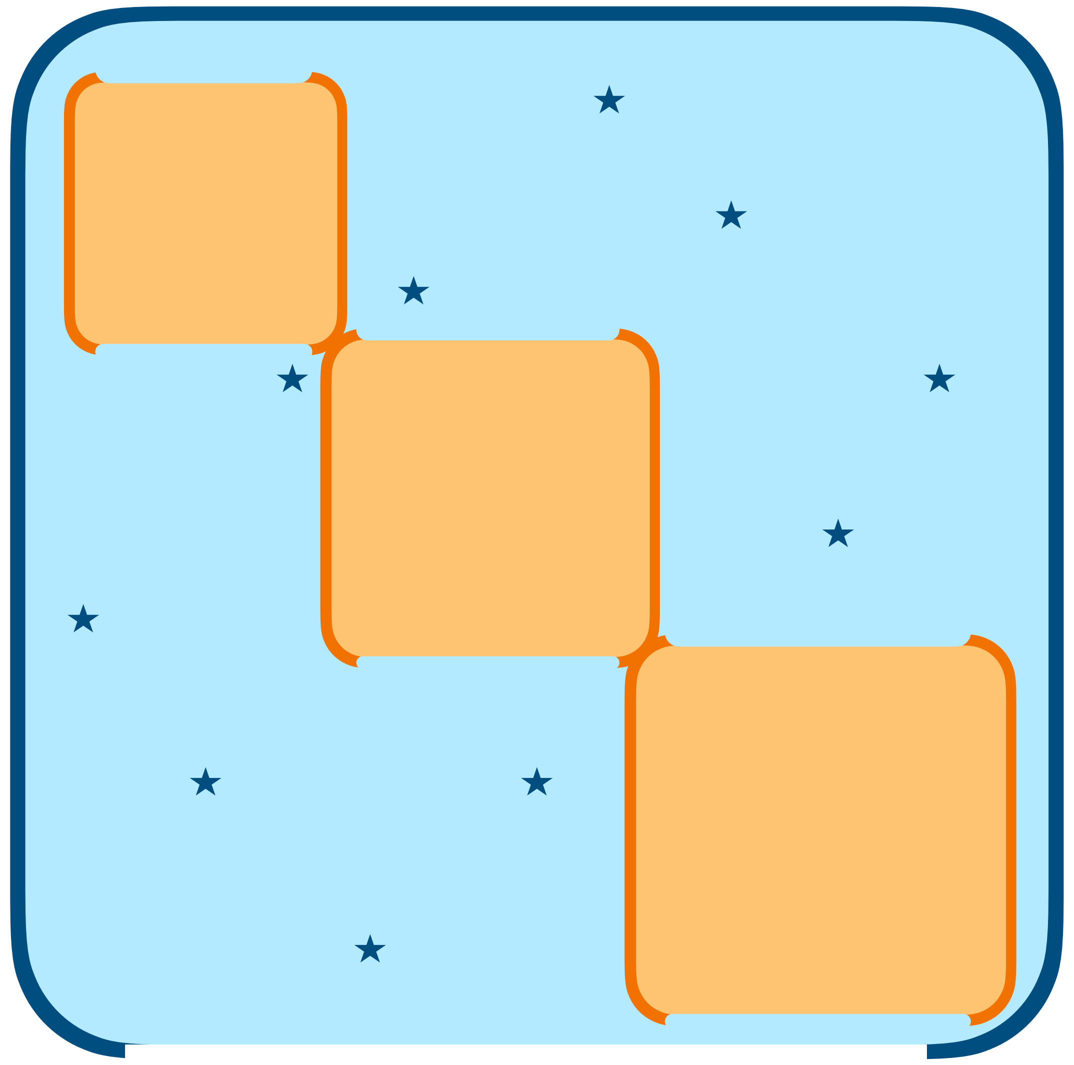
$$[\rho, Q] \neq 0 \Longrightarrow [\rho_A, Q_A] \neq 0$$

$\rho_A =$ 

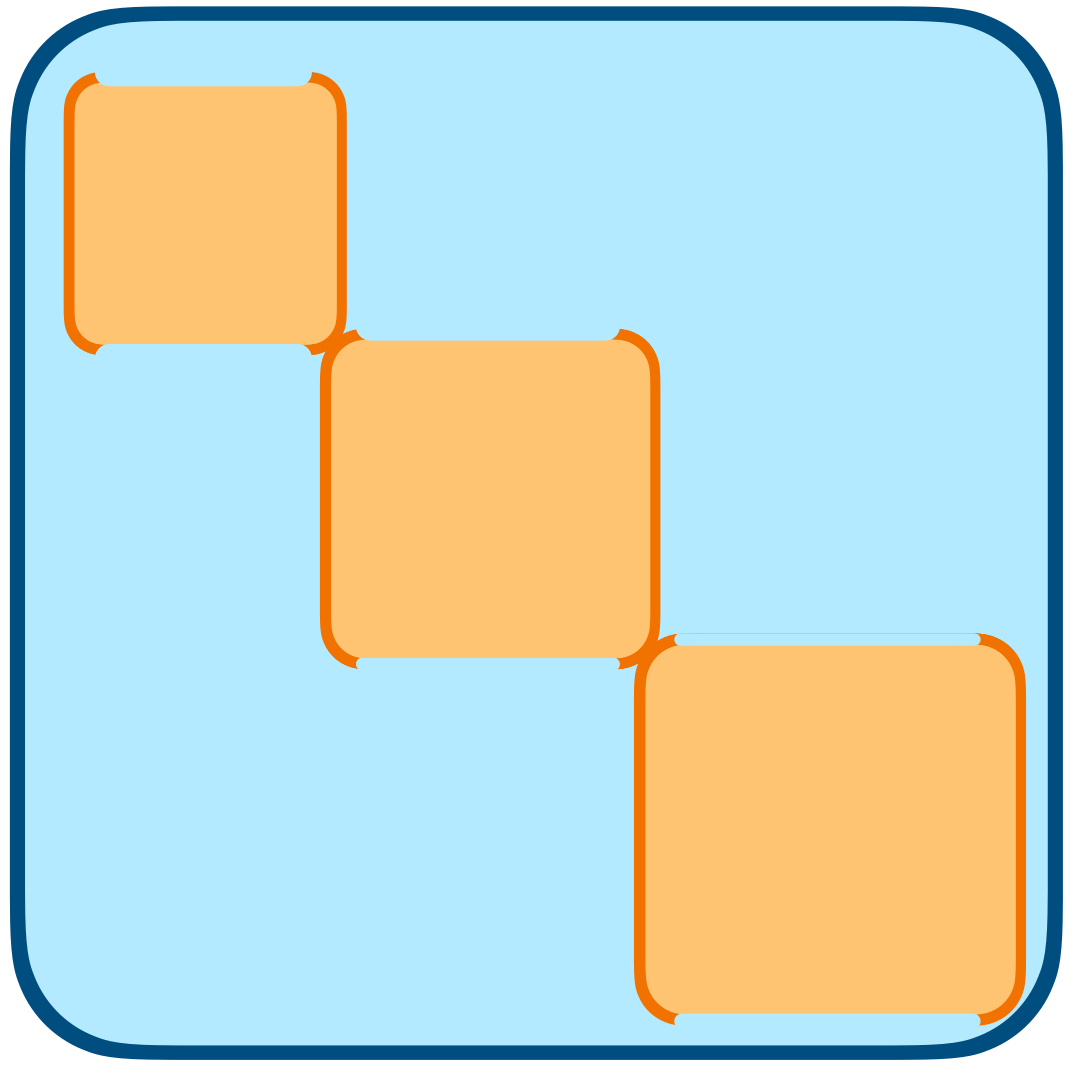
Entanglement asymmetry: a probe of symmetry breaking

F. Ares, S.Murciano P. Calabrese,, ArXiv:2207.14693

$\rho_A =$



$\rho_{A,Q} =$



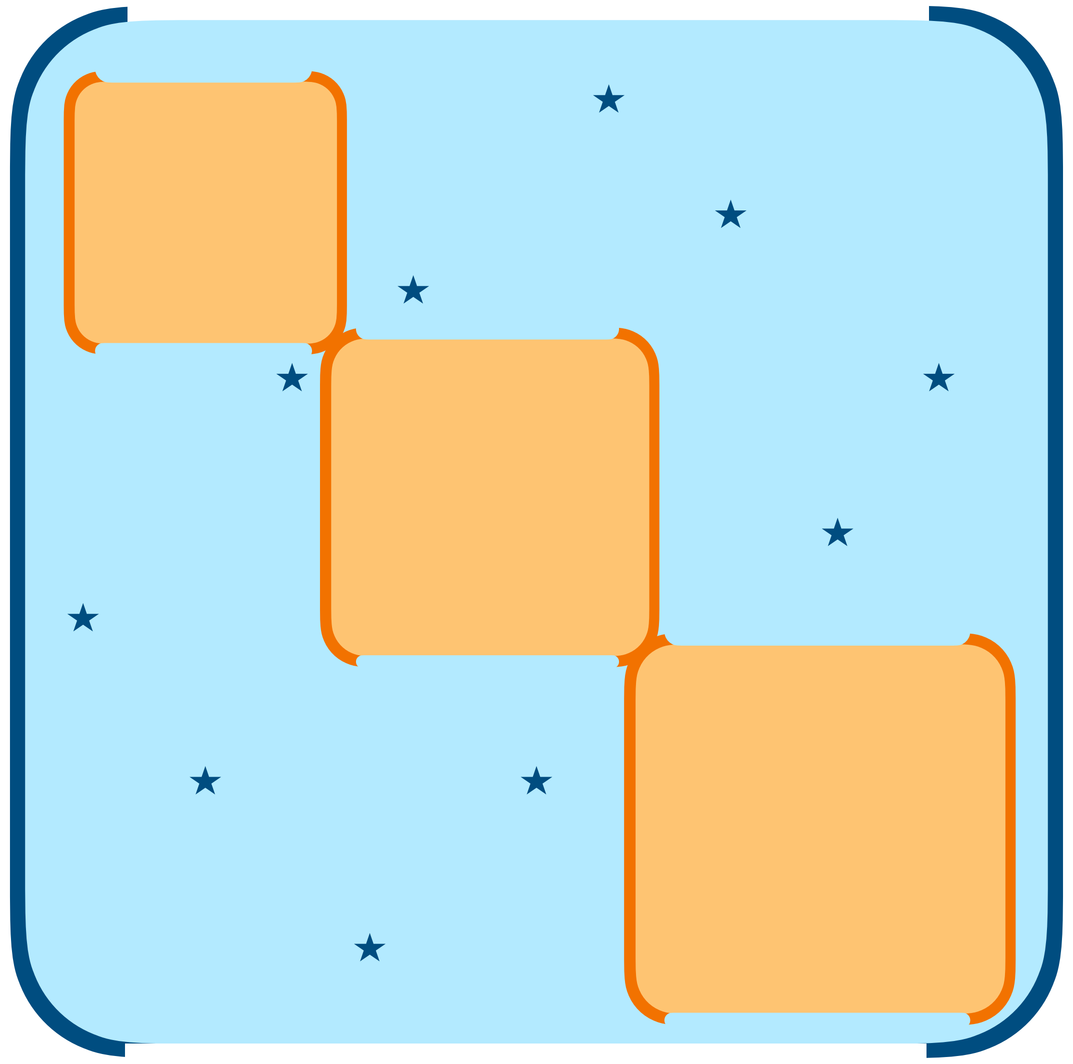
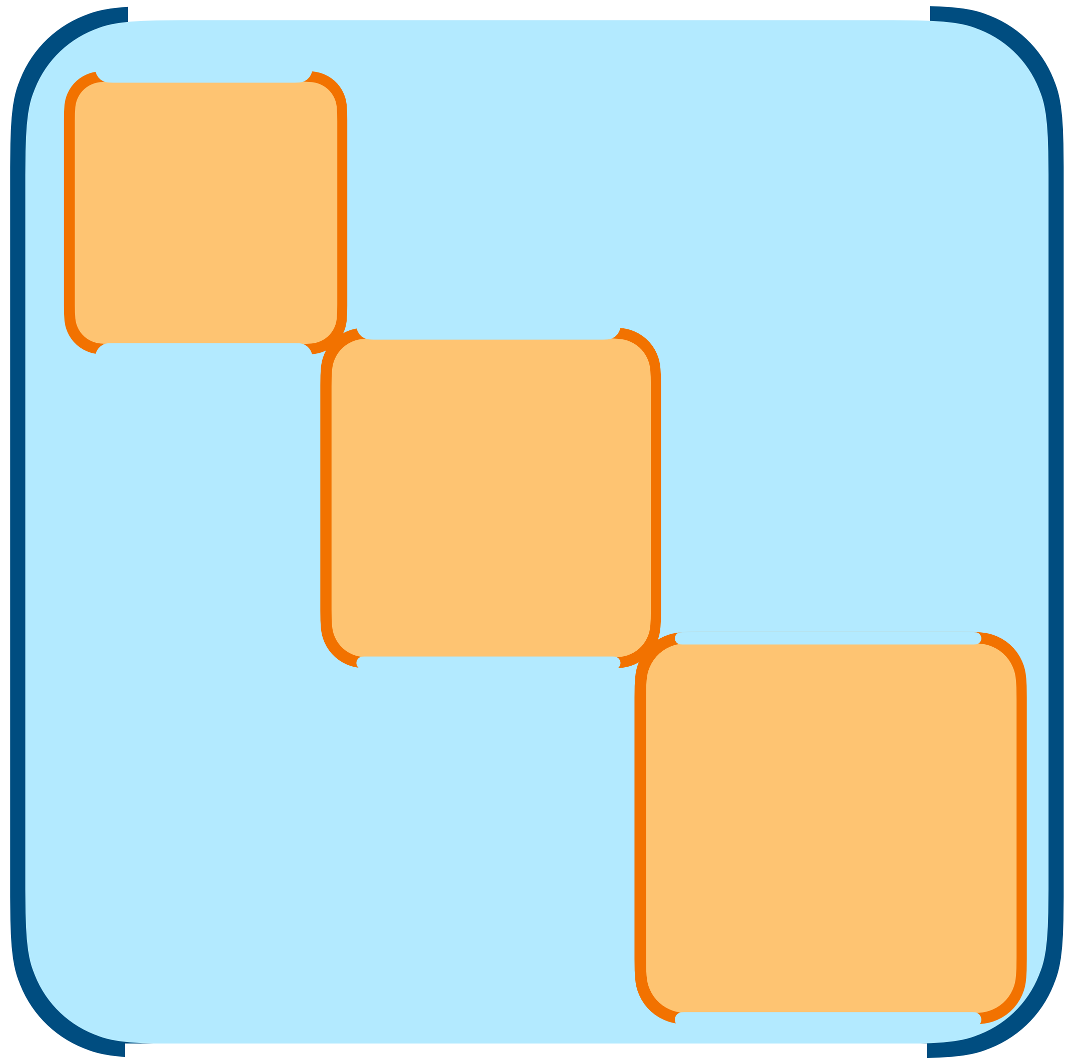
$= \sum_q \Pi_q \rho_A \Pi_q$

