Model for calorimetric measurements in an open quantum system

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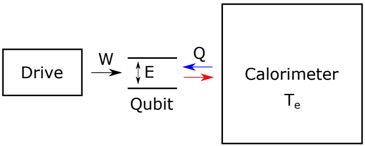
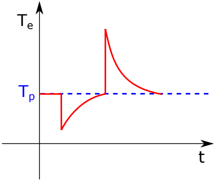
Introduction

Motivation: proposal for experimental setup to perform a calorimeteric measurement of work on a driven qubit[1]





Problem: How to model evolution of small quantum system while continuously measuring a macroscopic property of the bath
Two cases:

- Weak coupling, based on the Lindblad equation [2]
- Strong coupling: path integral formalism and filtering

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[1] Pekola et al (New. J. Phys. 2013)

[2] A. Kupiainen et al PRE (2016), B. D. et al, PRA (2018) and B. D. et al, PRA (2019)

Qubit-Calorimeter

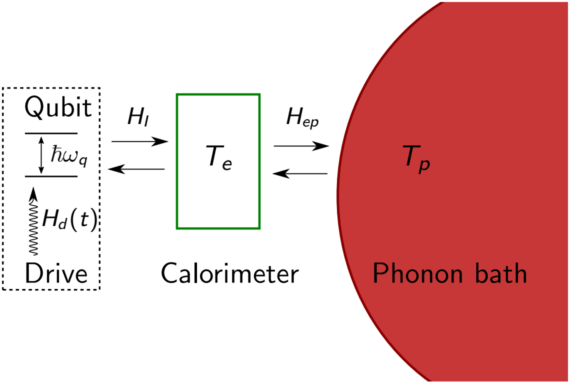

Figure: Experimental setup

$$H_I = g \sum_{k,l} (\sigma_+ + \sigma_-) a_k^\dagger a_l$$

Time scales

- $\tau_{ee} = O(10^0)$ns: Landau quasi-particle relaxation rate to Fermi–Dirac equilibrium in a metallic wire
- $\tau_{ep} = O(10^4)$: Electron-phonon interactions
- $\tau_R = 2 - 5 \times O(10^5)$ns: Transmon qubit relaxation times [3]
- $\tau_{eq} \approx g^{-2}$: Fermi's golden rule estimate of characteristic qubit-calorimeter time scale

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Time scale separations

$$\tau_{ee} \ll \tau_{eq} \ll \tau_{ep} \ll \tau_R$$

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Time scale separations

$$\tau_{ee} \lll \tau_{eq} \lll \tau_{ep} \lll \tau_R$$

We assume that the qubit is interacting with the calorimeter at a well-defined temperature

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Weak coupling: Stochastic Jump Process

The evolution of a closed quantum system is described by the Schrödinger equation

$$\psi(t + \mathrm{d}t) - \psi(t) = \mathrm{d}\psi(t) = -iH\psi \, \mathrm{d}t$$

For an open system the Schrödinger equation is modified

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- **Dissipative terms** are added to the Hamiltonian

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$$H\psi(t) \, \mathrm{d}t \to G(\psi(t)) \, \mathrm{d}t$$

- Addition of **jump terms**

$$\begin{aligned} (|+\rangle - \psi(t)) \, \mathrm{d}N_\uparrow, &\quad \mathrm{d}N_\uparrow = 0, 1, \\ (|-\rangle - \psi(t)) \, \mathrm{d}N_\downarrow, &\quad \mathrm{d}N_\downarrow = 0, 1, \end{aligned}$$

$$\mathtt{E}_\psi(\mathrm{d}N_{\uparrow/\downarrow}) = \gamma_{\uparrow/\downarrow} \|\sigma_\pm \psi\|^2 \, \mathrm{d}t$$

Temperature Process

Using the Sommerfeld expansion we find the dependence of the temperature on the change in internal energy E of the calorimeter

$$\mathrm{d}\,T_e^2 = \frac{\mathrm{d}E}{N\gamma}.$$

The qubit-electron interaction gives $\mathrm{d}E = \hbar\omega(\mathrm{d}N_\downarrow - \mathrm{d}N_\uparrow)$

