Quantum Transport and Universality
From Topological Materials to Quantum Hydrodynamics

Summer School in Rome, 16–20/09/2019
Università degli Studi Roma Tre, Dipartimento di Matematica e Fisica
Largo San Leonardo Murialdo 1, 00146 Roma (IT)
Room M3

Programme

		Monday	Tuesday	Wednesday	Thursday	Friday
8:30	9:00	*Registration*				
9:00	9:30	Doyon	Prosen	Graf	Graf	Doyon
9:30	10:00					
10:00	10:30					
10:30	11:00	*Coffee break*	*Coffee break*	*Coffee break*	*Coffee break*	*Coffee break*
11:00	11:30	Prosen	Doyon	Doyon	Graf	Porta
11:30	12:00					
12:00	12:30					
12:30	14:30	*Lunch break*	*Lunch break*	*Lunch break*	*Lunch break*	*Lunch break*
14:30	15:00	Prosen		Porta	Porta	Graf
15:00	15:30		Porta			
15:30	16:00					
16:00	16:30	*Coffee break*		*Coffee break*	*Coffee break*	
16:30	17:00	Contributed talks (see below)		Prosen	Contributed talks (see below)	
17:00	17:30					
17:30	18:00					

		Monday		Thursday
16:30	16:50	Donvil		Okumura
16:50	16:55	*Short break*		*Short break*
16:55	17:15	Suntajs		Marcelli
17:15	17:20	*Short break*		*Short break*
17:20	17:40	Moosavi		Ulčakar
17:40	17:45	*Short break*		
17:45	18:05	Mestyán		

Abstracts

School courses

Benjamin Doyon (King's College London)
Generalised hydrodynamics

Hydrodynamics is a powerful theory for describing the emergent out-of-equilibrium dynamics at large scales in space and time. The Navier-Stokes equation is an example of a hydrodynamic equation, with complex and beautiful phenomenology. But it is just an example, and the basic precepts of hydrodynamics can be applied to a much wider set of many-body systems, quantum and classical. An interesting family is that of integrable models: they admit a large number of conservation laws, and this strongly affects their thermalisation and out-of-equilibrium properties. Integrability is in a sense the most fundamental attribute of quasi-one-dimensional physics, and remains relevant up to measurable times in many experimentally realised, non-integrable models. The theory adapting hydrodynamics to integrability, dubbed "generalised hydrodynamics" (GHD), has been developed in recent years. It turns out to be extremely general, applicable to quantum and classical chains, field theories and gases.

These lectures will cover the fundamental concepts of this theory. I will start with an overview of what, fundamentally, hydrodynamics is about. I will then describe the most important aspects of integrability, from which I will derive the basic GHD equations. If time permits, I will explain one or two applications or more advanced concepts, such as the Riemann problem, correlation functions, large deviations, and diffusion. In order to have clear examples, I will focus on two simple models: the quantum Lieb-Liniger model, and the classical hard rod gas. The GHD of the Lieb-Liniger model has even been verified experimentally!

Only very basic knowledge of hydrodynamics and integrability is assumed, and I will keep everything as non-technical as possible.

Gian Michele Graf (ETH Zürich)
An introduction and an overview on topological matter

Topological insulators are materials which are conducting at their edges, though not in the bulk. Their essential physical properties take the form of an index, often associated to the Hamiltonian. "Topological" simply refers to the fact that indices remain invariant under continuous changes. An early type of such insulators is provided by the Quantum Hall effect. It will be described in its (basic) integer variant, both in terms of physical content as well as a source of various mathematical developments. Concepts such as vector bundles and Chern numbers (for periodic systems), as well as more general indices (including disordered systems) will be discussed in terms of their bulk and edge embodiments. The lecture will then ramify in various (possible) directions: First, by way of inclusion of symmetries, which will be presented in the case of time-reversal invariant topological insulators in dimension two, soon to be generalized to other symmetries and dimensions (Kitaev table and its derivation). Second, by way of disorder, such as in the case of chiral symmetry in dimension 1, where topology is reflected in the Lyapunov spectrum. Third, by extension to time-dependent (Floquet) systems, where a space-time duality complements the bulk-edge correspondence. And fourth, by extension to systems outside of quantum mechanics, such as a hydrodynamic model (shallow water waves), where the bulk-edge correspondence reaches its limits.

Marcello Porta (Eberhard-Karls Universität Tübingen)
Universal edge transport in interacting 2d topological insulators

One of the distinctive features of topological insulators is the presence of robust gapless edge currents flowing on their boundaries. The stability of these edge modes against external perturbations has a deep topological interpretation: the bulk-edge duality relates the topological classification of the system with no boundaries to the number of edge modes appearing once a boundary is introduced. Much of our rigorous understanding of topological edge modes is based on the single-particle approximation, in which many-body interactions are neglected. The effect of interactions is usually studied via integrable effective theories, such as the chiral Luttinger model for the edge states of 2d quantum Hall systems.