Entanglement asymmetry: $\Delta S_A = S(\rho_{A,Q}) - S(\rho_A) \begin{cases} \geq 0; \\ 0 \iff \rho_A = \rho_{A,Q} \end{cases}$

Recall: $S(\rho) = -\mathrm{Tr}(\rho \log \rho)$

Replica trick for asymmetry

F. Ares, S.Murciano P. Calabrese, ArXiv:2207.14693

$\rho_A =$  $\rho_{A,Q} =$ 

$$\Delta S_A^{(n)} = \frac{1}{1-n}\left[\log \mathrm{Tr}(\rho_{A,Q}^n) - \log \mathrm{Tr}(\rho_A^n)\right]$$

$$\Delta S_A^{(n)} \geq 0 \qquad \Delta S_A^{(n)} = 0 \Leftrightarrow [\rho_A, Q_A] = 0$$

Replica trick for asymmetry (II)

$$\Delta S_A^{(n)} = \frac{1}{1-n}\left[\log \mathrm{Tr}(\rho_{A,Q}^n) - \log \mathrm{Tr}(\rho_A^n)\right]$$

We use $\rho_{A,Q} = \sum_q \Pi_q \rho_A \Pi_q$ and $\Pi_q = \int_{-\pi}^{\pi} \frac{d\alpha}{2\pi} e^{i(Q_A - q)\alpha} \Rightarrow \rho_{A,Q} = \boxed{\int_{-\pi}^{\pi} \frac{d\alpha}{2\pi} e^{-i\alpha Q_A} \rho_A e^{i\alpha Q_A}}$

Replica trick for asymmetry (II)

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The moments $\mathrm{Tr}(\rho_{A,q}^n)$ can be accessed from the generalized charged moments

$$Z_n(\boldsymbol{\alpha}) = \mathrm{Tr}\left[\prod_{j=1}^{n} \rho_A e^{i(\alpha_j - \alpha_{j+1})Q_A}\right] = \mathrm{Tr}\left[\rho_A e^{i(\alpha_1-\alpha_2)Q_A} \rho_A e^{i(\alpha_2-\alpha_3)Q_A} \ldots \rho_A e^{i(\alpha_n - \alpha_1)Q_A}\right]$$

$$\mathrm{Tr}(\rho_{A,Q}^n) = \int_{-\pi}^{\pi} \frac{d\alpha_1 d\alpha_2 \ldots d\alpha_n}{(2\pi)^n} Z_n(\boldsymbol{\alpha})$$

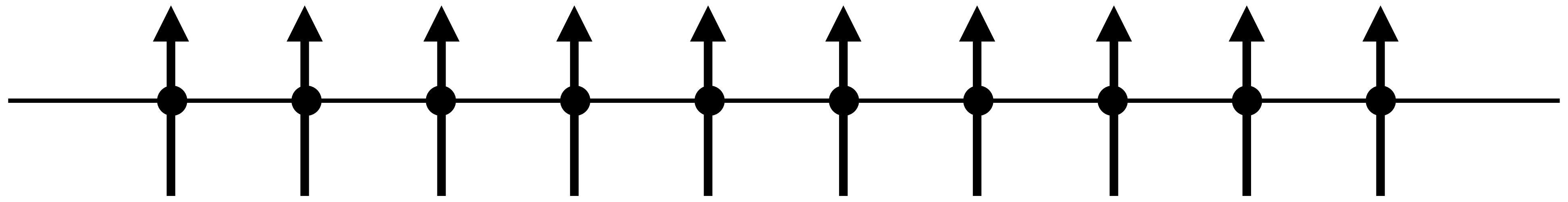
Sanity check: $[\rho_A, Q_A] = 0 \Rightarrow Z_n(\boldsymbol{\alpha}) = Z_n(\mathbf{0}) = \mathrm{Tr}\rho_A^n \Rightarrow \Delta S_A^{(n)} = 0$