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The final result is the set of coupled equations

$$\boxed{\begin{cases} \mathrm{d}\psi(t) = -iG(\psi(t))\,\mathrm{d}t \\ \qquad\qquad + \Big(|+\rangle - \psi(t)\Big)\,\mathrm{d}N_\uparrow + \Big(|-\rangle - \psi(t)\Big)\,\mathrm{d}N_\downarrow \\ \mathrm{d}\, T_e = \frac{\hbar\omega}{N\gamma}(\mathrm{d}N_\downarrow - \mathrm{d}N_\uparrow) \end{cases}}$$

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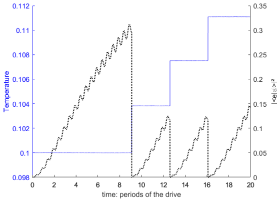

Add the substrate

A Fröhlich electron-phonon interaction leads to extra terms

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[4] Kaganov, Lifshitz and Tanatarov (1956)

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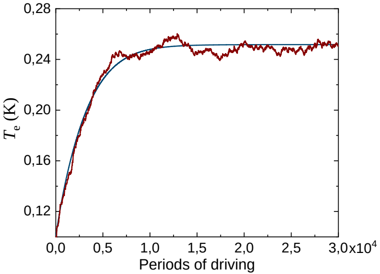

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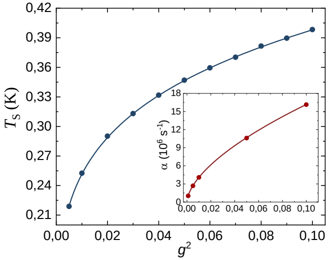
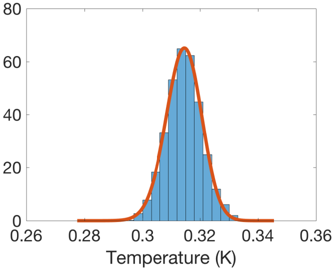
Effective temperature process

Performing multi-timescale analysis eliminates the jumps process and adds a correction to the drift and noise

$$\mathrm{d}T_e^2 = \frac{1}{\gamma}\left(\Sigma V(T_p^5 - T_e^5)\,\mathrm{d}t + J(T_e^2)\,\mathrm{d}t + \sqrt{10\Sigma V k_B}\,T_p^3\,\mathrm{d}w_t + \sqrt{S(T_e^2)}\,\mathrm{d}w_t\right).$$

$$J(T_e^2) = \text{Average heat dissipated by the qubit in a thermal state } T_e + O(\epsilon)$$





Strong Coupling: Central fermion (Work in Progress)

Issues with the strong coupling spin-fermion models:

- Quadratic coupling
- Performing spin path integrals requires "Non Interacting Blip Approximation" (NIBA)[6] or other approximations

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To avoid making non-trivial approximations, we consider the "central-fermion"-model

$$H = \underbrace{\omega_0 c^\dagger c}_{\text{central fermion}} + \sum_k g_k (b_k^\dagger c + c^\dagger b_k) + \sum_k \omega_k b_k^\dagger b_k$$

This Hamiltonian can be used to model a quantum dot

[6] Leggett et al., Rev. Mod. Phys. (1987)

Central fermion

Our goal is to find a set of equations which describes the evolution of the density matrix of the central fermion $\rho(t)$ and the energy of the bath $E(t)$

$$\begin{cases} \mathrm{d}\rho(t) = \mathcal{L}_t \rho(t) \\ \mathrm{d}E(t) = \mathrm{tr}(\mathcal{E}(t)\rho(t)) \end{cases}$$

We calculate both operators separately

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- $\mathcal{L}_t$ we find by exactly integrating the bath and qubit dynamics

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- $\mathcal{E}(t)$ is obtained from performing the partial trace $\mathrm{tr}_B(\sum_k \omega_k b_k^\dagger b_k \rho_T(t))$