In this course I will discuss renormalization group methods for interacting condensed matter systems, that allow in particular to make rigorous sense of the chiral Luttinger liquid description for the edge currents of a class of 2d topological insulators. I will show how lattice and emergent conservation laws can be used to constrain the form of the imaginary-time response functions, and to prove universality of edge transport. Finally, I will prove a reconstruction theorem, that allows to recover the real-time transport coefficients starting from their imaginary time counterparts.

Based on joint works with V. Mastropietro and G. Antinucci.

Tomaž Prosen (University of Ljubljana)
Exactly solvable models of transport and many-body quantum chaos

My lectures will be divided in two parts. In both I will focus on exact solutions of dynamics of interacting classical and quantum 1+1 dimensional lattice systems.

In the first part, I will discuss a class of integrable classical reversible cellular automata which model interacting particles and for which exact solutions of time-dependent and non-equilibrium steady states exist in terms of explicit matrix products *ansätze*. The model, known also as “Rule 54”, exhibits generic physics with coexistence of ballistic and diffusive transport and is amenable to a rigorous analytic treatment.

In the second part, I will turn to the quantum cellular automata in 1+1 dimensions, which can also be interpreted as Floquet (periodically driven) spin chains. In particular, I will focus on a class of systems which exhibit a property of dual unitarity, meaning that upon the flip of space and time axes the propagator (transfer matrix) remains unitary. I will show that this class of systems hosts a rich variety of dynamical behaviours, ranging from the integrable one, where the dynamics is dominated by conserved quantities, to the maximally chaotic (ergodic and mixing) one, where all dynamical correlations decay exponentially while the Floquet spectrum exhibits fluctuation properties which are given by random matrix theory at all scales. The paradigmatic example treated in detail in this part of the lectures will be the self-dual kicked Ising model.

Contributed talks

Brecht Donvil (University of Helsinki)
Hybrid master equations for calorimetric measurements

I present a mathematical study of an experimental setup proposed by [J. P. Pekola *et al.*, *New J. Phys.* **15** (2013), 115006] for calorimetric measurements of thermodynamic indicators in an open quantum system. The goal of the experiment is to detect energy quanta exchanged between a driven qubit and a thermal bath by measuring changes in the temperature of the bath.

We model the setup as a driven qubit interacting with a large, but finite, thermal bath of electrons, the calorimeter. Under weak-coupling assumptions it is possible to express the evolution of the qubit-calorimeter system as a hybrid master equation for the state of the qubit and the temperature of the calorimeter.

In the asymptotic regime of long driving, the hybrid master equation can be reduced to a Fokker-Planck equation for the temperature. From the aforementioned equations, we can obtain numerical information about the temperature behaviour in terms of the intensity of the drive and qubit-calorimeter coupling.

Giovanna Marcelli (Eberhard-Karls Universität Tübingen)

Non-equilibrium almost-stationary states and linear response for gapped non-interacting quantum systems

We prove the validity of linear response theory at zero temperature for gapped infinitely extended quantum systems within the one-particle approximation. A gapped Hamiltonian, which is not necessarily periodic, is perturbed by switching on adiabatically in time a constant electric field of intensity $\varepsilon \ll 1$, modeled by a linear potential. It is shown that the initial Fermi projection evolves adiabatically into a *non-equilibrium almost-stationary state* (NEASS), once the perturbation, which closes immediately the spectral gap of the unperturbed Hamiltonian, is turned on. We prove formulas for linear and higher order response coefficients, including the (spin) conductivity tensor for the quantum (spin) Hall effect (in the case of spin transport, we use a proper definition of the spin current operator [J. Shi *et al.*, *Phys. Rev. Lett.* **96** (2006), 076604]).

We follow the strategy implemented in [S. Teufel, *Commun. Math. Phys.*, Online First (2019)], but for both discrete and continuum models. Two new technical difficulties occur: to establish the trace class property and to deal with domain issues of some relevant unbounded operators (e.g. the domain of the perturbed Hamiltonian does depend on time). Finally, we provide a rigorous comparison between our approach, in which a uniform electric field is modeled by a linear (unbounded) potential, and the more traditional one where a time-dependent gauge transformation is performed (e.g. [J.-M. Bouclet *et al.*, *J. Funct. Anal.* **226** (2005), 301]).

Based on joint works with D. Monaco (Università degli Studi Roma Tre), G. Panati ("Sapienza" Università di Roma), and S. Teufel (Universität Tübingen).