A case study

F. Ares, S.Murciano P. Calabrese, ArXiv:2207.14693

Ferromagnet

$$|\Psi\rangle =$$

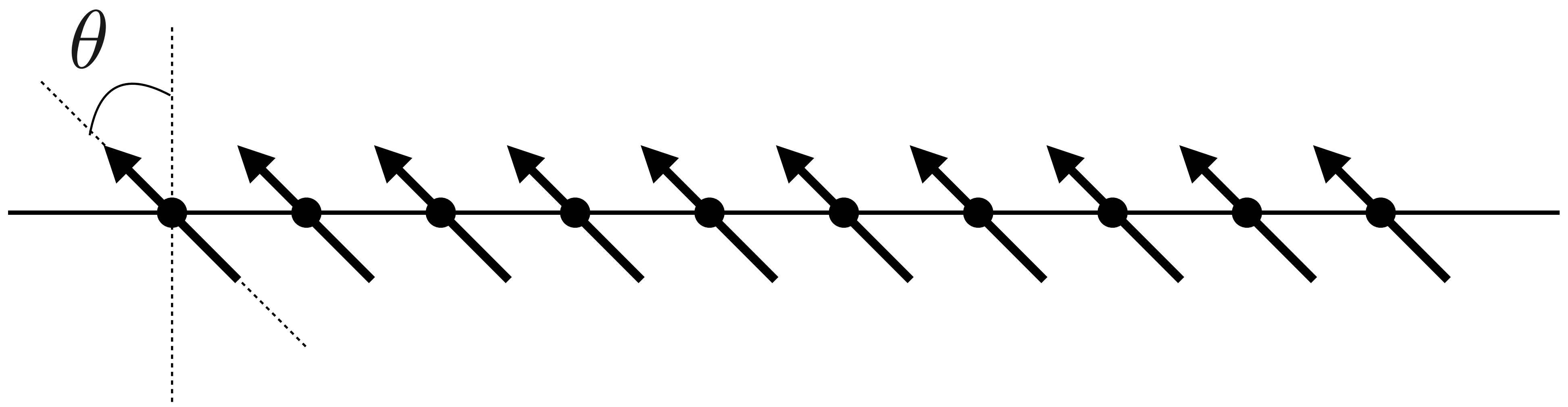


$$Q = \frac{1}{2}\sum_j \sigma_j^z \qquad [|\Psi\rangle\langle\Psi|, Q] = 0$$

$$\Delta S_A = 0$$

A case study

Tilted Ferromagnet

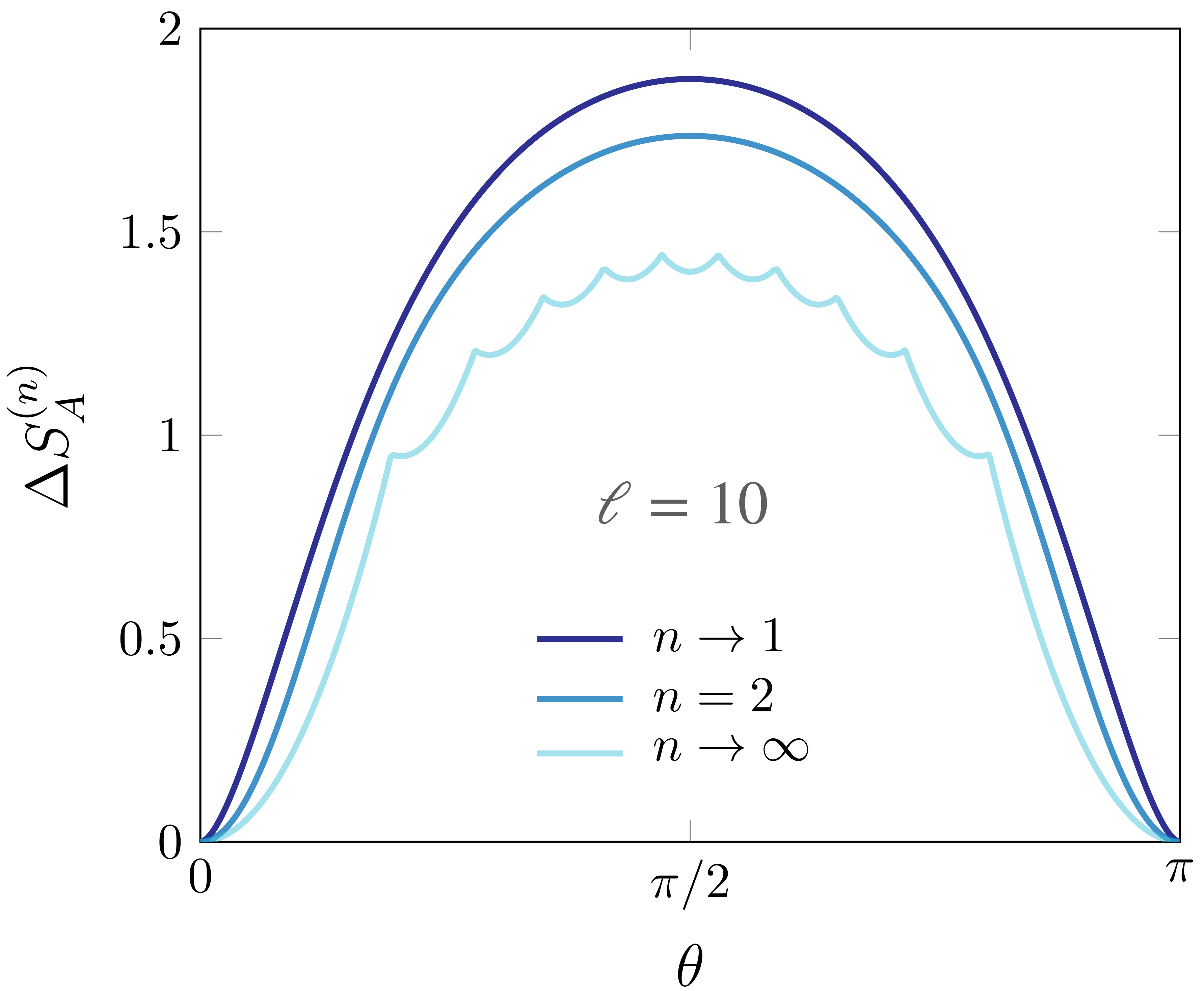
$|\theta\rangle =$ 

$Q = \frac{1}{2}\sum_j \sigma_j^z \qquad [|\Psi\rangle\langle\Psi|, Q] \neq 0$

$\Delta S_A \neq 0$

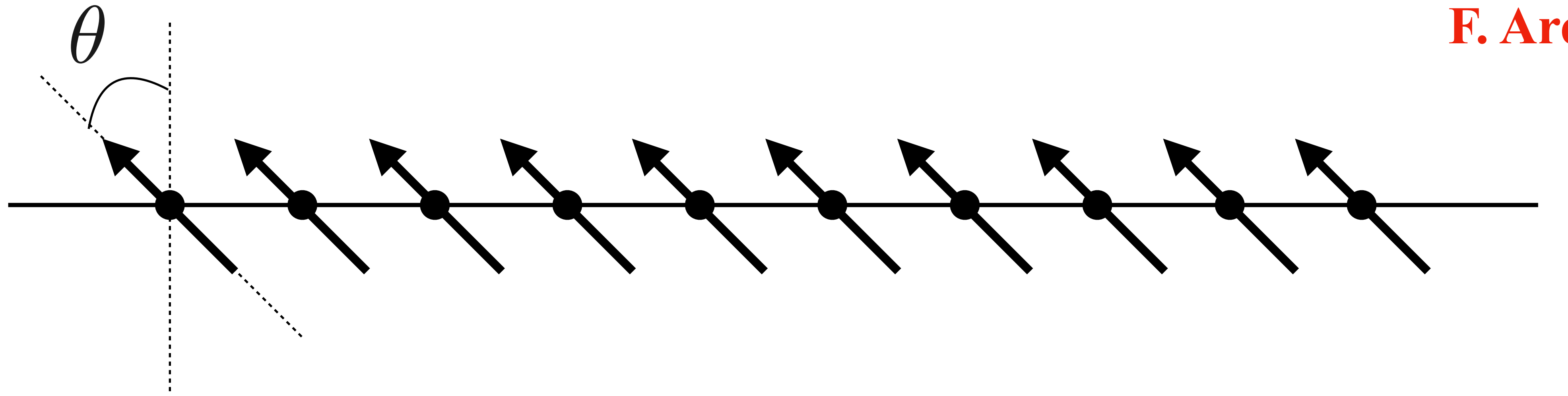
$$\Delta S_A^{(n)} = \frac{1}{1-n}\log\left[\cos^{2n\ell}\left(\frac{\theta}{2}\right)\sum_{p=0}^{\ell}\binom{\ell}{p}^n \tan^{2np}\left(\frac{\theta}{2}\right)\right]$$

$$= \frac{1}{2}\log\ell + \frac{1}{2}\log\frac{\pi n^{\frac{1}{n-1}}\sin^2\theta}{2} + O(\ell^{-1})$$



Asymmetry after a quench

F. Ares, S.Murciano P. Calabrese, ArXiv:2207.14693

$t = 0$ $\quad |\Psi\rangle =$  $\quad [|\Psi\rangle\langle\Psi|, Q] \neq 0$

$t > 0$ $\quad |\Psi(t)\rangle = e^{-itH}|\Psi(0)\rangle \qquad H = -\frac{1}{4}\sum_{j=-\infty}^{\infty}\left[\sigma_j^x\sigma_{j+1}^x + \sigma_j^y\sigma_{j+1}^y\right]$

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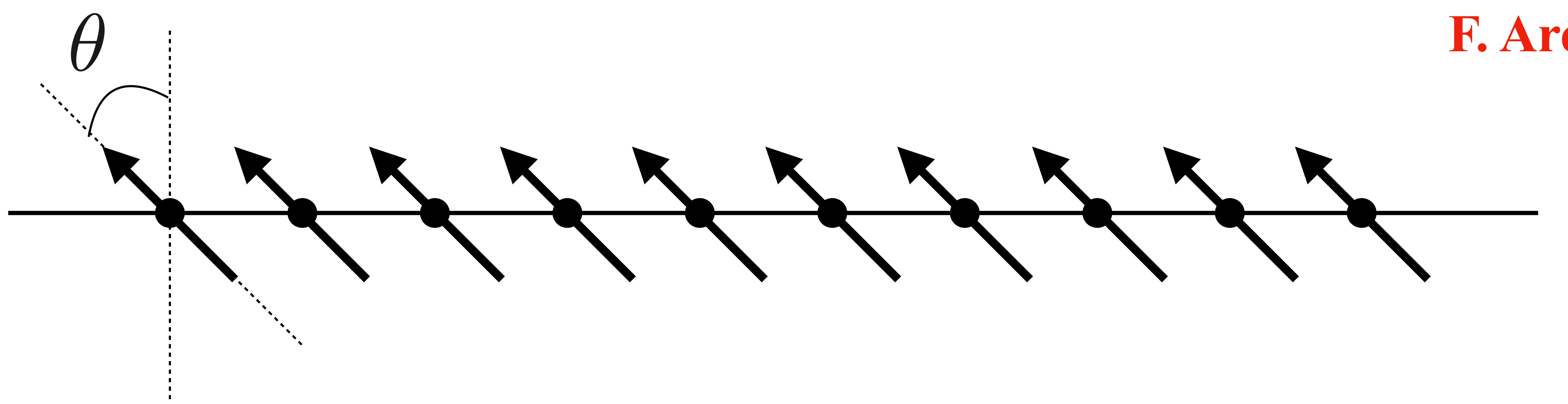

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A lot of algebra!

$$\boxed{Z_n(\boldsymbol{\alpha}, t) = Z_n(\mathbf{0}, t)e^{\ell(A_n(\boldsymbol{\alpha}) + B_n(\boldsymbol{\alpha}, t/\ell))}}$$

$$B_n(\boldsymbol{\alpha}, t = \infty) = -A_n(\boldsymbol{\alpha})$$

$$B_n(\boldsymbol{\alpha}, t/\ell) = -\int_{-\pi}^{\pi}\frac{\mathrm{d}k}{2\pi}\min(2t/\ell|v_k|, 1)f_n(\boldsymbol{\alpha}, k, \theta)$$

$$f_n(\boldsymbol{\alpha}, k, \theta) = \log\prod_{j=1}^{n}\left[ie^{i\Delta(k,\theta)}\sin\left(\frac{\alpha_j - \alpha_{j+1}}{2}\right) + \cos\left(\frac{\alpha_j - \alpha_{j+1}}{2}\right)\right]$$

Asymmetry after a quench

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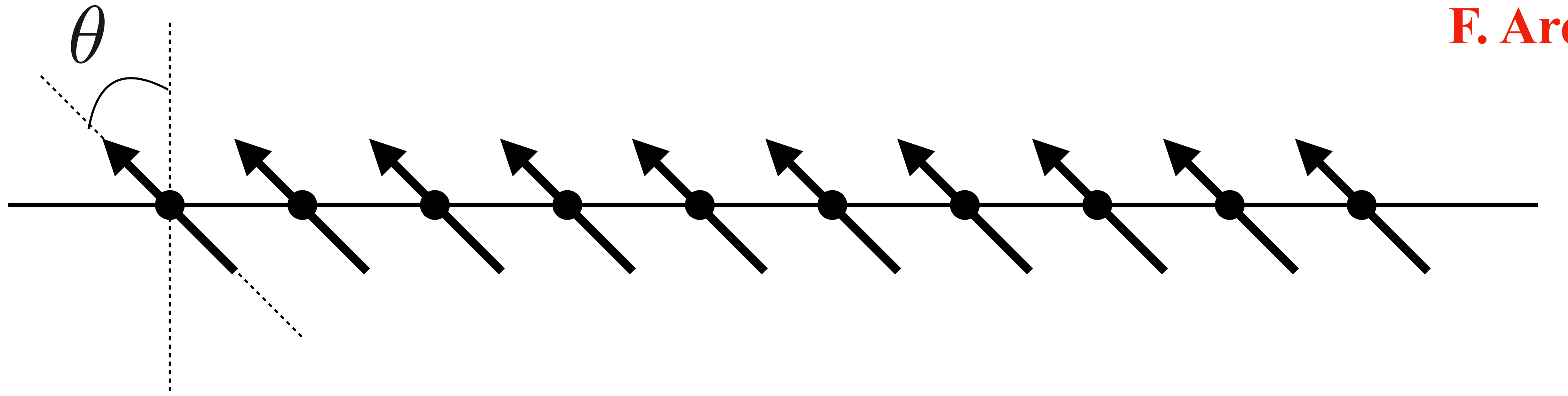

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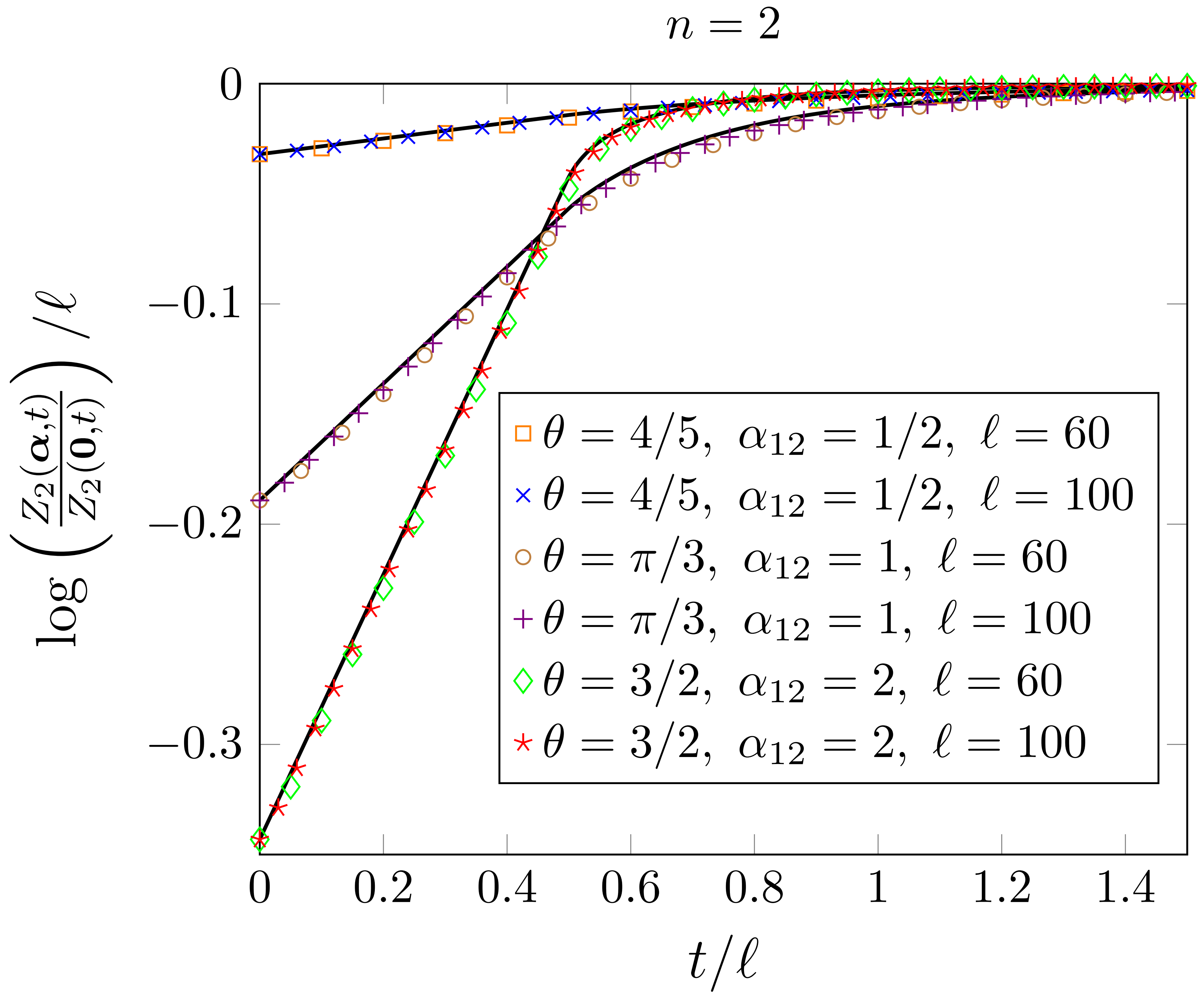