Fermionic path integral

The dynamics of the full central fermion-bath system can be represented in terms of a path integral

$$\rho_{TOT}(t) = \int d[x', X', x, X]\Phi_{TOT}(x', X'|x, X, t)\rho_0(x, X)$$

$$\Phi(x'|x, t) = \int \mathcal{D}[x_t, X_t]e^{iS_T[x_t, X_t]}$$

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For **linear system-bath coupling**, similar to the Caldeira-Leggett model, the bath fields X_t can be integrated over

$$\rho(t) = \int d(x, x) \Phi(x'|x, t) \rho_0(x)$$

with

$$\Phi(x'|x, t) = \int \mathcal{D}[x_t] e^{iS[x_t]}$$

The action $S[x_t]$ is now time non-local

Central fermion: dynamics

Solving the qubit path-integral gives an expression for the propagator

$$\Phi(x'|x,t) = \frac{1}{N(t)} e^{x' K(t) x}$$

Differentiating the propagator leads to a master equation for the qubit state [7]

$$\begin{aligned}\dot{\rho}(t) =& \mathcal{L}_t \rho(t) \\ =& \Omega[c^\dagger c, \rho(t)] + f(t)(c^\dagger c \rho(t) + \rho(t) c^\dagger c) \\ & + g(t) c \rho(t) c^\dagger + h(t) c^\dagger \rho(t) c + k(t)\rho(t)\end{aligned}$$

[7] Tu and Zang, PRB, 78 (2008)

Energy of the electron bath

The energy of the electron bath is given by the operator $\sum_k \omega_k b_k^\dagger b_k$. Using similar path integral techniques as before, we find $A(t)$ such that

$$E(t) = \mathrm{tr}(\sum_k \omega_k b_k^\dagger b_k \rho_T(t)) = \mathrm{tr}_S(A(t)\rho(t))$$

Thus, we have the set of equations

$$\begin{cases} \partial_t \rho(t) = \bar{\mathcal{L}}_t \rho_t \\ \partial_t E(t) = \mathrm{tr}((\partial_t + \mathcal{L}_t^\dagger) A(t) \rho(t)) \end{cases}$$

Continuous measurement of the bath: filtering

Consider the energy operator $A(t)$ continuously being measured. The measurement is imperfect and distributed as

$$P(a,t) = \sqrt{8k}\exp(-4k(\langle A(t)\rangle - a)^2)$$

where k is the measurement rate.

[8] Jacobs and Steck, Cont. Phys., 47 (2006)

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The continuous measurement induces a back action on the fermion [8]

$$\begin{aligned} d\rho(t) =& \bar{\mathcal{L}}_t \rho(t) dt \\ =& \mathcal{L}\rho(t)dt - k[A(t),[A(t),\rho(t)]dt \\ & + (2k)^{1/2}(A(t)\rho(t) + \rho(t)A(t) - 2\langle A(t)\rangle\rho(t))dw_t \end{aligned}$$

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Our final result is

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Summary

- In case of the weak coupling, we modelled the qubit-calorimeter system as two coupled jump processes
- We studied the setup for physically relevant parameters
- For strong coupling, we used path integral methods to obtain a joint evolution for the energy of the bath
- Using filtering methods, we introduced the continuous energy measurement of the bath

Fermionic coherent states

A bosonic coherent state is defined as

$$|\phi\rangle = \exp(\phi a^\dagger)|0\rangle \quad \text{such that} \quad a|\phi\rangle = \phi|\phi\rangle$$

for a complex number ϕ

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In this case ψ is not a complex number, but a Grassmann number.
Grassmann numbers ψ, χ have the properties

- $\psi\chi = -\chi\psi$ ($cb = -bc$)
- $\psi^2 = 0$ ($c^2 = 0$)

Central fermion: dynamics

We want to obtain a differential equation for $\rho(t) = \mathrm{tr}_B(\rho_T(t))$. Integrating out the bath gives a path integral representation for

$$\rho(t) = \int d(\psi, \chi, \phi, \xi)\Phi(\psi, \chi, \phi, \xi, t)|\psi\rangle\langle\chi| \langle\phi|\rho|\xi\rangle$$

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with action

$$S[\psi^+, \psi^-] = \int_{t_i}^{t_f} dt\ \underbrace{\bar{\psi}^+(t)(i\partial_t - \omega_0)\psi^+(t) - \bar{\psi}^-(t)(i\partial_t - \omega_0)\psi^-(t)}_{\text{free fermion dynamics}}$$

$$+ i\int_{t_i}^{t_f} dtds\ \underbrace{\begin{pmatrix}\bar{\psi}^+(t)\\ \bar{\psi}^-(t)\end{pmatrix}^T G(t-s)\begin{pmatrix}\psi^+(s)\\ \psi^-(s)\end{pmatrix}}_{\text{interaction with the bath}}$$