Márton Mestyán (SISSA)

Molecular dynamics simulation of entanglement spreading in generalized hydrodynamics

The so-called flea gas is an elementary yet very powerful method that allows the simulation of the out-of-equilibrium dynamics after quantum quenches in integrable systems. Here we show that, after supplementing it with minimal information about the initial state correlations, the flea gas provides a versatile tool to simulate the dynamics of entanglement-related quantities. The method can be applied to any quantum integrable system and to a large class of initial states. Moreover, the efficiency of the method does not depend on the choice of the subsystem configuration. Here we implement the flea gas dynamics for the gapped anisotropic

Heisenberg XXZ chain, considering quenches from globally homogeneous and piecewise homogeneous initial states. We compute the time evolution of the entanglement entropy and the mutual information in these quenches, providing strong confirmation of recent analytical results obtained using the Generalized Hydrodynamics approach. The method also allows us to obtain the full-time dynamics of the mutual information after quenches from inhomogeneous settings, for which no analytical results are available.

Based on `arXiv:1905.03206`.

Per Moosavi (KTH Stockholm)

Anomalous transport in random conformal field theory

Conformal field theory (CFT) has recently been used to study closed (1+1)D quantum many-body systems out of equilibrium. However, standard CFT is known to exhibit only ballistic transport and no diffusion. In this talk, I will present exact analytical results for inhomogeneous CFT (in 2D Minkowski space) with a random position-dependent velocity that emerges as an effective description of quantum many-body systems with certain static random impurities. We refer to such inhomogeneous CFT as random CFT and show that it exhibits diffusion. Our methods are based on projective unitary representations of diffeomorphisms and a connection with wave propagation in random media, and our results elucidate how purely ballistic transport in standard CFT is supplemented by normal and anomalous diffusive contributions in random CFT.

Satoshi Okumura (Tohoku University Graduate School of Mathematics)

Magnetic Weyl Quantization and Semiclassical Limit

I will talk about magnetic Weyl quantization and semi classical limit.

Magnetic Weyl quantization was defined by [M. Mantoiu and R. Purice, *J. Math. Phys.* **45** (2004), 1394]. The magnetic Weyl quantization of Classical observable $f \in C^{\infty}(\Xi)$ is given as the operator valued integral :

$$\mathfrak{Op}^A[f] := \frac{1}{(2\pi)^d} \int_{\Xi} dY \, \mathscr{F}_\sigma[f](Y) \, W^A_\varepsilon(Y).$$

This Ξ is $\mathbb{R}^{2d}$ which equipps symplectic form and $\mathscr{F}_\sigma$ is symplectic Fourier Transform. $W^A_\varepsilon(Y)$ is a unitary operator constructed by the position operator $\hat{x}$ and the momentum operator $\hat{p}^A$ including a magnetic field A.

The Egorov-type theorem is known as a theorem that compares classical systems and quantum systems through Weyl quantization. Since I proved the case including the magnetic field of this theorem, I will introduce it.

Jan Suntajs (Jožef Stefan Institute)

Quantum chaos challenges many-body localization

Characterizing states of matter through the lens of their ergodic properties is a fascinating new direction of research. In the quantum realm, the many-body localization (MBL) was proposed to be the paradigmatic nonergodic phenomenon, which extends the concept of Anderson localization to interacting systems. At the same time, random matrix theory has established a powerful framework for characterizing the onset of quantum chaos and ergodicity (or the absence thereof) in quantum many-body systems. Here we study a paradigmatic class of models that are expected to exhibit MBL, i.e., disordered spin chains with Heisenberg-like interactions. Surprisingly, we observe that exact calculations show no evidence of approaching MBL while increasing disordered strength in the ergodic regime. Moreover, a scaling analysis suggests that quantum chaotic properties survive for any disorder strength in the thermodynamic limit. Our results are based on calculations of the spectral form factor, which provides a powerful measure for the emergence of many-body quantum chaos.

Based on joint work with J. Bonča, T. Prosen, and L. Vidmar, `arXiv:1905.06345`.

Lara Ulčakar (Jožef Stefan Institute)

Slow quenches in topological insulators

We investigate slow quenches in Chern insulators in ribbon geometry. We consider the Qi-Wu-Zhang model and slowly ramp the parameters (large time of the quench τ) from a non-topological (Chern number $= 0$) to a topological regime (Chern number $\neq 0$). In contrast to the Haldane model considered in [L. Privitera and G. E. Santoro, *Phys. Rev. B* **93** (2016), 241406(R)] earlier, the in-gap state degeneracy point is pinned to an inversion symmetric momentum, which changes the behaviour drastically. The density of excitations in the in-gap states scales as $\tau^{-1/2}$ and the Kibble-Zurek mechanism applies. The calculated Hall conductance oscillates around that of the final Hamiltonian, with deviations of the average value that drop as the ramp becomes slow. Despite the different scaling of the bulk and the in-gap excitations the deviations of the Hall conductance from the ground state value scale as τ^{-1}.