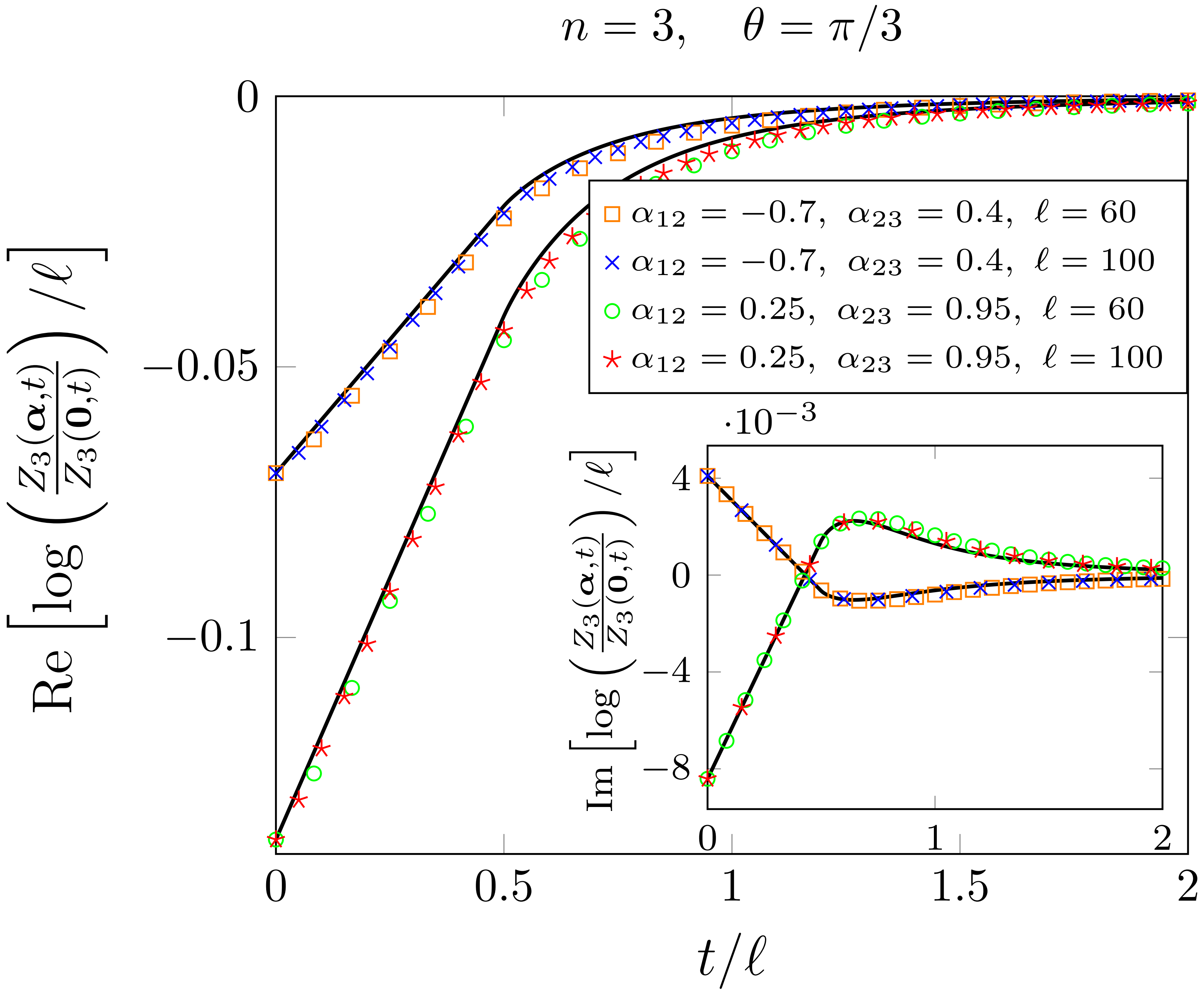

Quantum Mpemba effect

F. Ares, S.Murciano P. Calabrese, ArXiv:2207.14693

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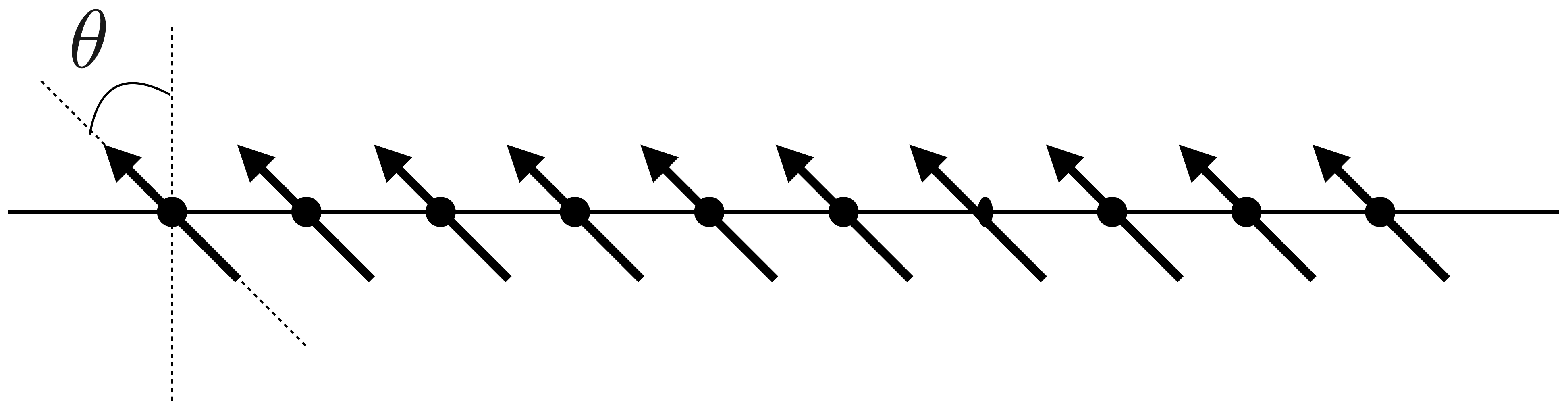

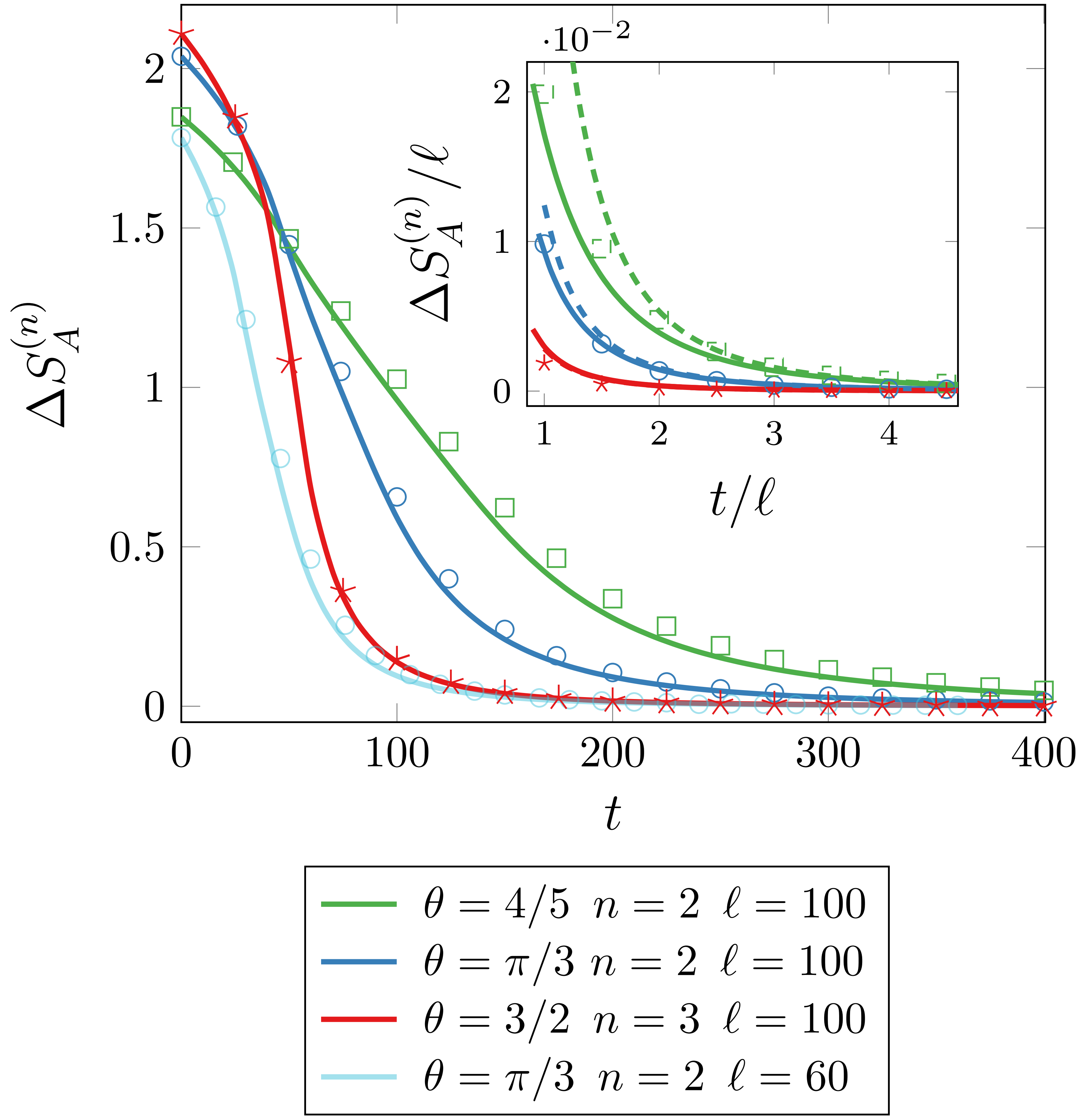

- $\Delta S_A^{(n)}(t)$ tends to zero for large t and the U(1) symmetry is restored

$$\boxed{\Delta S_A^{(n)}(t) \simeq \frac{\pi}{1152}\left(1 + 8\frac{\cos^2\theta}{\sin^4\theta}\right)\frac{\ell}{\zeta^3}}$$

$$\zeta = \frac{t}{\ell}$$

Quantum Mpemba effect

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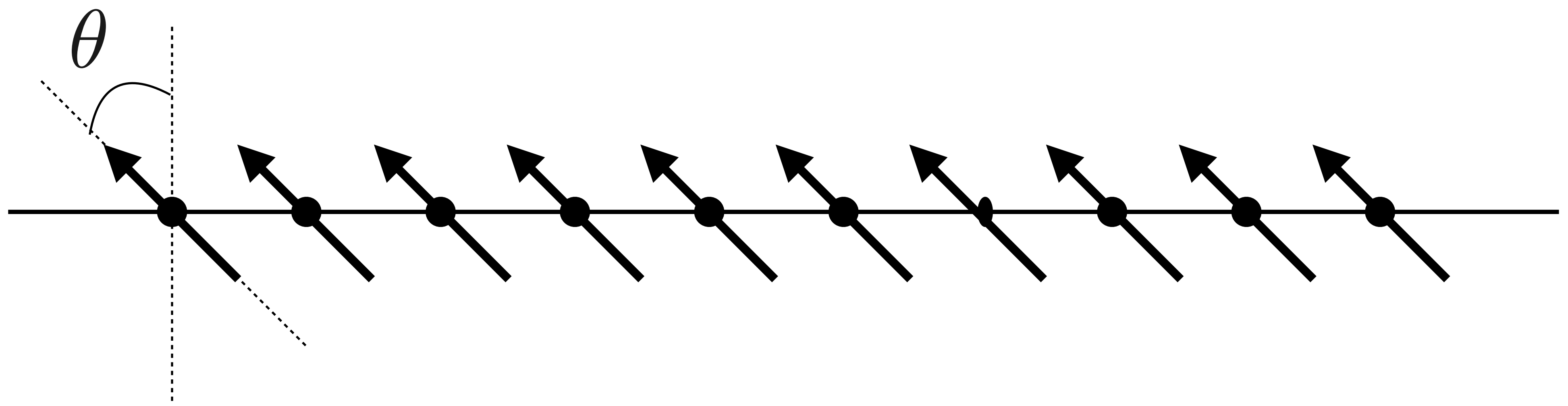

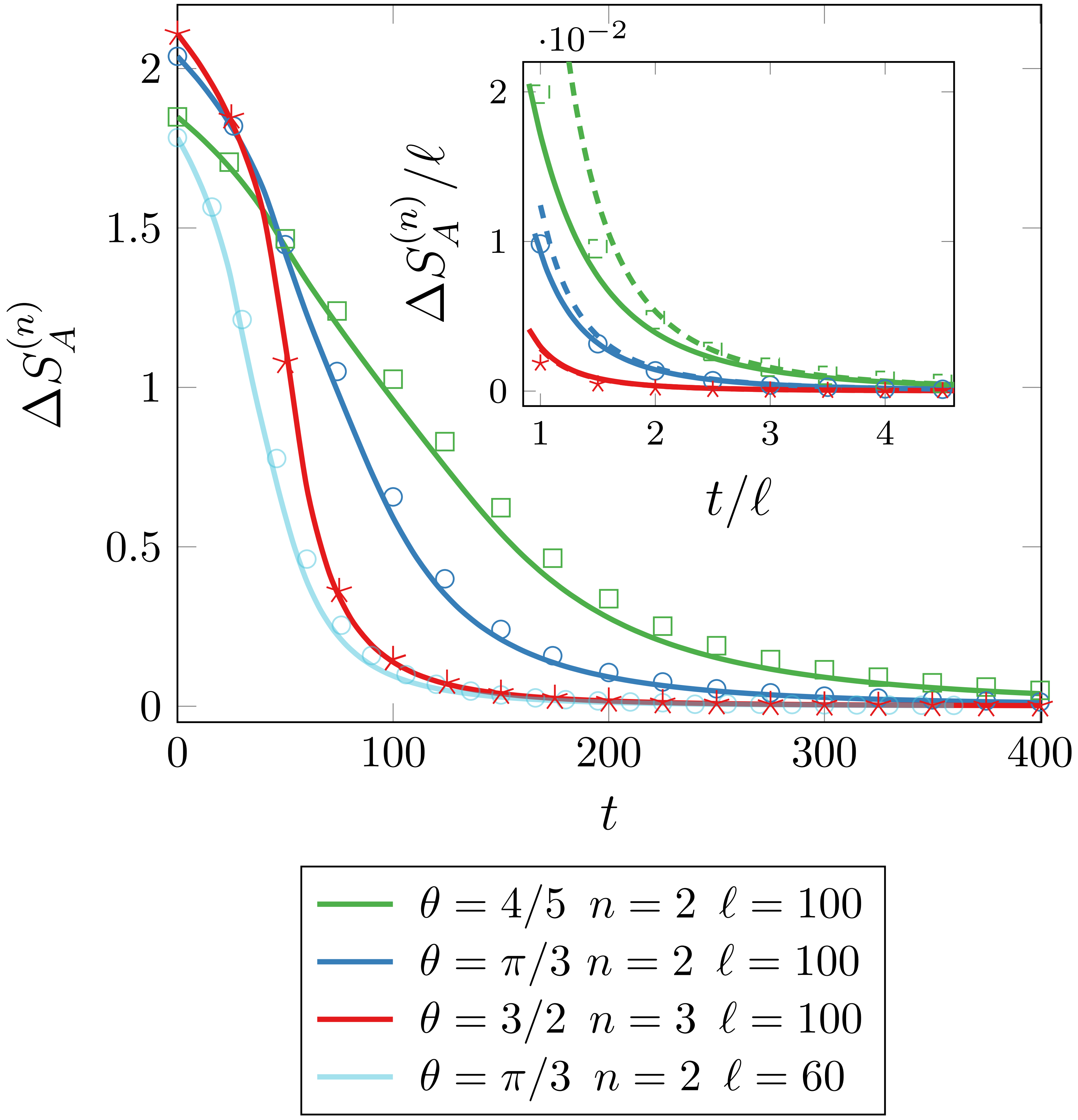

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- Larger subsystems require more time to recover the symmetry

Quantum Mpemba effect

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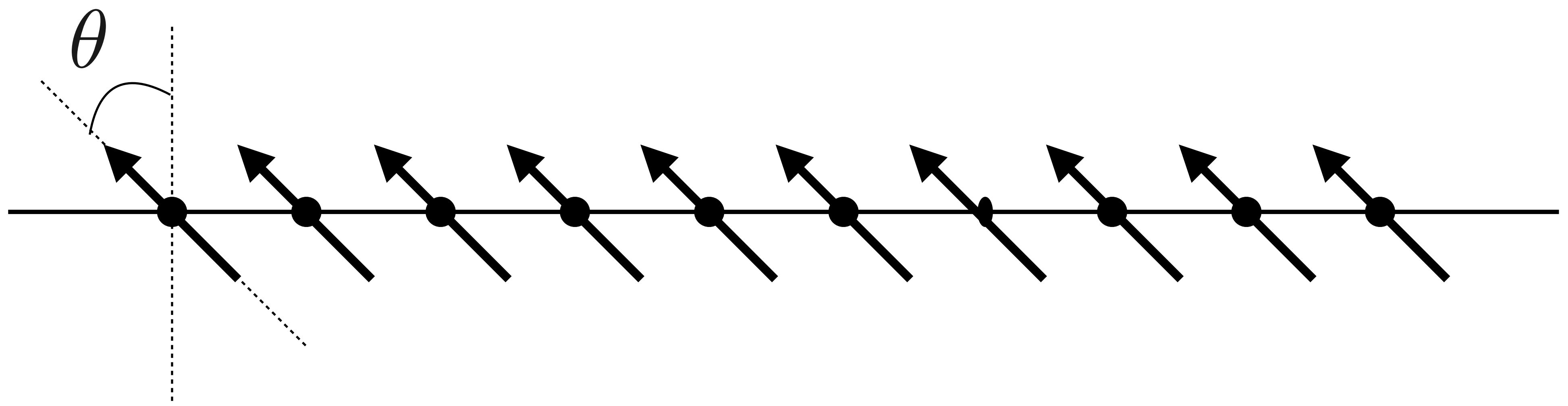

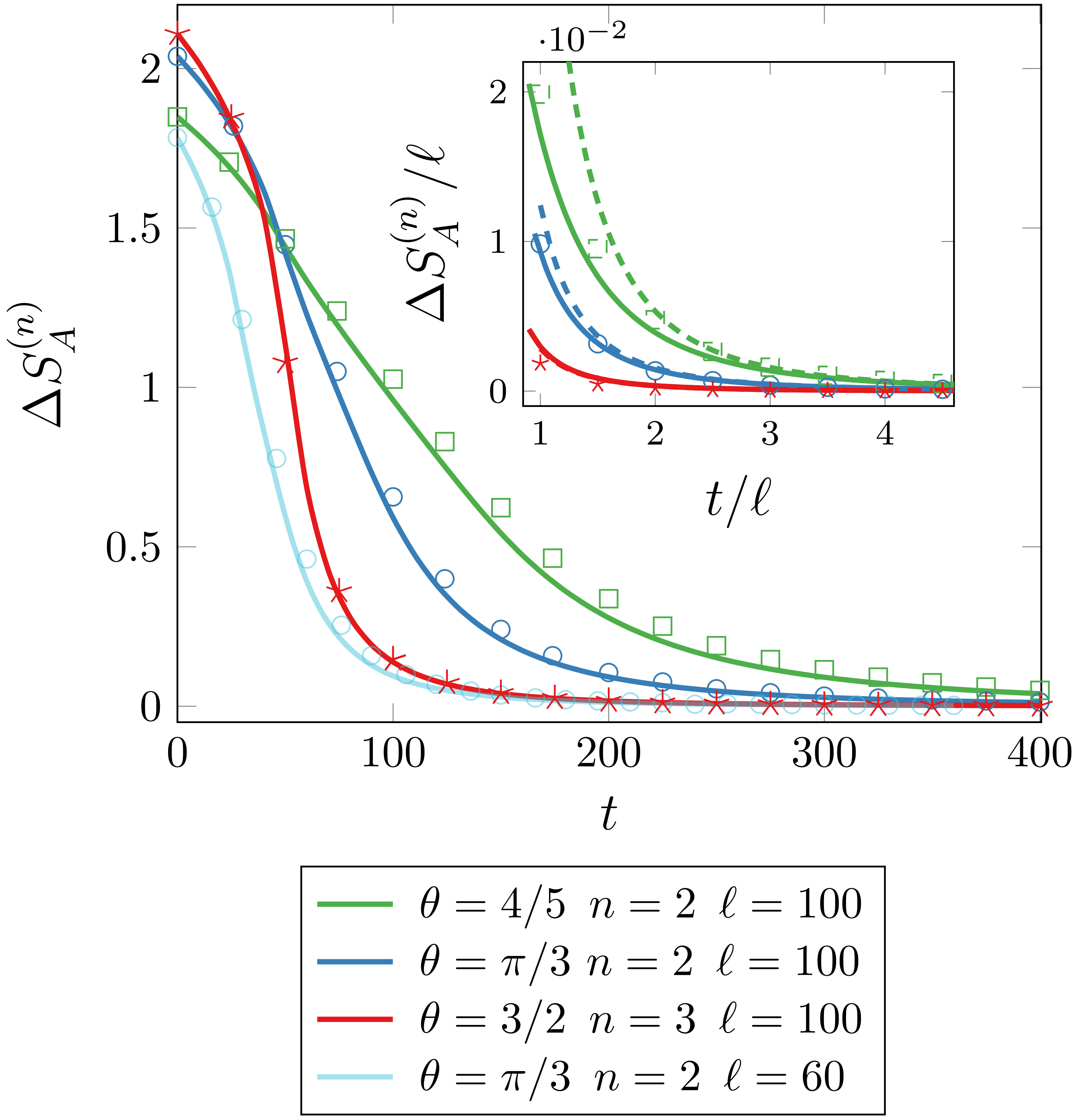

- $\Delta S_A^{(n)}(t)$ tends to zero for large t and the $\mathrm{U}(1)$ symmetry is restored

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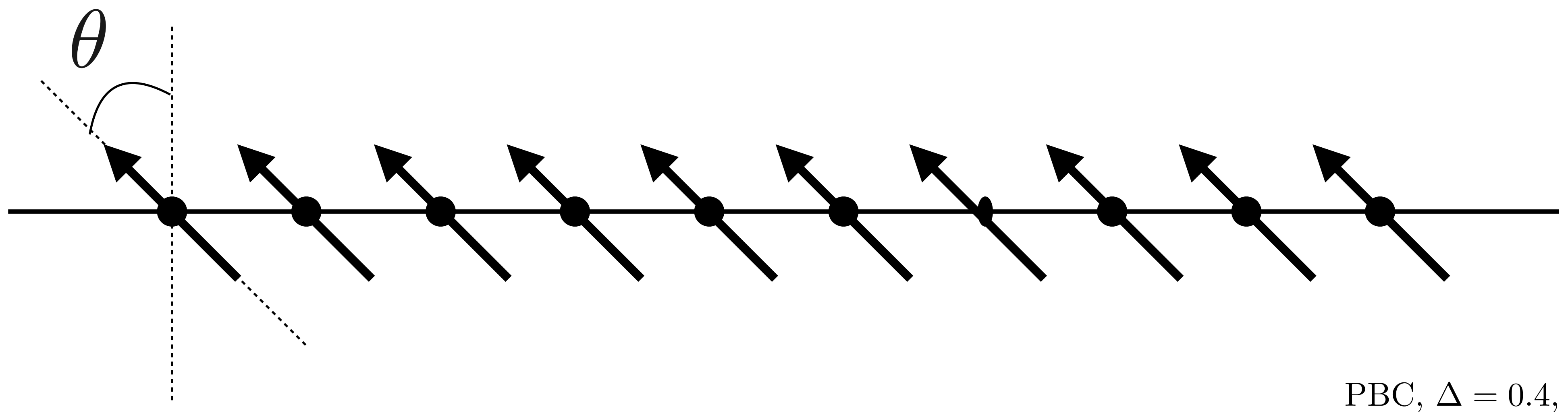
- Larger subsystems require more time to recover the symmetry

- Quantum Mpemba effect: The more the symmetry is initially broken, shorter is the time to restore it

Robustness of Quantum Mpemba effect

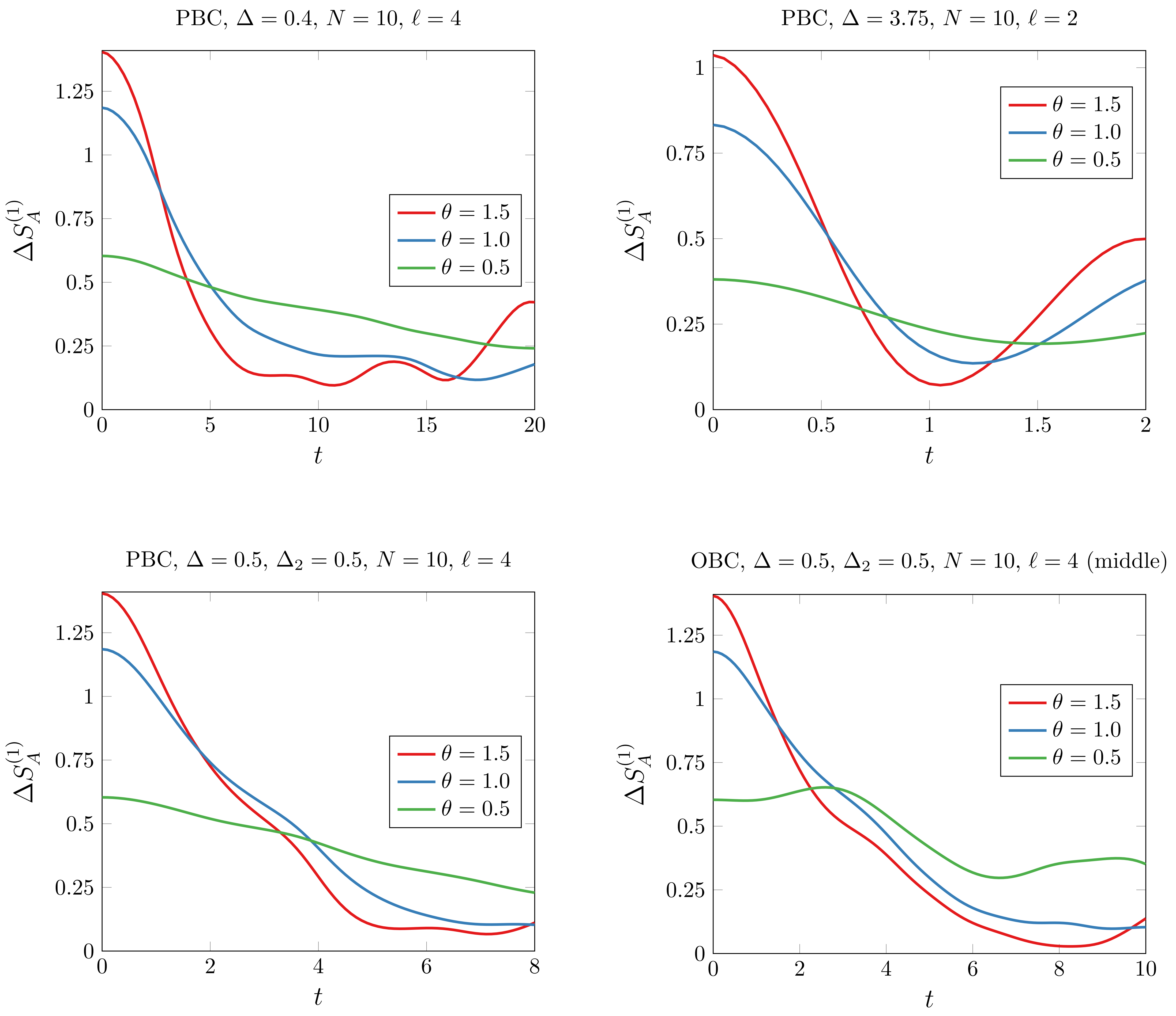
F. Ares, S.Murciano P. Calabrese, ArXiv:2207.14693

$t = 0$ $\quad |\Psi\rangle =$



$t > 0$

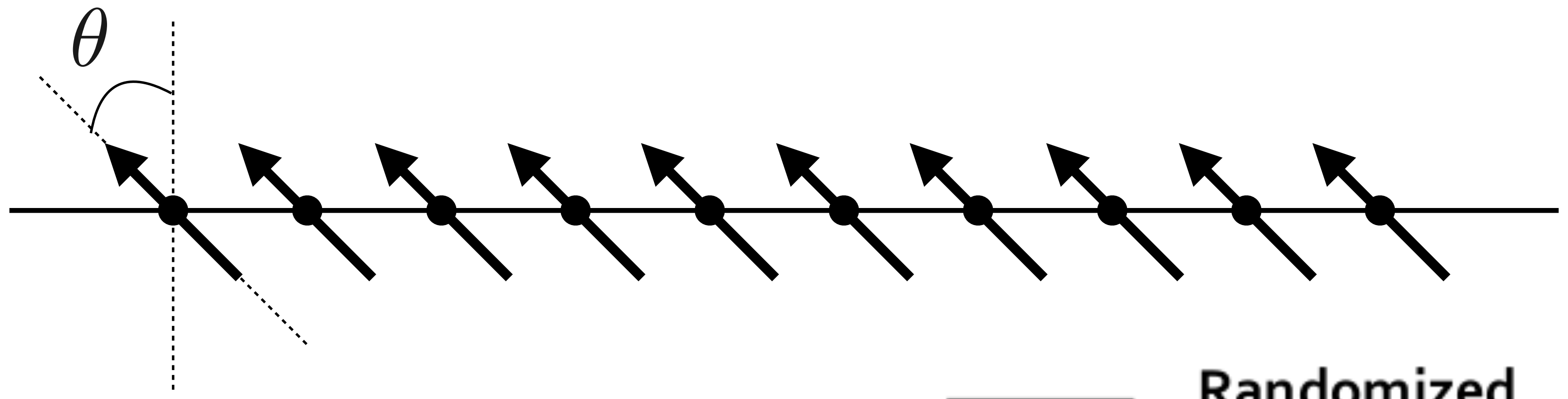
$$H = -\frac{1}{4}\sum_{j=1}^{N}\left[\sigma_j^x\sigma_{j+1}^x + \sigma_j^y\sigma_{j+1}^y + \Delta\sigma_j^z\sigma_{j+1}^z\right]$$
$$- \frac{J_2}{4}\sum_{j=1}^{N}\left[\sigma_j^x\sigma_{j+2}^x + \sigma_j^y\sigma_{j+2}^y + \Delta_2\sigma_j^z\sigma_{j+2}^z\right],$$



Quantum Mpemba effect in trapped ion experiment

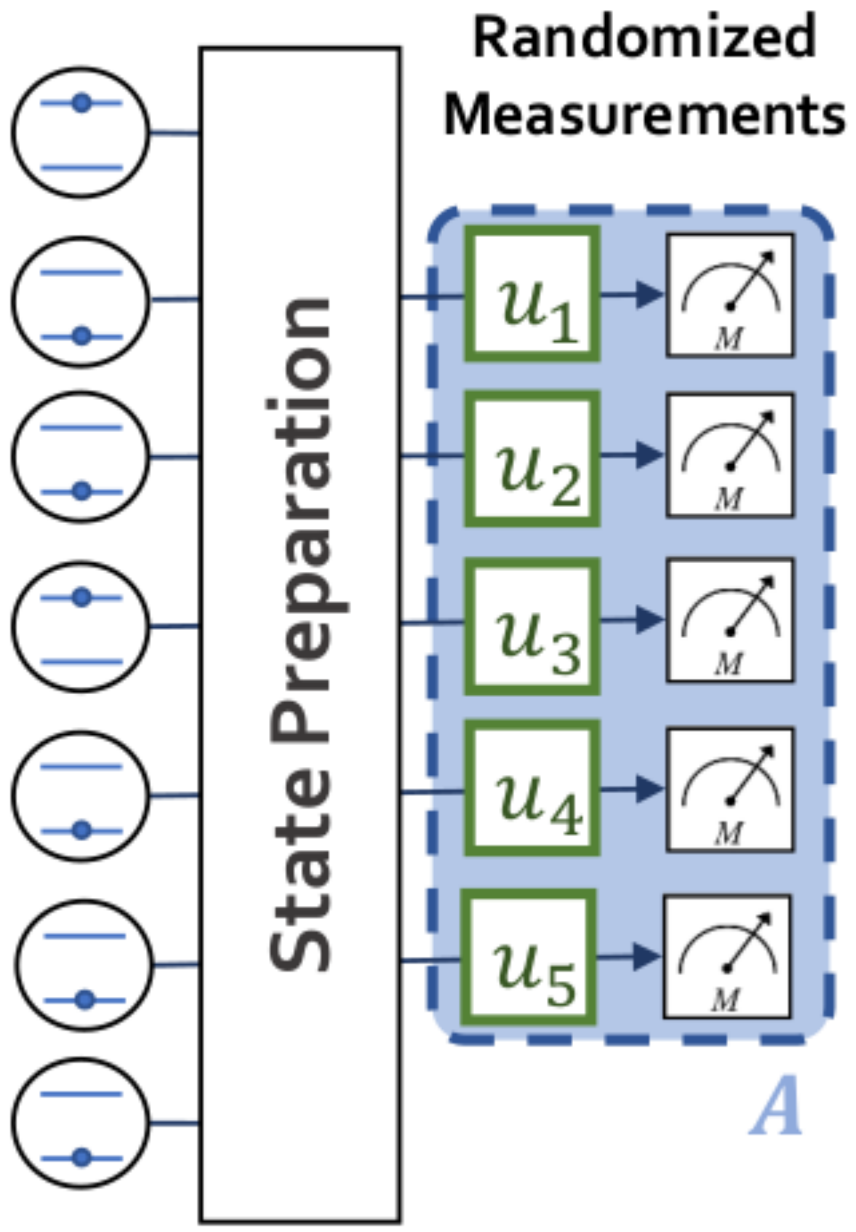
F. Ares, P. Calabrese, J. Franke, F. Kranzi, J.Manoj, L. Manoj, S.Murciano, A. Rath, C. Roos, B. Vermesch, P. Zoller

Protocol:

1. Prepare a tilted ferromagnet $|\Psi\rangle =$ 

Quench with: $H_{XY} = \hbar \sum_{i<j} J_{ij}(\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+) + \hbar B \sum_j \sigma_j^z$

2. Perform randomised measurements

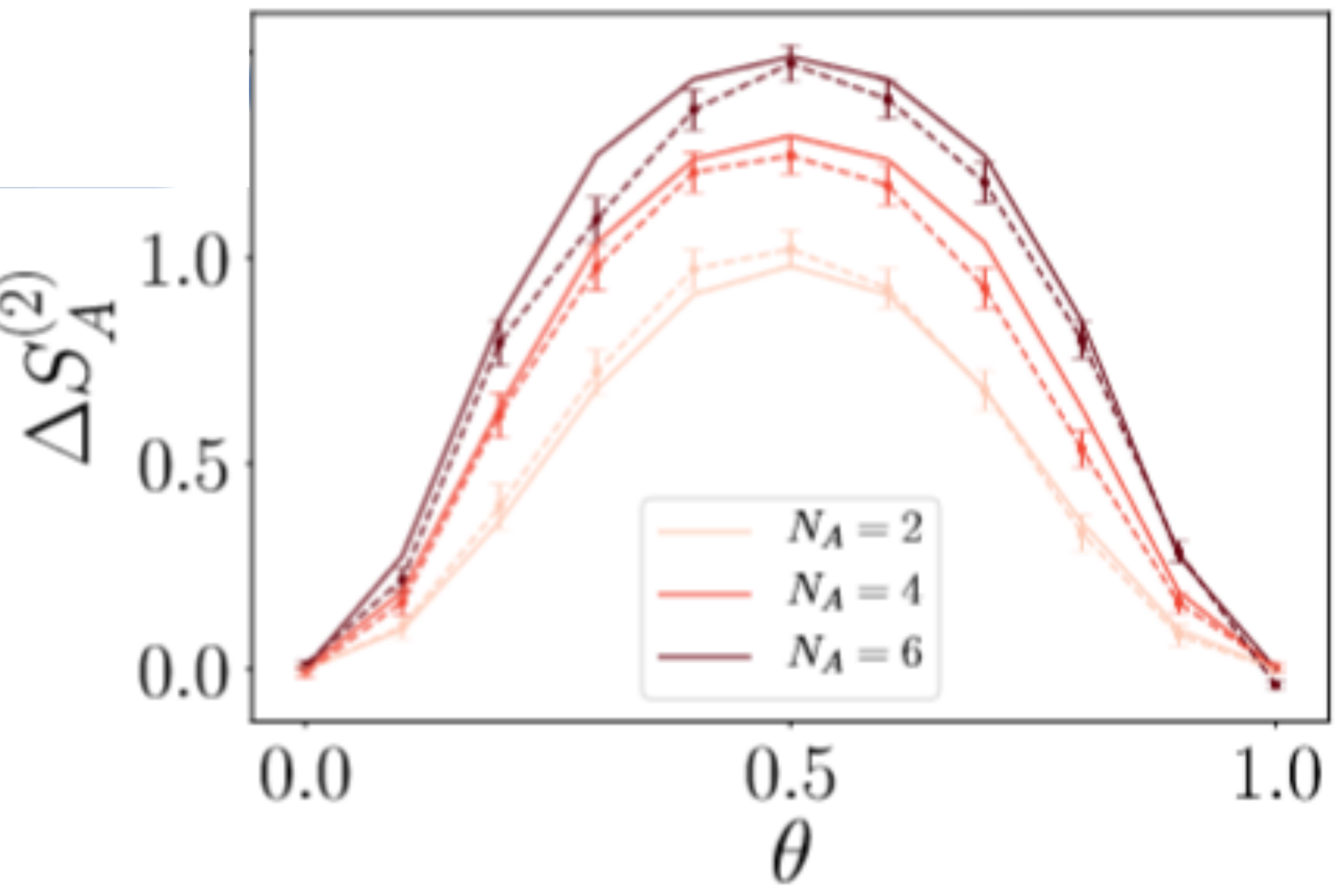


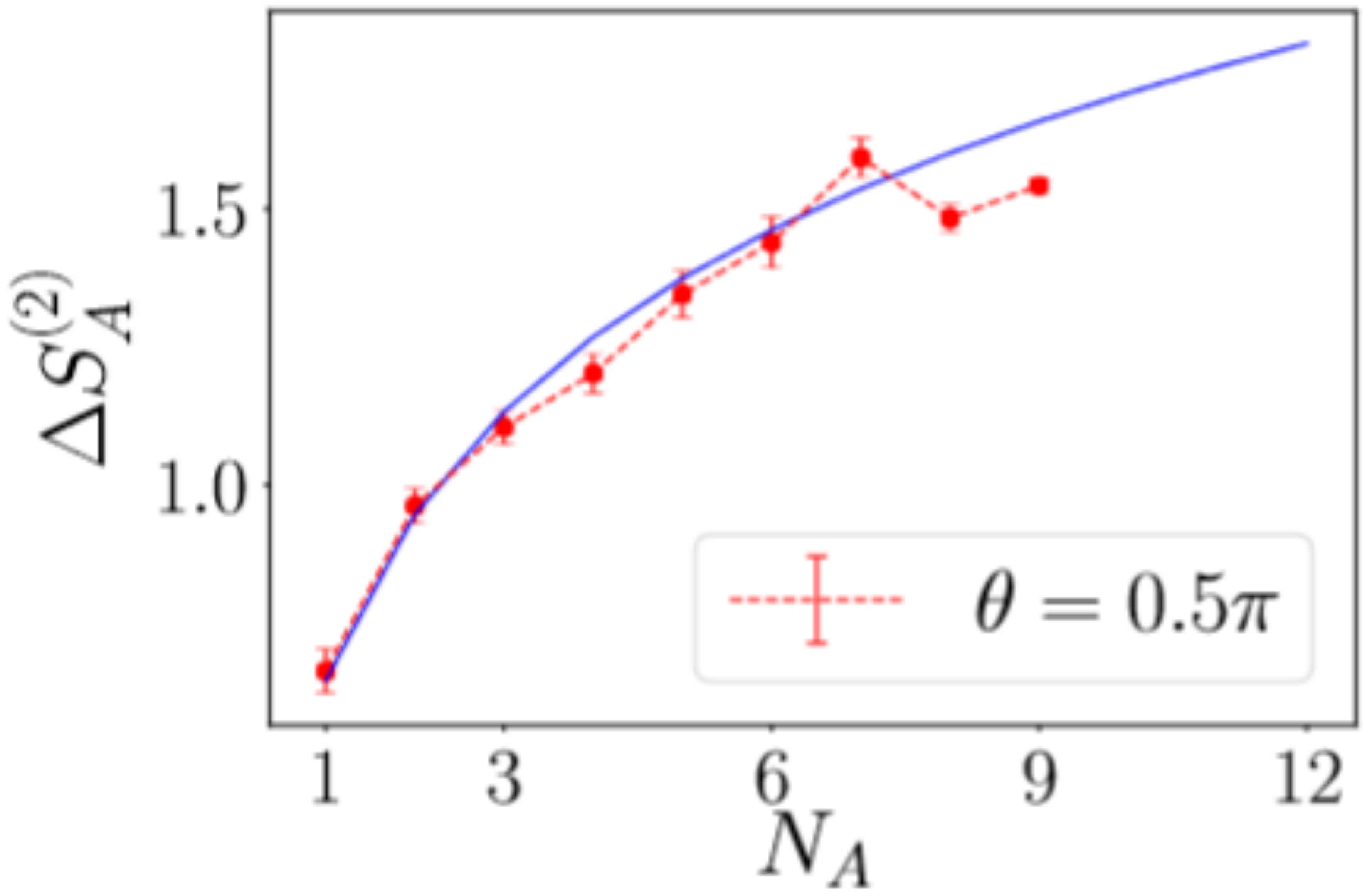
3. Estimate entanglement asymmetry with classical shadows

Quantum Mpemba effect in trapped ion experiment: results

F. Ares, P. Calabrese, J. Franke, F. Kranzi, J.Manoj, L. Manoj, S.Murciano, A. Rath, C. Roos, B. Vermesch, P. Zoller

Theta dependence in the initial state:





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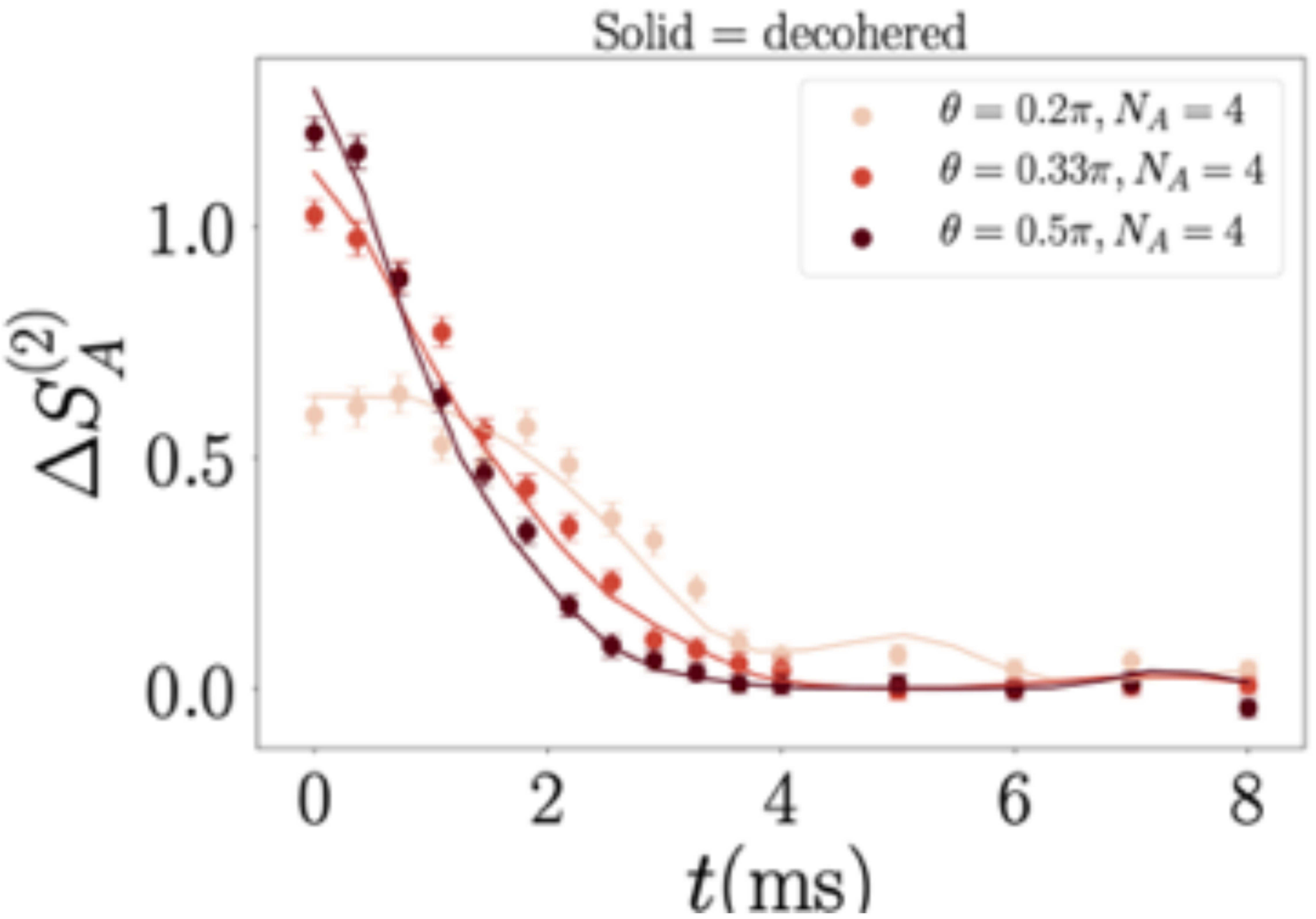
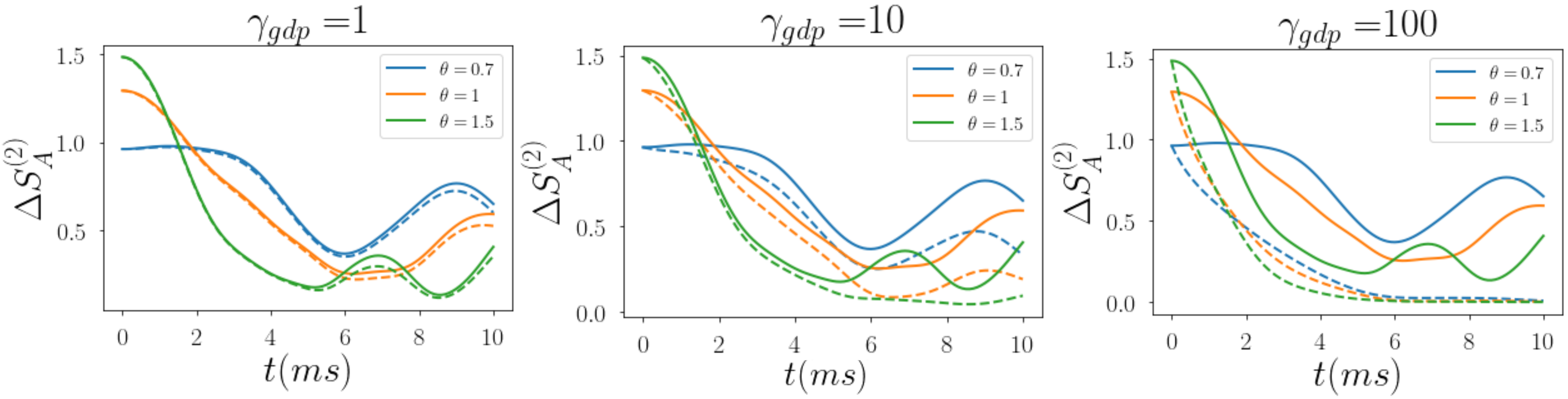
Entanglement asymmetry for 12 ions:



Curiosity: effect of dephasing

F. Ares, P. Calabrese, J. Franke, F. Kranzi, J.Manoj, L. Manoj, S.Murciano, A. Rath, C. Roos, B. Vermesch, P. Zoller

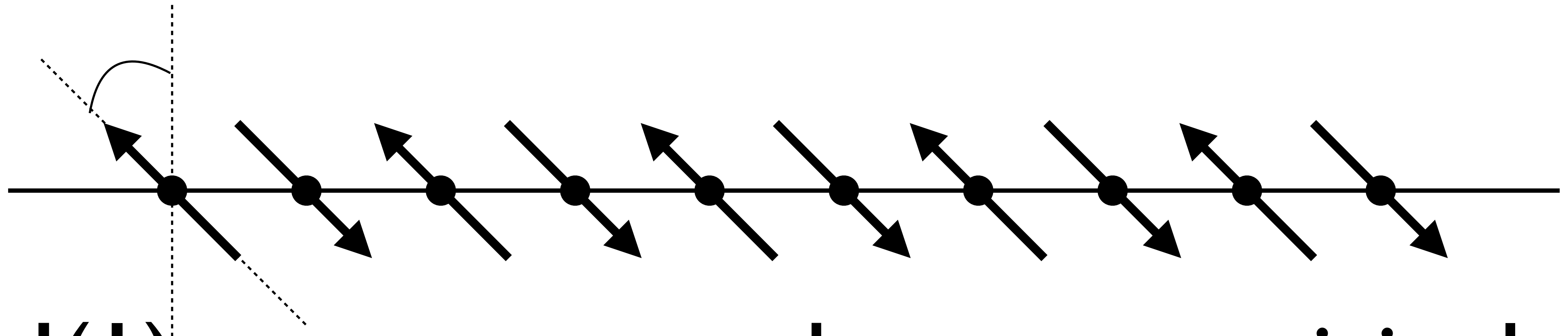


Dephasing makes Mpemba stronger

Non-symmetry restoration: the tilted Neel

F. Ares, S.Murciano, E. Vernier, P. Calabrese

Let us now consider as initial state a tilted Neel

$|\theta\rangle =$ 

$|\theta\rangle$ breaks the U(1) symmetry and very surprisingly it is not restored by time evolution in the XX spin chain

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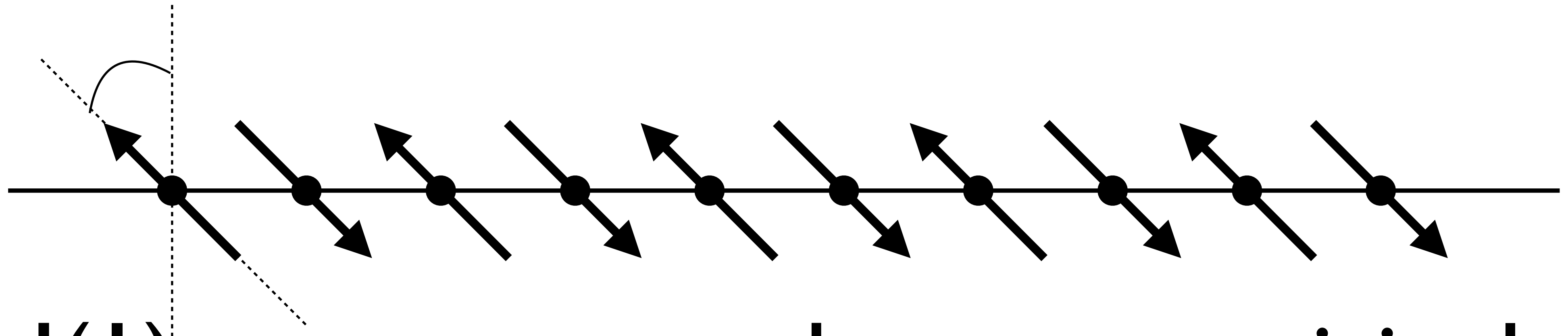
$|\theta\rangle =$ 

$|\theta\rangle$ breaks the U(1) symmetry and very surprisingly <u>it is not restored by time evolution</u> in the XX spin chain

Because of the activation of the non-abelian charges $Q_-^m = \sum_j (-1)^j c_j c_{j+m}$ M. Fagotti. '14

This fact attracted no attention in the literature, but it is valid for all gapless XXZ

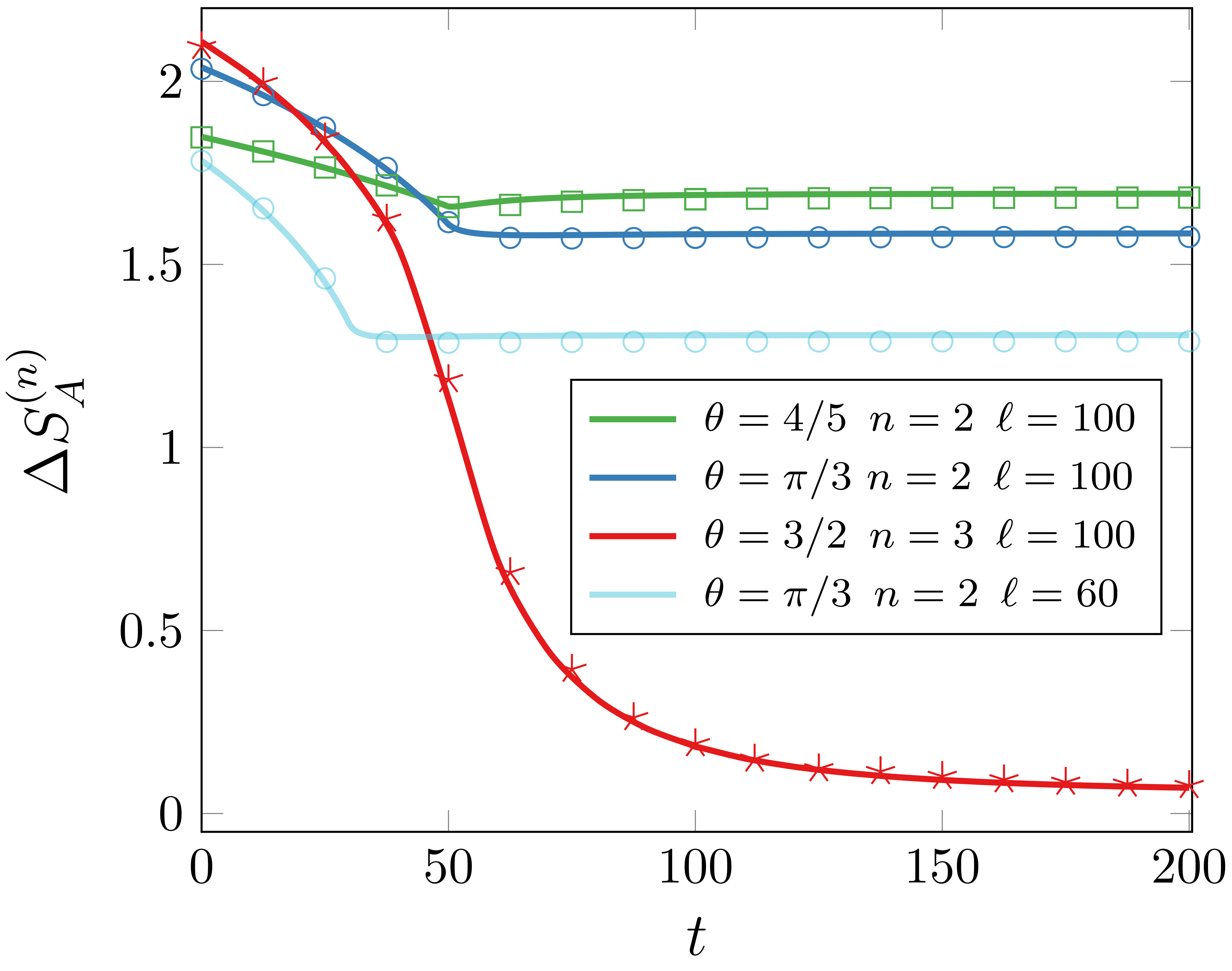
The entanglement asymmetry is the ideal quantity to study the lack of symmetry restoration

Non-symmetry restoration: the tilted Neel II

F. Ares, S.Murciano, E. Vernier, P. Calabrese

The entanglement asymmetry: theory vs exact numerics

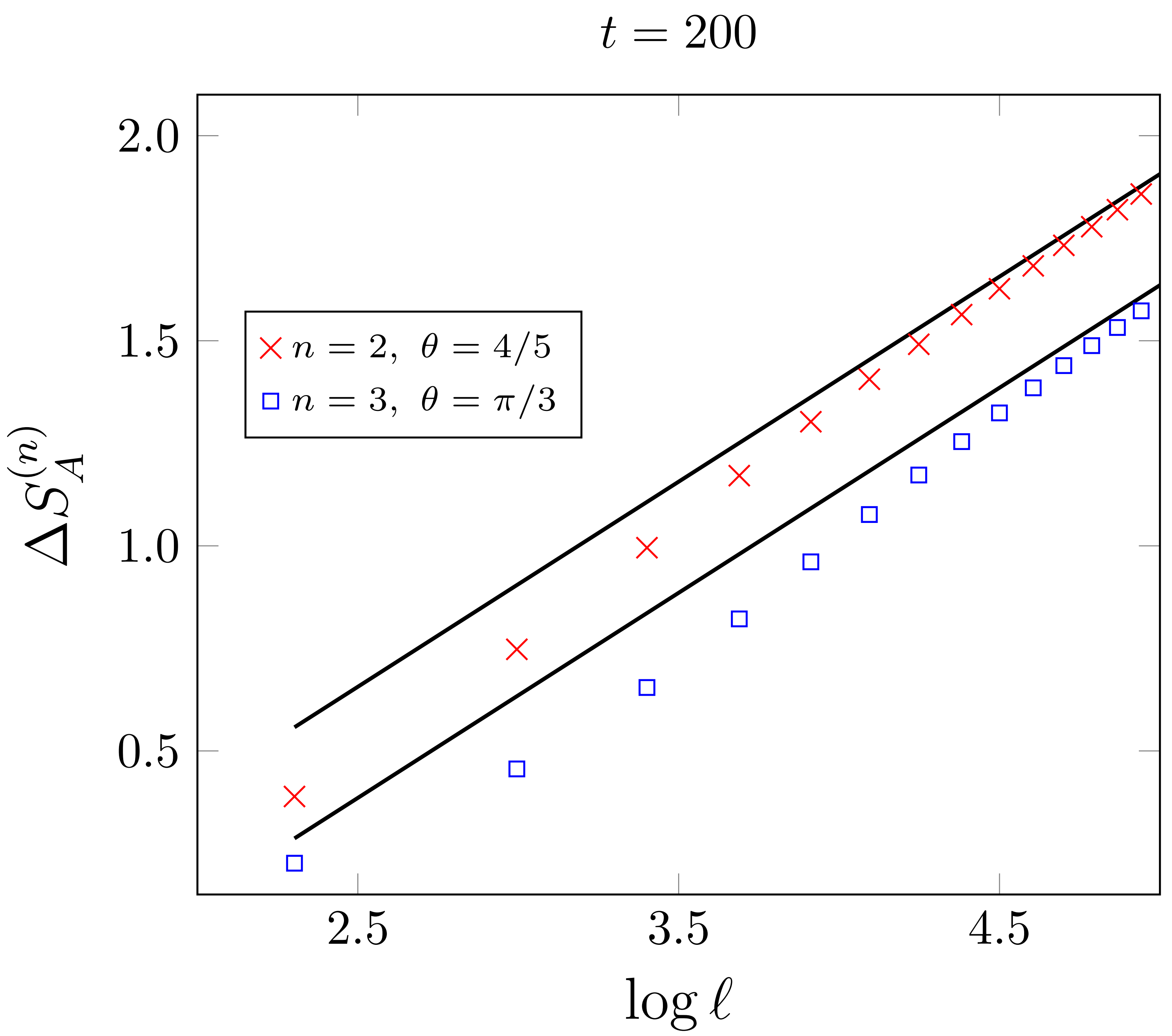
A lot of algebra!

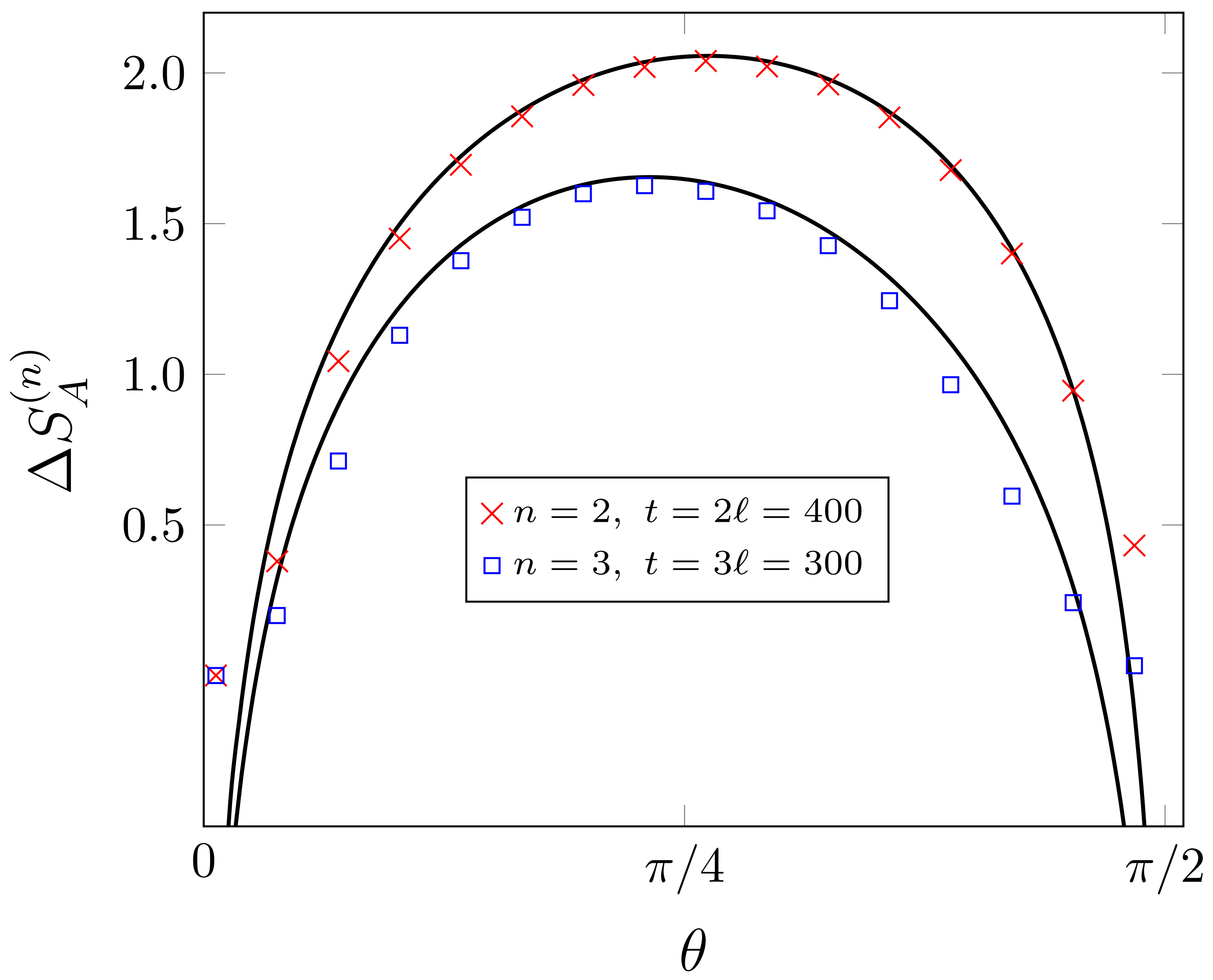


Non-symmetry resolution: the tilted Neel II

F. Ares, S.Murciano, E. Vernier, P. Calabrese

$$\Delta S_A^{(n)}(t\to\infty)=\frac{1}{2}\log\ell+\frac{1}{2}\log\frac{\pi n^{1/(n-1)}g_\infty^{(n)}(\theta)}{4}+O(\ell^{-1})$$





To conclude:

We introduced a new experimentally accessible quantity, the entanglement asymmetry, which measure the symmetry breaking and allowed us to discover a new unexpected phenomenon, the quantum Mpemba effect

Some current research directions:

- Entanglement asymmetry in ground states
- Breaking of non-abelian symmetries
- Random unitary circuits and integrable models
- Disordered models and MBL